Real People 

People as interacting “atoms”

- states: binary, multistate, discrete/continuous, multidimensional
- connections: random, structured, heterogeneous, time dependent
- dynamics: consensus, contrarianism, (ir)reversible, exogeny, latent variables

A model should be as simple as possible, but no simpler — A. Einstein
Some General Perspectives

Strengths: *application of sophisticated theoretical tools from condensed-matter physics to agent-based models for discovery and elucidation of collective behaviors*

Critiques: *idealized models; sometimes tenuous connections to data or phenomenology*

Some Current Applications:

- cellphone data mining to infer travel patterns & location
- social data mining to predict trends
- optimize social networks to encourage specific outcomes
Outline:

• (binary) voter models  
  consensus

• multistate voter models  
  non-consensus/consensus

• bounded compromise models  
  fragmentation/consensus

• innovation adoption models  
  role of reinforcement
Voter Model

Clifford & Sudbury (1973)
Holley & Liggett (1975)

0. Binary voter variable at each site i
1. Pick a random voter
2. Assume state of randomly-selected neighbor
   \textit{individual has no self-confidence & adopts neighbor’s state}
Voter Model

Example update:

0. Binary voter variable at each site $i$
1. Pick a random voter
2. Assume state of randomly-selected neighbor

*individual has no self-confidence & adopts neighbor's state*
0. Binary voter variable at each site $i$

1. Pick a random voter

2. Assume state of randomly-selected neighbor

individual has no self-confidence & adopts neighbor’s state
0. Binary voter variable at each site i
1. Pick a random voter
2. Assume state of randomly-selected neighbor *individual has no self-confidence & adopts neighbor’s state*
3. Repeat 1 & 2 until consensus *necessarily* occurs in a finite system

**Voter Model**

Example update:

- **3/4**
- **1/4**

Clifford & Sudbury (1973)
Holley & Liggett (1975)
## Lattice Voter Model: Consensus Time

<table>
<thead>
<tr>
<th>dimension</th>
<th>consensus time</th>
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<tbody>
<tr>
<td>1</td>
<td>$N^2$</td>
</tr>
<tr>
<td>2</td>
<td>$N \ln N$</td>
</tr>
<tr>
<td>$&gt;2$</td>
<td>$N$</td>
</tr>
</tbody>
</table>
Voter Model on Complex Networks

magnetization *conserved* on regular networks

magnetization *not conserved* on complex networks

information “flow” from high degree to low degree

what is conserved: degree-weighted magnetization

Suchecki, Eguiluz & San Miguel (2005)

Sood & SR (2005)
Consensus Time for Complex Networks

with \( n_k \sim k^{-\nu} \)

Sood & SR (2005)

\[
T_N \sim \begin{cases} 
N & \nu > 3 \\
N / \ln N & \nu = 3 \\
N^2(\nu-2)/(\nu-1) & 2 < \nu < 3 \\
(\ln N)^2 & \nu = 2 \\
\mathcal{O}(1) & \nu < 2 
\end{cases}
\]

fast consensus due to hubs
Non-Consensus & Social Fragmentation

If people are reasonable, why is consensus unachievable?

Possibilities:
- insufficient communication
- individual diversity
- stubbornness
- ..... 

Models for Social Fragmentation:

3 voting states (left.center/right): left/right do not speak

Bounded confidence: compromise when close
Three Voting States:

- leftist
0
+ centrist
rightist

voter model interactions

Three Voting States:

- leftist
0
+ centrist
rightist


Voter model interactions

No interaction
The Phase Diagram
(for well-mixed population)

centrist consensus

leftist consensus

extremist stasis

rightist consensus

all centrist

all leftist

all rightist
Bounded Compromise Model

Deffuant, Neau, Amblard & Weisbuch (2000)
Hegselmann & Krause (2002)

\[ \frac{x_1 + x_2}{2} \]

If \(|x_2 - x_1| < 1\) compromise

If \(|x_2 - x_1| > 1\) no interaction
The Opinion Distribution

\[ P(x,t) = \text{probability that agent has opinion } x \text{ at time } t \]

Basic parameter: \( \Delta \) diversity (initial opinion range)

\( \Delta < 1: \) consensus \quad \Delta > 1: \) fragmentation

Evolution of the probability distribution:

\[
\frac{\partial P(x,t)}{\partial t} = \int \int dx_1 dx_2 \, P(x_1,t) P(x_2,t) \times \left[ \delta \left( x - \frac{1}{2} (x_1 + x_2) \right) - \delta (x - x_1) \right]
\]

\(|x_1 - x_2| < 1 \]

gain by averaging opinions \quad loss by interaction
Early time evolution (for $\Delta=4.2$)
integrate master equation rather than simulate!
Early time evolution (for $\Delta=4.2$)
integrate master equation rather than simulate!
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integrate master equation rather than simulate!
Early time evolution (for $\Delta=4.2$)
integrate master equation rather than simulate!
Fragmentation Sequence

final opinion

Δ (diversity)
Fragmentation Sequence

final opinion vs. $\Delta$ (diversity)

- major
- minor
- central

Note: The text in the image is not clearly visible due to the quality of the image.
### A Possible Realization

**1993 Canadian Federal Election**

<table>
<thead>
<tr>
<th>year</th>
<th>PQ</th>
<th>NDP</th>
<th>L</th>
<th>PC</th>
<th>SC</th>
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<td>177</td>
<td>2</td>
<td>52</td>
<td></td>
</tr>
</tbody>
</table>
Modeling the Adoption of Innovations

Young 2009

Generic mechanisms:

• Social learning:
  adopt when perceived benefit exceeds a threshold

• Social influence:
  adopt when bandwagon has formed

• Contagion:
  adopt by contact with local neighbors (epidemics)
Traditional Models for Innovation Adoption


2 states: 0 - susceptible density $n_0$
           1 - innovator density $n_1$

innovation adoption processes:

$1 + 0 \rightarrow 1 + 1$  contagion

$0 \rightarrow 1$  social benefit
Contagion-Driven Innovation \((1 + 0 \rightarrow 1 + 1)\)

\[
n_1(t) = \left[(N-1)e^{-t} + 1\right]^{-1}
\]

\[
n_0(t) = 1 - n_1(t)
\]
Example: cars/person in the US

\[ 0.9(1 + 60 e^{-t/15})^{-1} \]
Example 2: Planting Hybrid Corn

\[ (1 + 500e^{-(yr-1929)/1.2})^{-1} \]
Extensions

Heterogeneity: different individual thresholds

Backsliding: become less enamored

Transience: abandon the innovation (fad)

Reinforcement: multiple prompts to adopt

Centola (2010)
Volovik & SR (2011)
Outlook

Theoretical physics tools provide insights on collective behavior of many agent-based social models.

Connecting models with data: Great potential (*big data*) but with ambiguities (*causation/correlation conundrum*)

Major challenge for the future: Exploit modeling/data to be predictive rather than descriptive.
• Provide an overview of what opinion dynamics modeling is, what it is based on, its main constructs (including how “opinion” is usually defined), and why it is useful.
• What are the strengths and weaknesses of an opinion dynamics approach?
• When is this approach most useful (what types of problems)?
• Share examples of how opinion dynamics has been applied successfully (examples related to health and empirical data would be especially helpful).
• How can empirical data can be incorporated (and why hasn’t it been incorporated yet in most of the examples out there)?
• How has opinion dynamics modeling been used to inform policy? How could it be used in the future?