MATHEMATICAL FRONTIERS

The National Academies of SCIENCES ENGINEERING MEDICINE

nas.edu/MathFrontiers

Board on Mathematical Sciences & Analytics

MATHEMATICAL FRONTIERS
2018 Monthly Webinar Series, 2-3pm ET

February 13: ***Recording posted***
Mathematics of the Electric Grid

March 13: ***Recording posted***
Probability for People and Places

April 10:
Social and Biological Networks

May 8:
Mathematics of Redistricting

June 12:
Number Theory: The Riemann Hypothesis

July 10: *Topology*

August 14:
Algorithms for Threat Detection

September 11:
Mathematical Analysis

October 9: *Combinatorics*

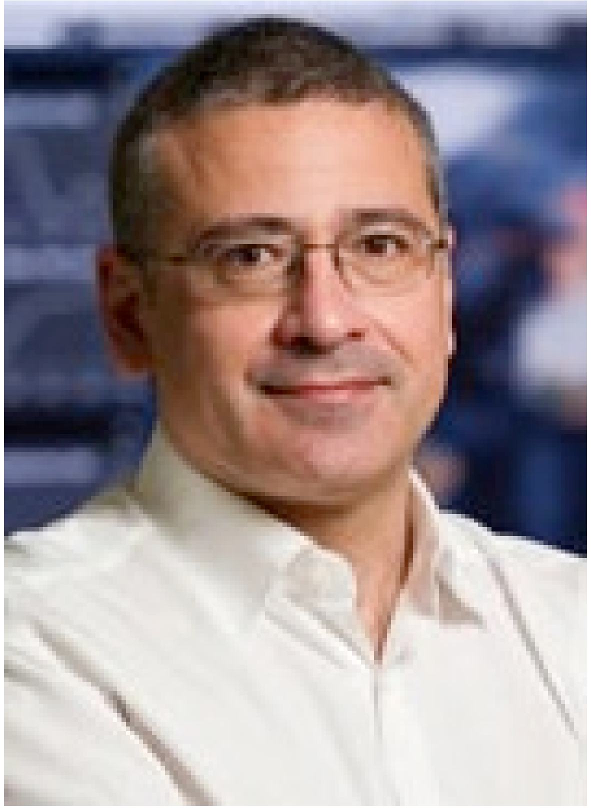
November 13:
Why Machine Learning Works

December 11:
Mathematics of Epidemics

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers

MATHEMATICAL FRONTIERS
Social and Biological Networks

Nina H. Fefferman,
University of Tennessee, Knoxville



Alessandro Vespignani,
Northeastern University

James H. Curry,
University of Colorado, Boulder

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers

MATHEMATICAL FRONTIERS
Social and Biological Networks

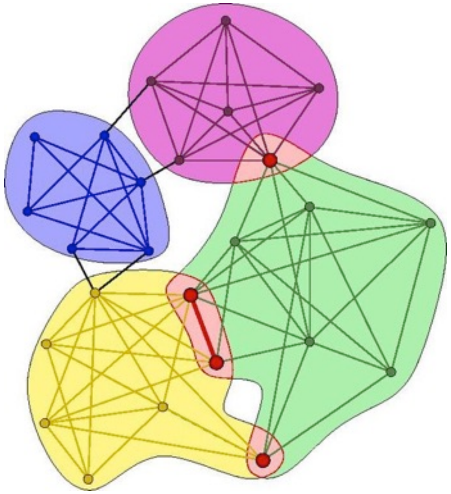
Nina H. Fefferman,
University of Tennessee, Knoxville

Associate Professor
in the departments of
Ecology and Evolutionary Biology
& Mathematics

The Evolution of Social Systems

Animal Social Networks Questions

- How do groups form?
- Who is important/in charge?
- Are there distinct communities?
- How does the social network affect ongoing biological processes?
  - Mating
  - Disease spread*
- How do groups change over time?
- How did group behaviors evolve?



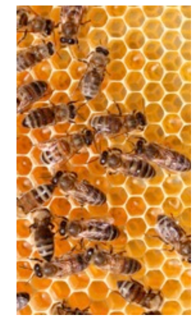
How Groups of Animals Organize

Solitary

Gregarious

Social

Eusocial



There are different costs and benefits to each strategy

Who Benefits and How?

- Benefits of Group Success
  - Diffusion of risk from predators
  - Increased foraging success
  - Better engineering
- Costs of Group Participation
  - Attract predators
  - Competition for food/mates
  - Disease transmission

From telegraph.co.uk

Biology Question, Math Answer

How do you Evolve these different strategies?

- Main idea of evolution:

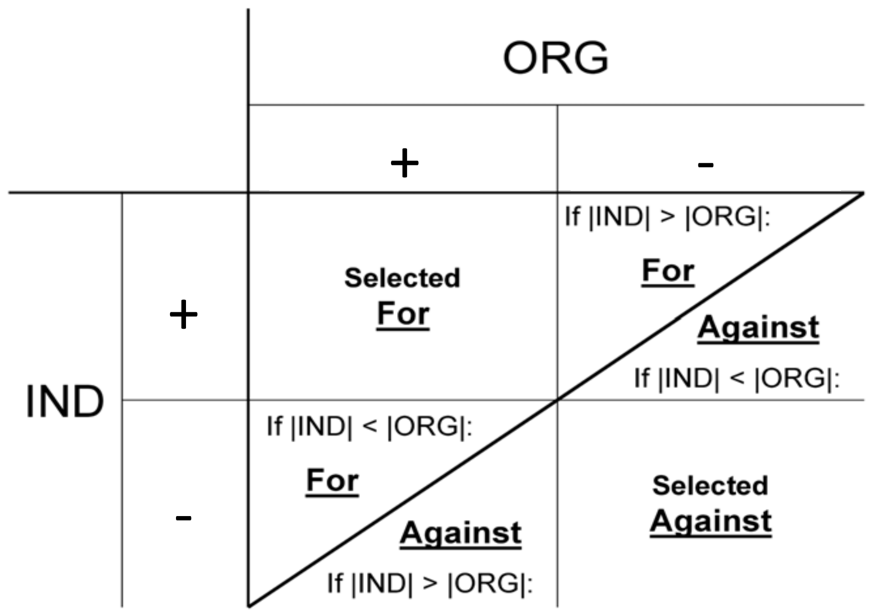
  Traits survive and spread if they improve survival and/or reproduction of individuals that carry them

  Social systems = many individuals, many roles

- How to study them?

But Genes are Carried by Individuals

- We need Multilevel Selection:



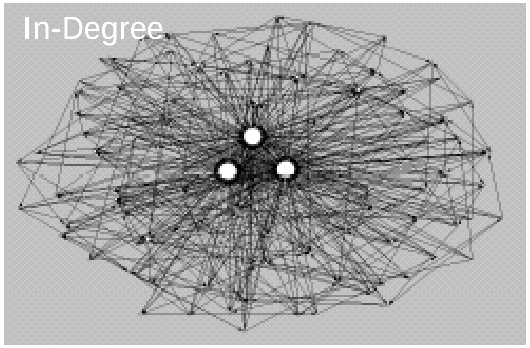
Reproduced from Hock, Ng and Fefferman, 2010

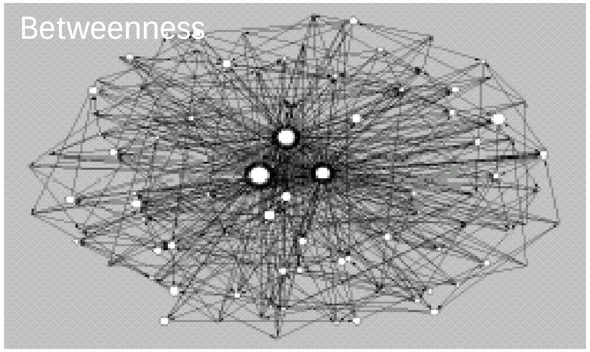
- For social systems, this means NETWORKS!

We build a mathematical abstraction and use it for computational experiments

Assumption: Individuals make genetically determined “selfish” social affiliation choices (with no regard for group-level effects)

We can compare networks that emerge from different beliefs about what is selfishly best

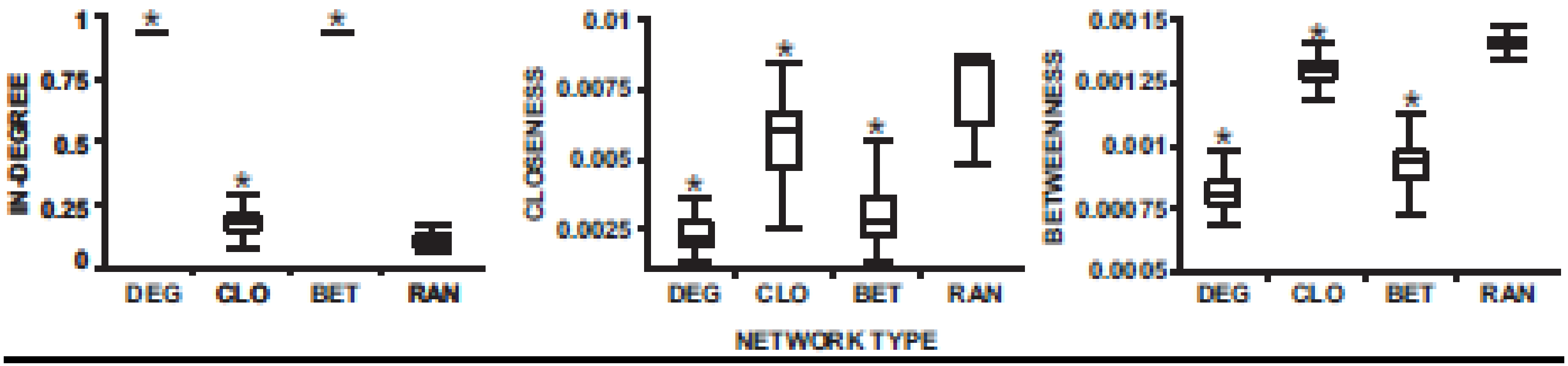




Reproduced from Fefferman and Ng, 2007a

Result: Different Individual Strategies Work to Accomplish Different Tasks

D ≈ B > C > R　　R > C > B > D　　R > C > B > D

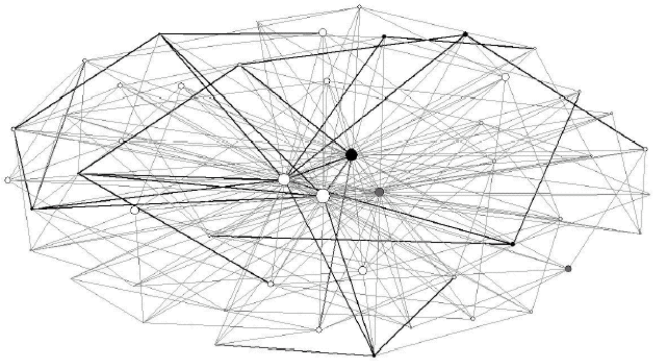


Discussed in Fefferman and Ng, 2007a and Hock and Fefferman 2011, and 2012

Already gives us some insight into evolutionary pressures on self-organizing social behaviors

Those are just the benefits, what about the costs?

- What happens if there is disease on these networks?



	3-Way Test	B-population Dynamic	B-population Static	C-population Dynamic	C-population Static	D-population Dynamic	D-population Static
B	Dynamic¶	B static > B dynamic†		<*	<*	>	>*
C				C static > C dynamic†		>*	>*
D	Static¶					D static > D dynamic†	
		Overall: Dynamic C>B>D; Static C>B>D					

Reproduced from Fefferman and Ng, 2007b

Now we can ask about costs and benefits!

Social Benefits:	**D-Preference** D ≈ B > C > R	**C-Preference** R > C > B > D	**D-Preference** R > C > B > D

Disease Risks:

Overall: Dynamic C>B>D; Static C>B>D

Reproduced from Fefferman and Ng, 2007b
and from Hock and Fefferman 2012

Sometimes more organizational efficiency also means more disease, but not always – it depends on what type of organizational task the population needs!

Only the beginning…

These are just scratching the surface of the questions we’ve already asked with networks and the evolution of social systems

- ✓ Populations with mixed social preferences
- ✓ Learned social preferences
- ✓ Friends vs. Family
- ✓ More about diseases and how to prevent them

More to come!

MATHEMATICAL FRONTIERS
Social and Biological Networks

Sternberg Family Distinguished Professor and Director of Physics, Bouve College of Health Sciences, College of Computer and Information Science

Contagion processes in social networks

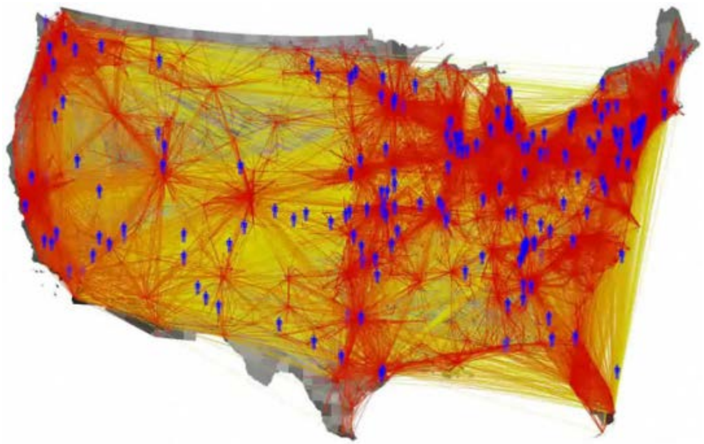
Alessandro Vespignani, Northeastern University @alexvespi

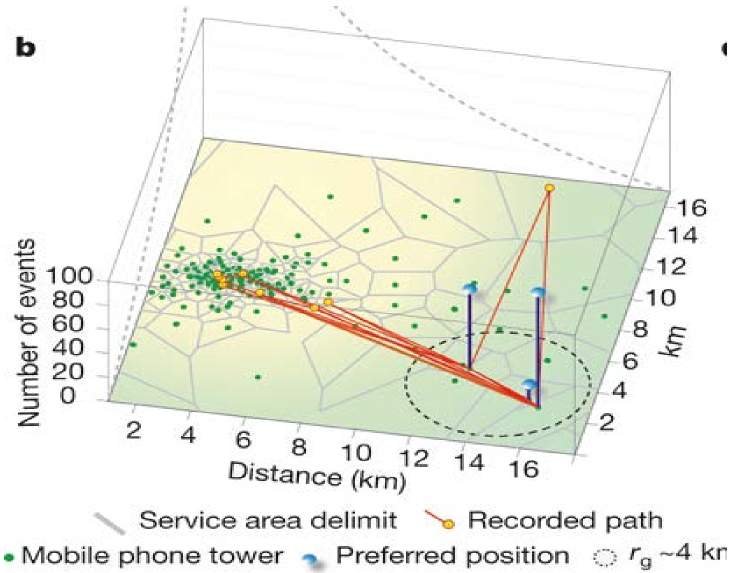
CENTER FOR INFERENCE & DYNAMICS OF INFECTIOUS DISEASES

Northeastern

MOBS LAB
LABORATORY FOR THE MODELING OF BIOLOGICAL AND SOCIO-TECHNICAL SYSTEMS

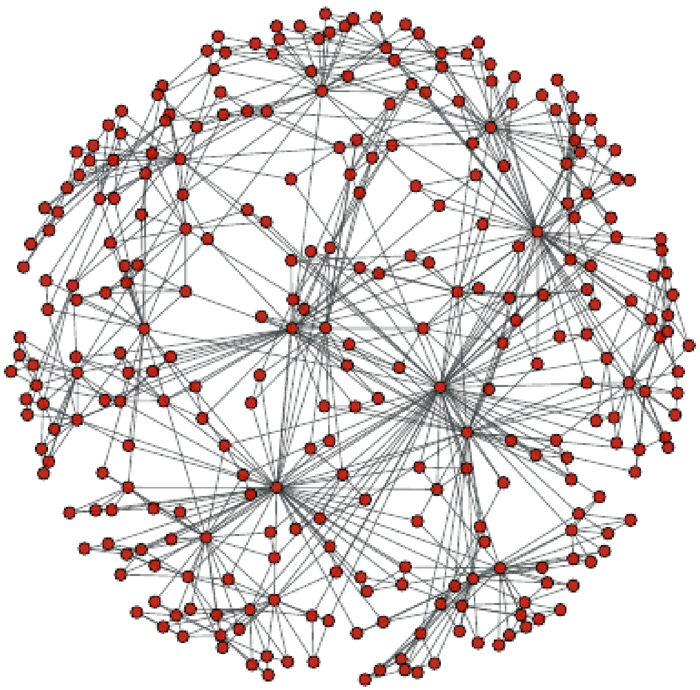
Contagion:
From geography to social space

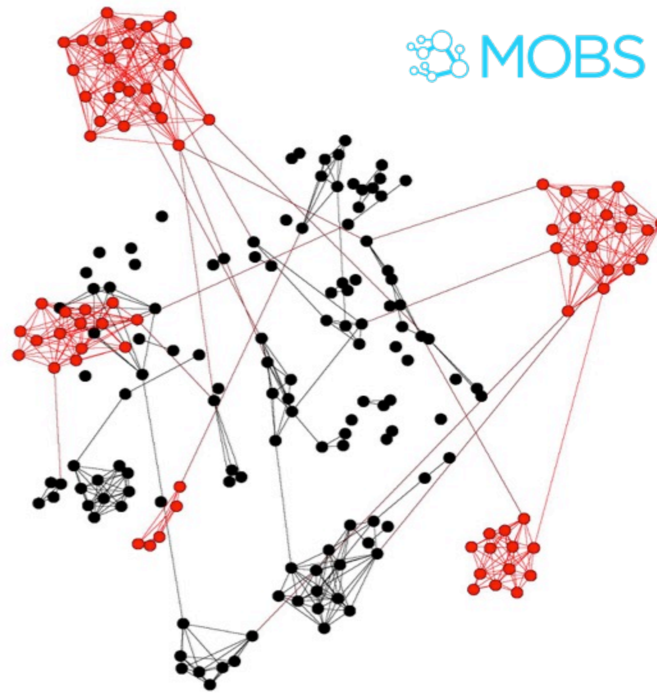




Graphical areas/census
Mobility

Contagion: From geography to social space



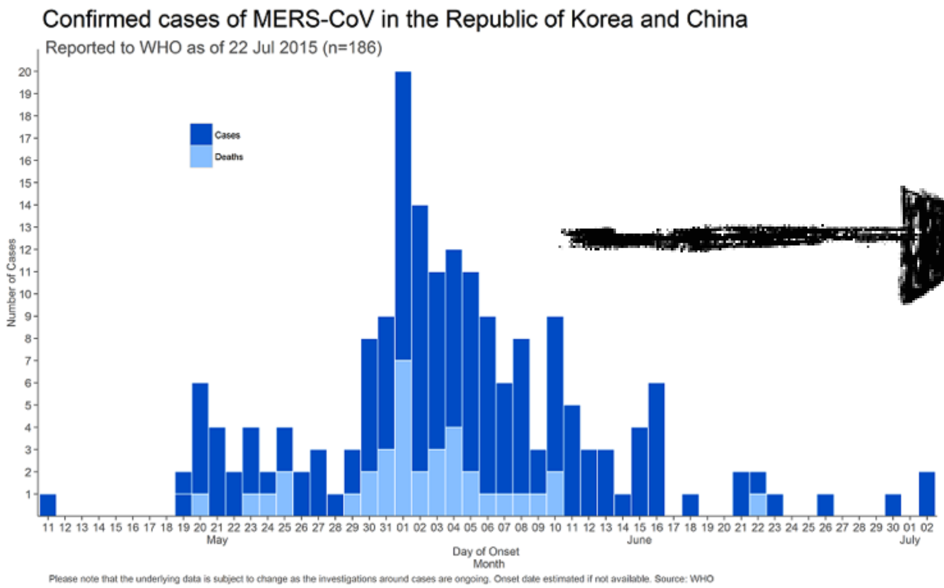


MOBS LAB

Structured communities in the abstract social space defined by knowledge and information

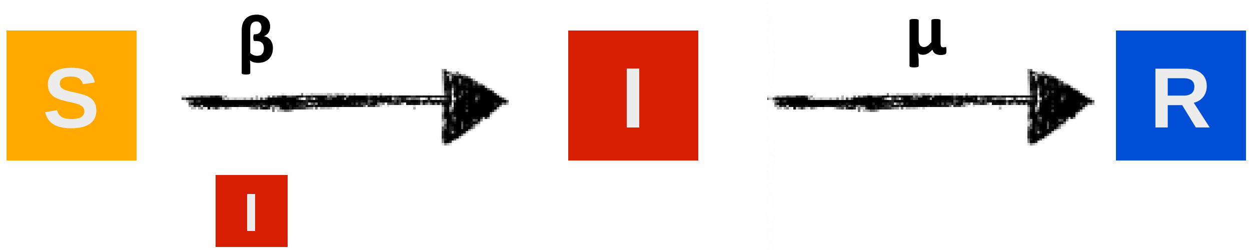
Epidemic/contagion modeling

“I simply wish that, in a matter which so closely concerns the wellbeing of the human race, no decision shall be made without all the knowledge which a little analysis and calculation can provide”
Daniel Bernoulli ~1760



Mathematics of contagion processes ~1766

- Based on the disease compartmental structure: Individuals are characterized by the disease stage: susceptible, infectious, recovered, exposed, etc.



- The mathematical description is based on the so called homogenous assumption

Reaction rate equations

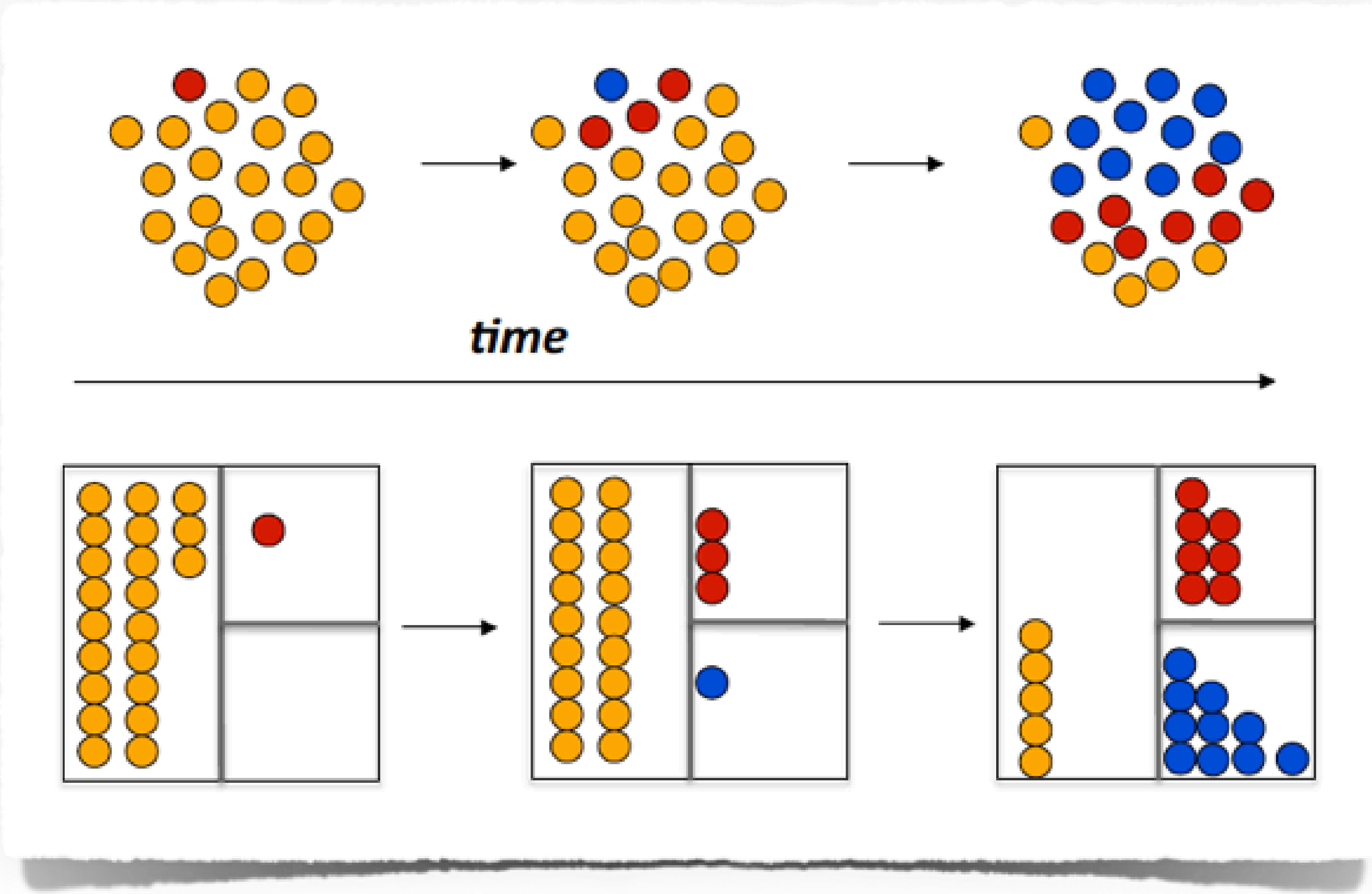
$$\partial_t S(t) = -\beta \frac{I(t)S(t)}{N}$$

$$\partial_t I(t) = \beta \frac{I(t)S(t)}{N} - \mu I(t)$$

$$\partial_t R(t) = \mu I(t)$$

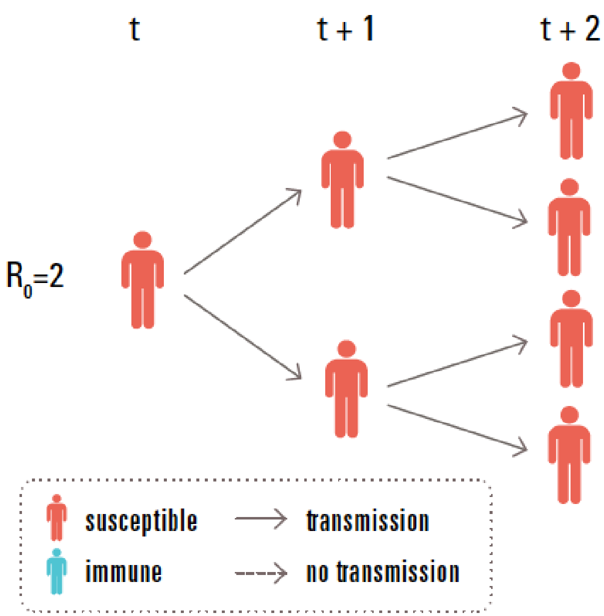
MOBS LAB

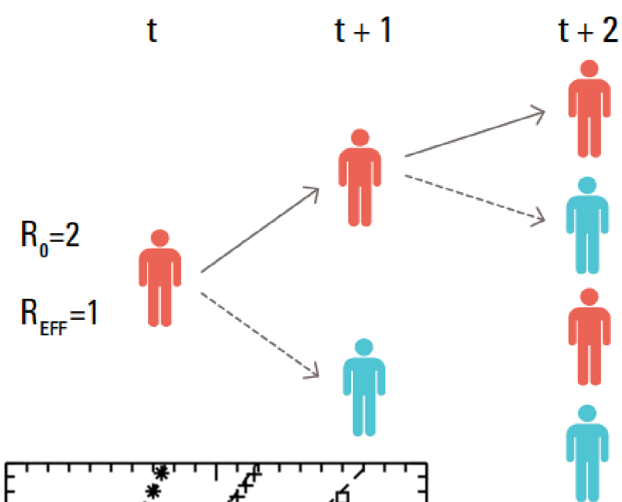
Indistinguishable + homogeneity

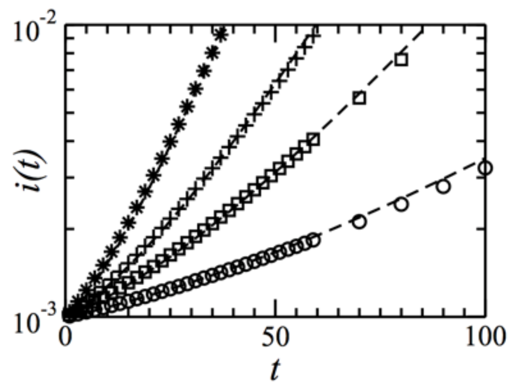


Basic reproductive number R_0

R_0 is the average number of individuals infected directly by an infected individuals during his infectious period in a fully susceptible population.



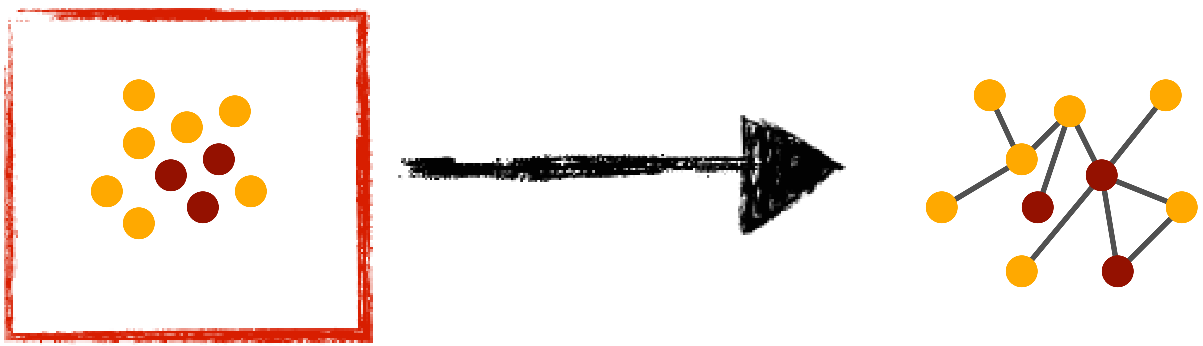




Exponential growth

MOBS LAB

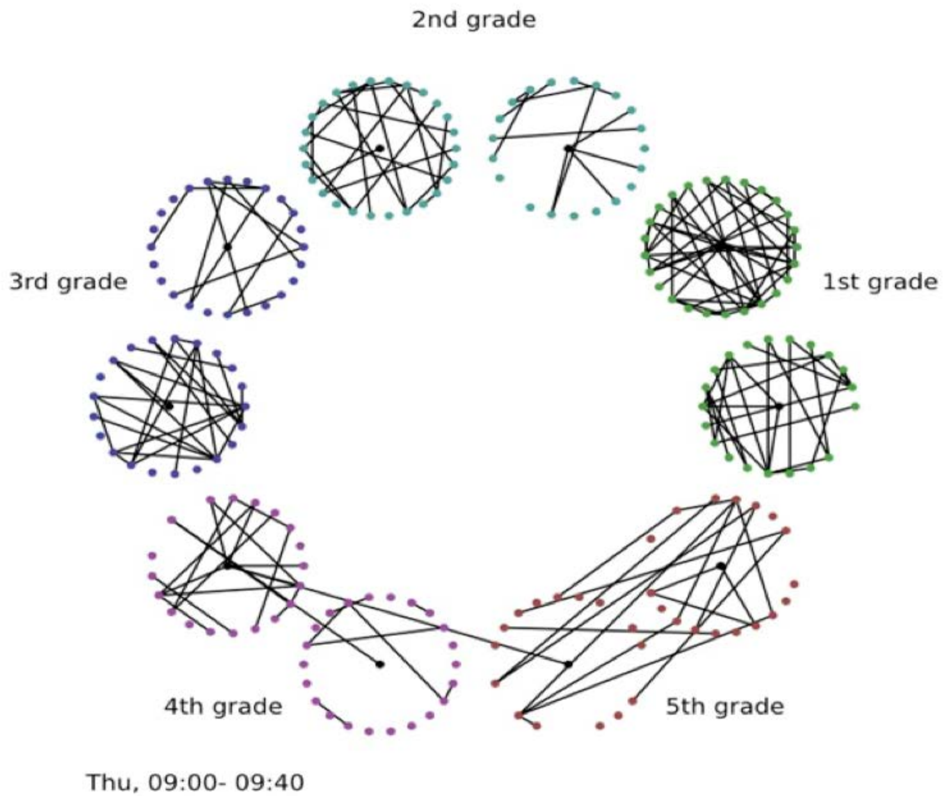
Homogenous assumption limits

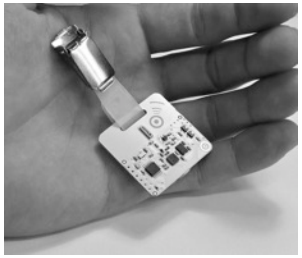


Random mixing Networks

MOBS LAB

Micro-scale: proximity interactions in confined environments

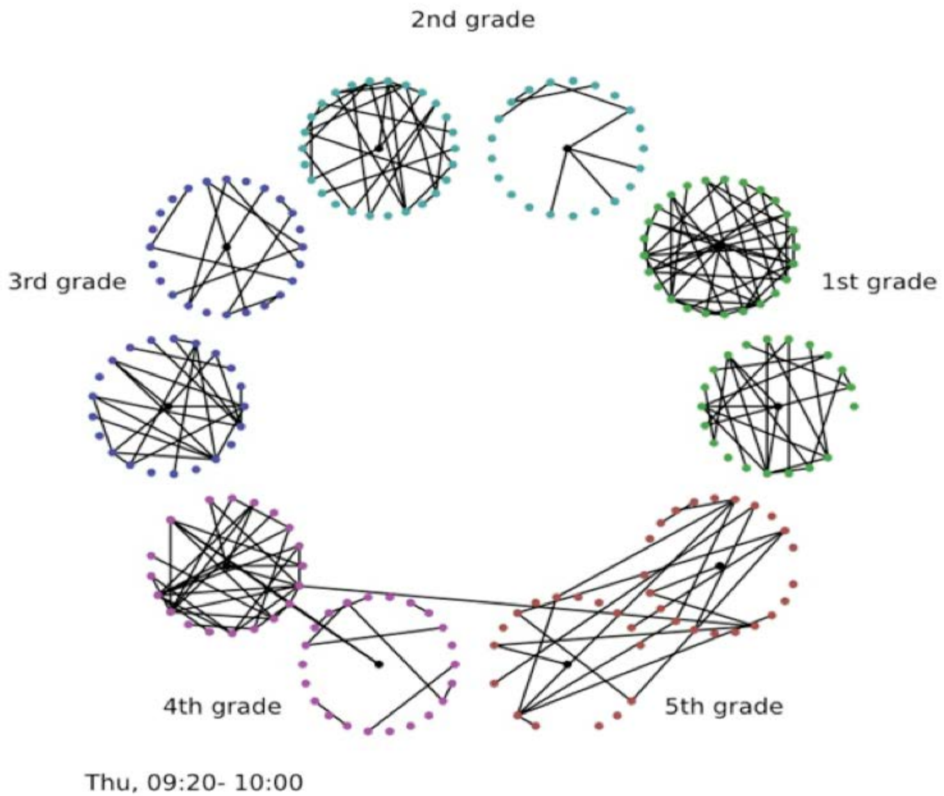


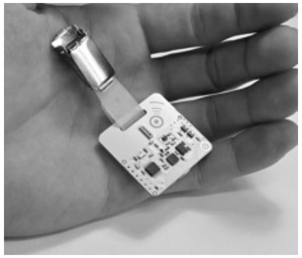


Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

Micro-scale: proximity interactions in confined environments



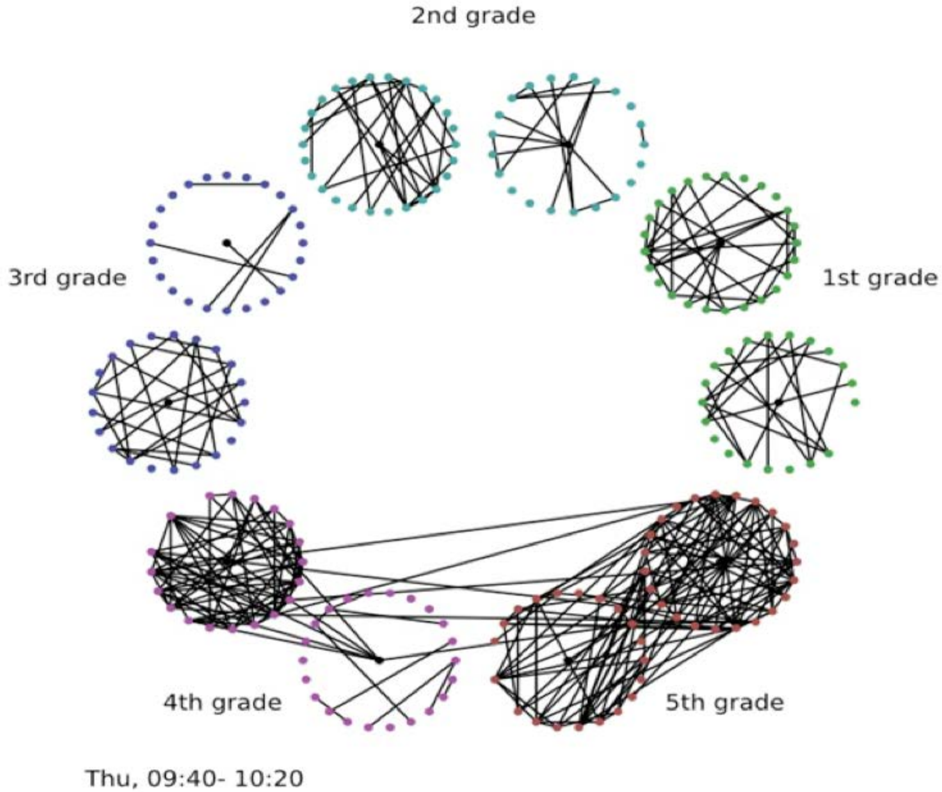


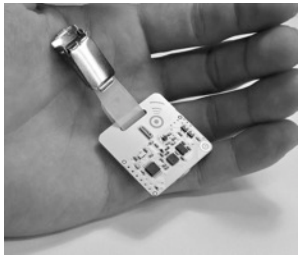
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



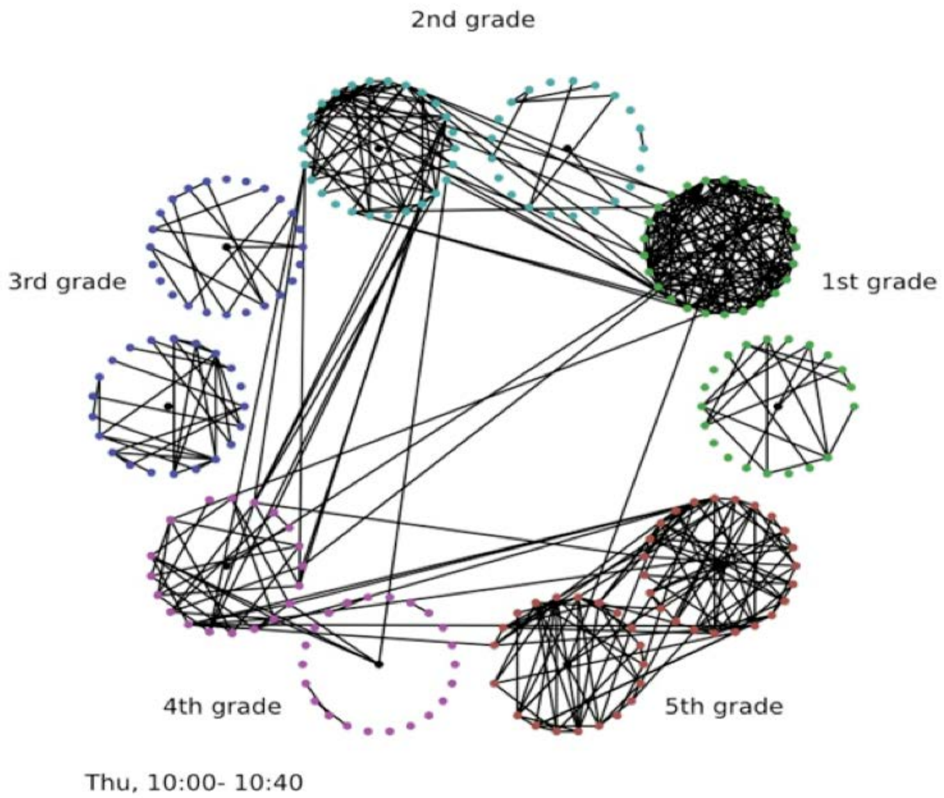


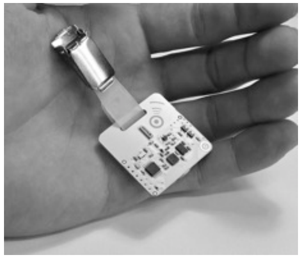
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



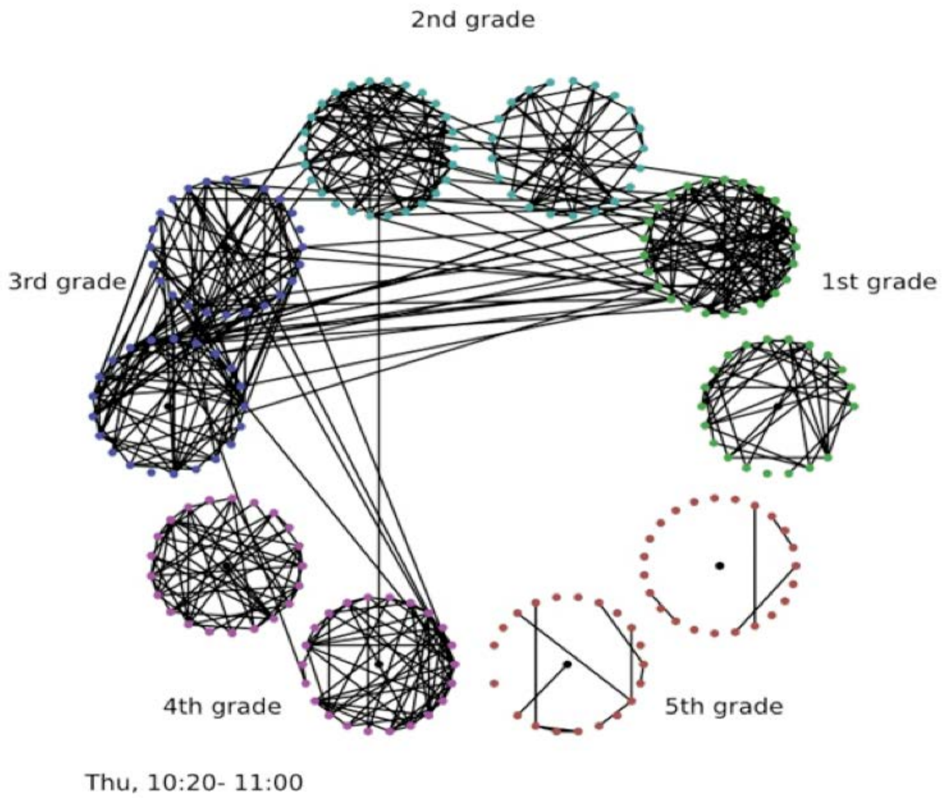


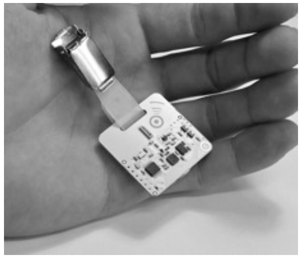
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



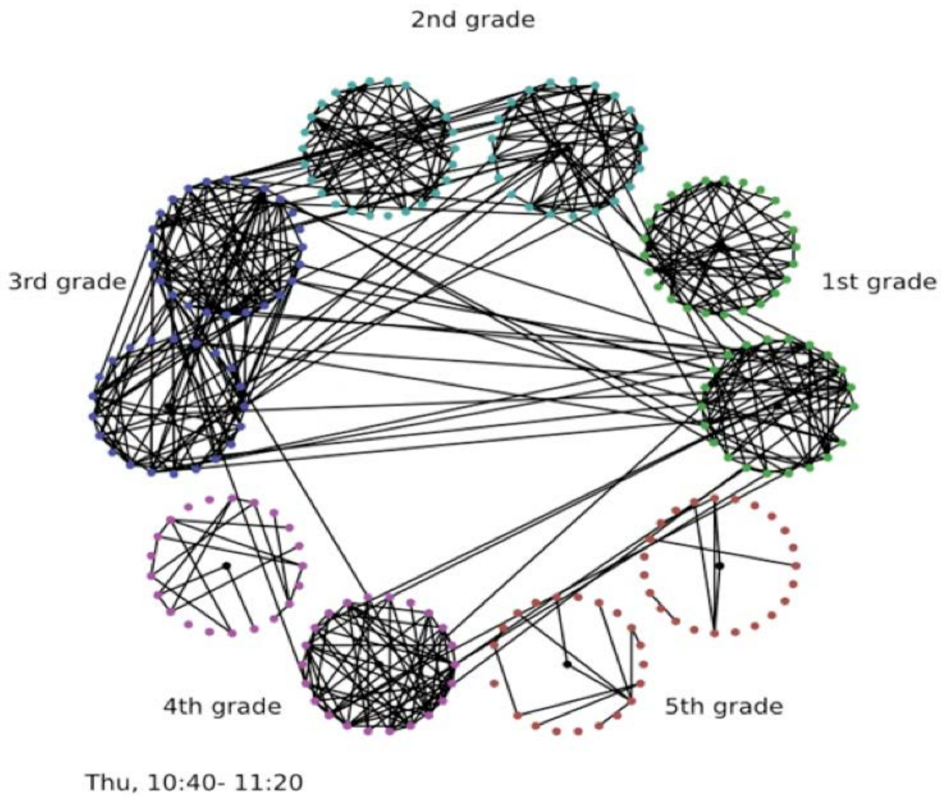


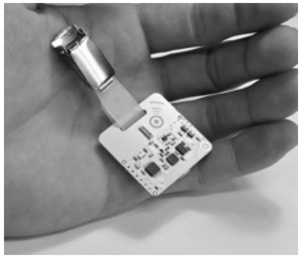
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



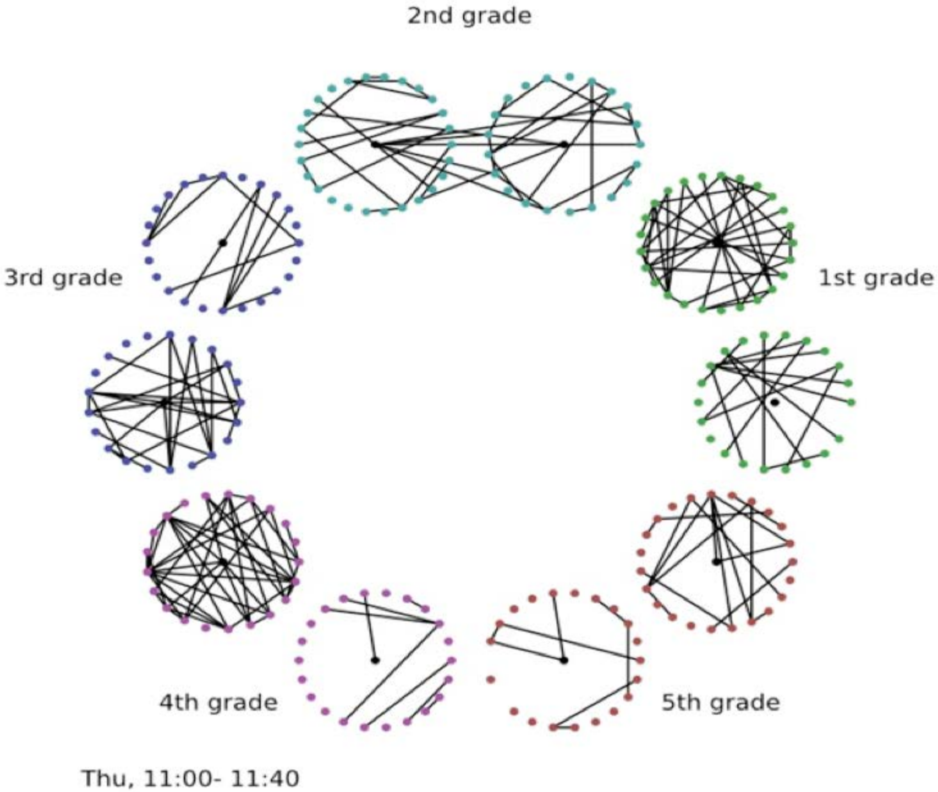


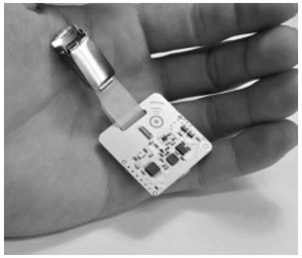
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments

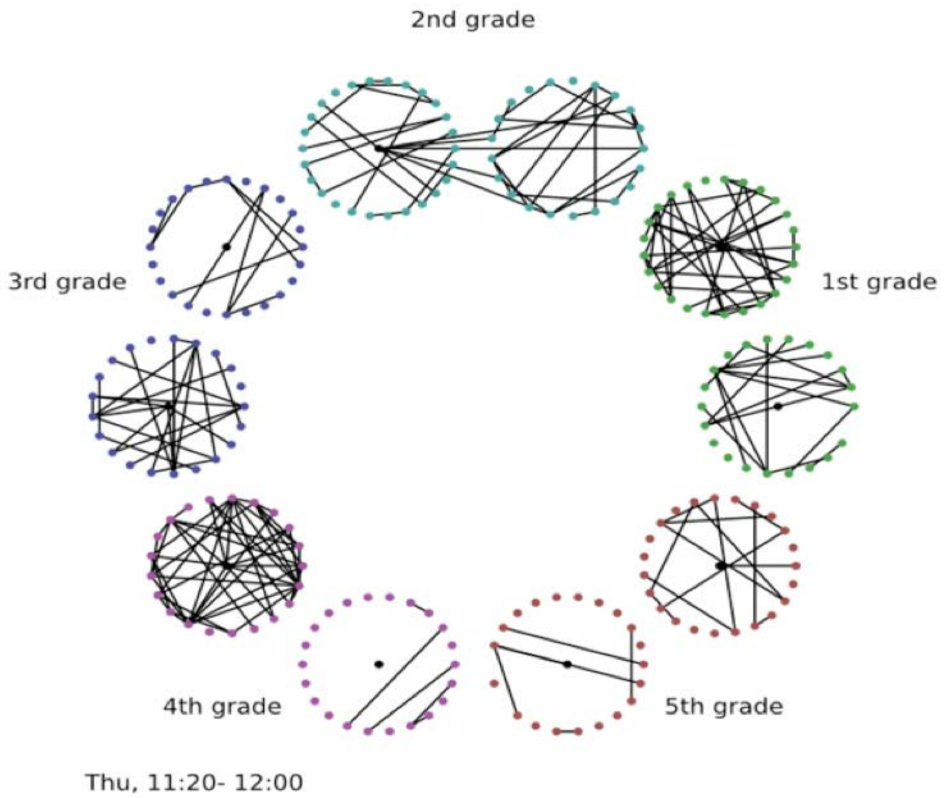


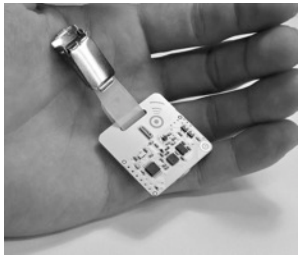


Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

Micro-scale: proximity interactions in confined environments



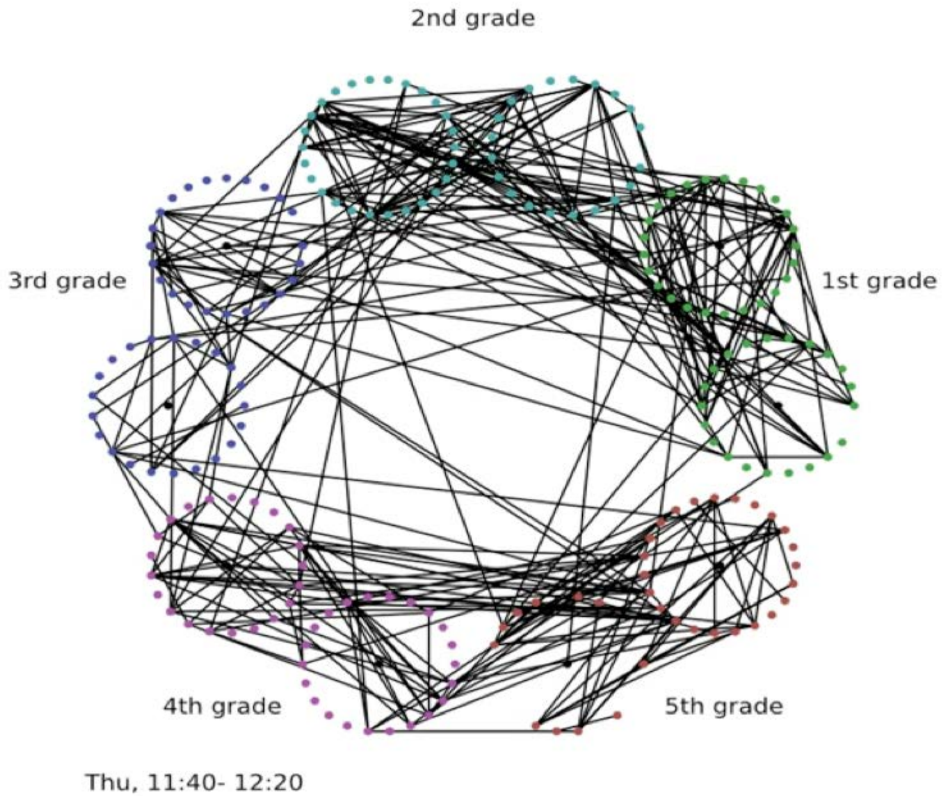


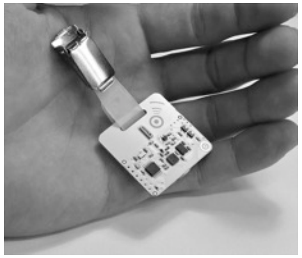
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010))

MOBS LAB

Micro-scale: proximity interactions in confined environments



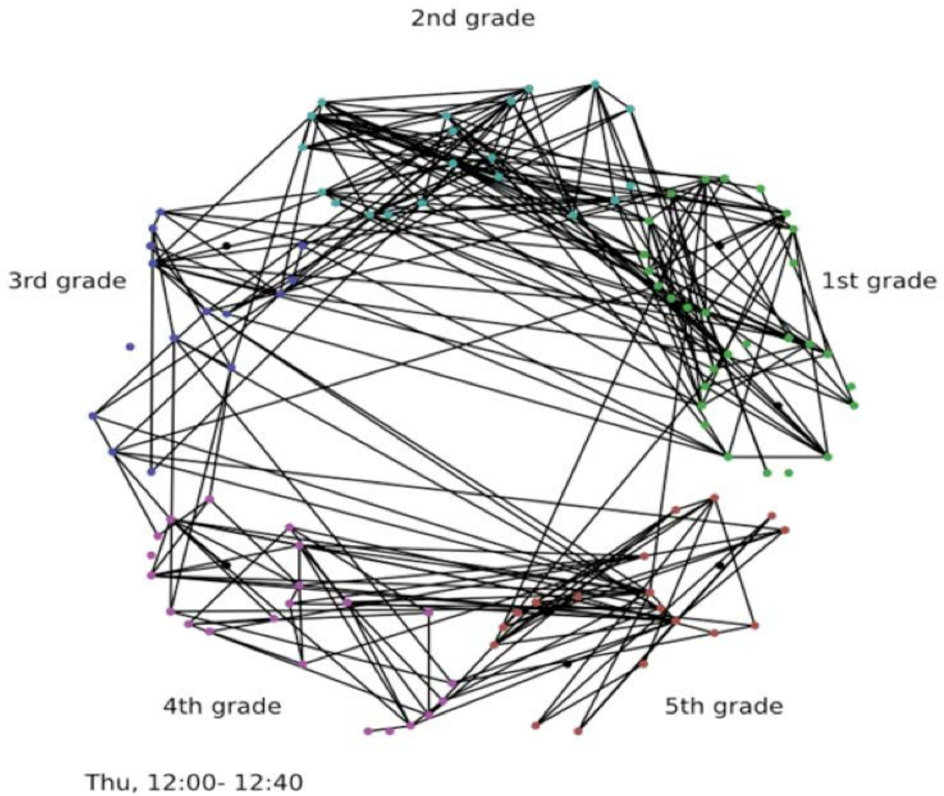


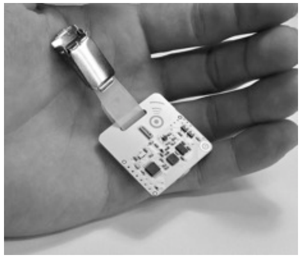
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



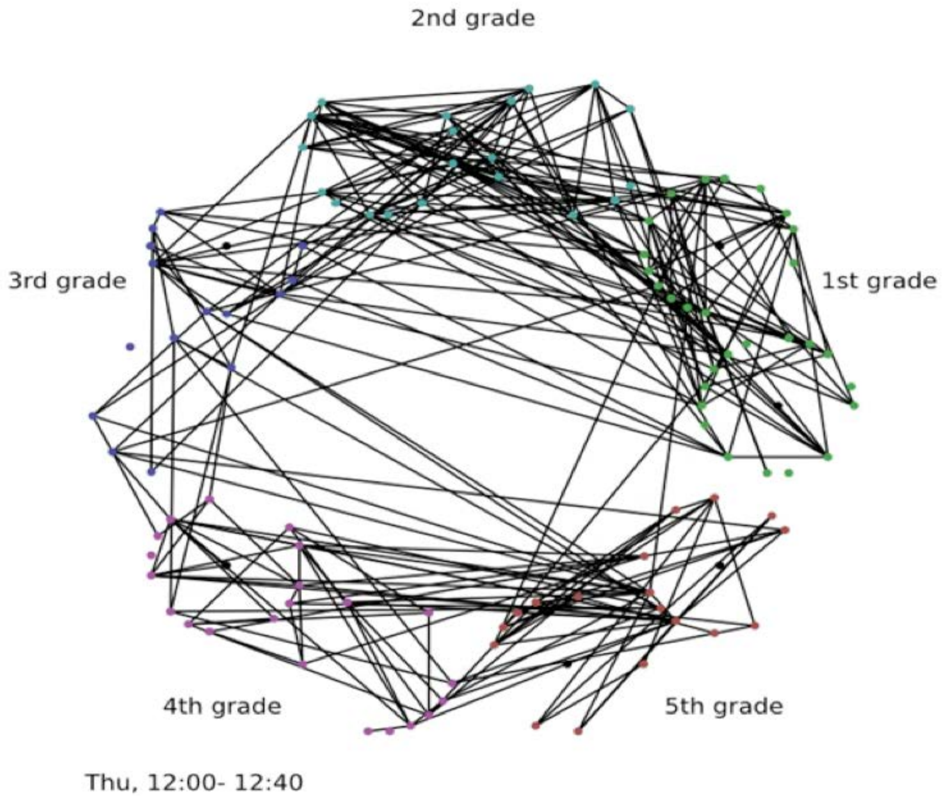


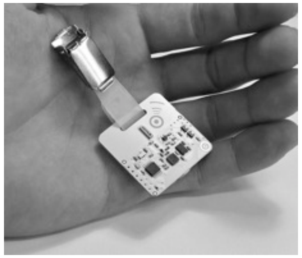
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010))

MOBS LAB

Micro-scale: proximity interactions in confined environments



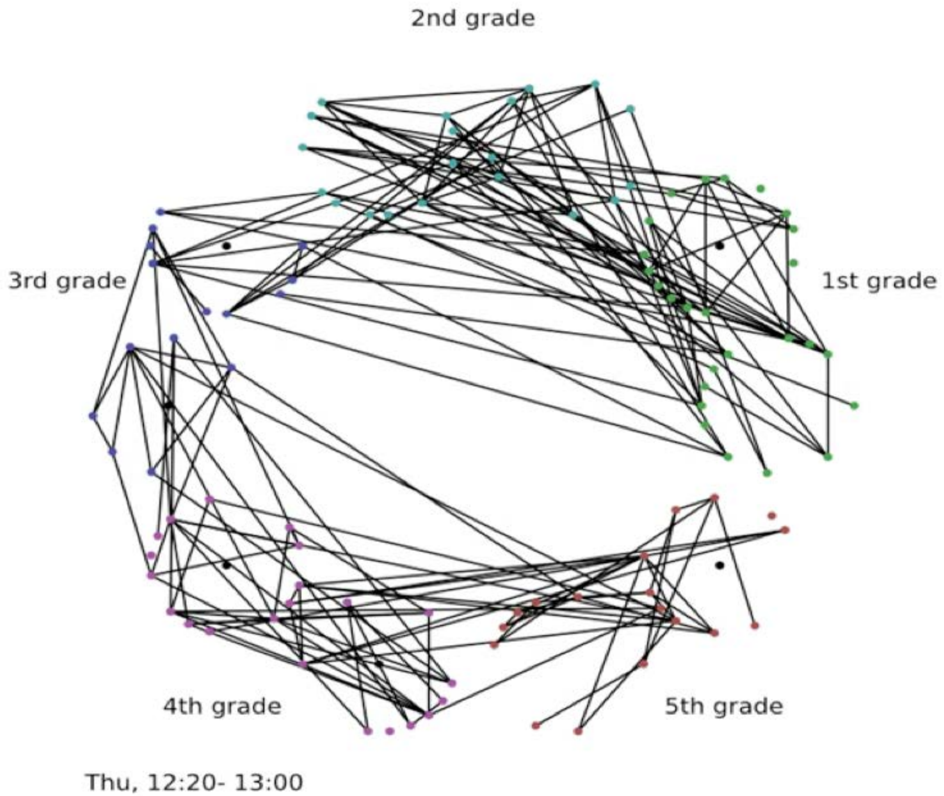


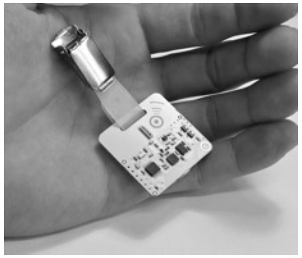
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010))

MOBS LAB

Micro-scale: proximity interactions in confined environments



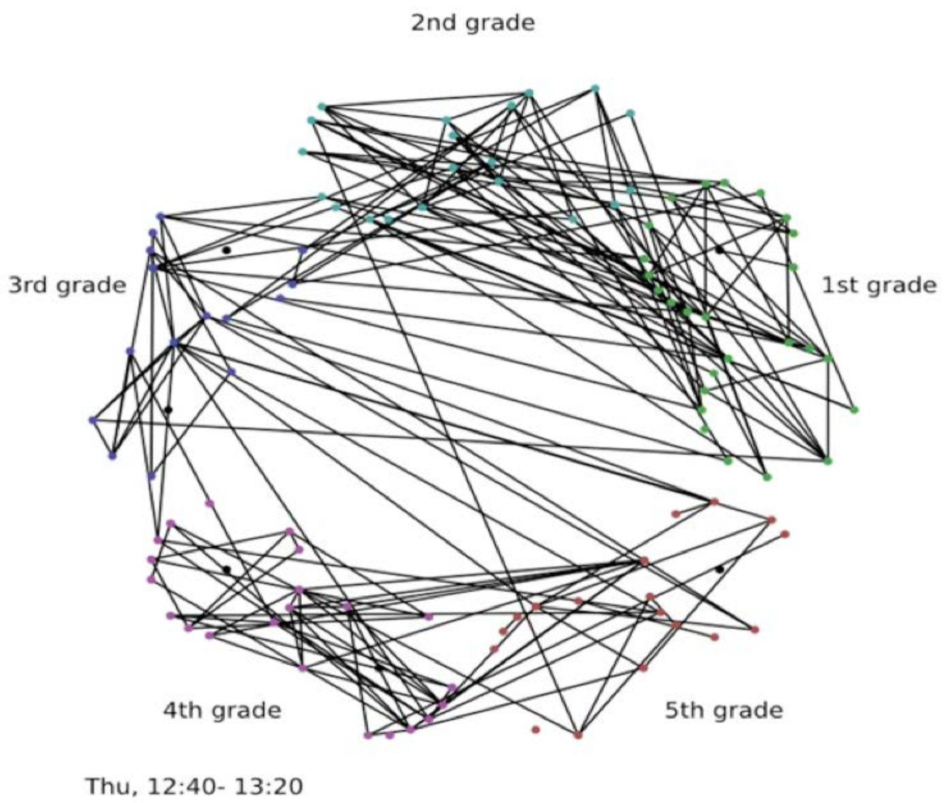


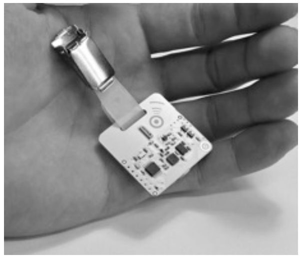
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



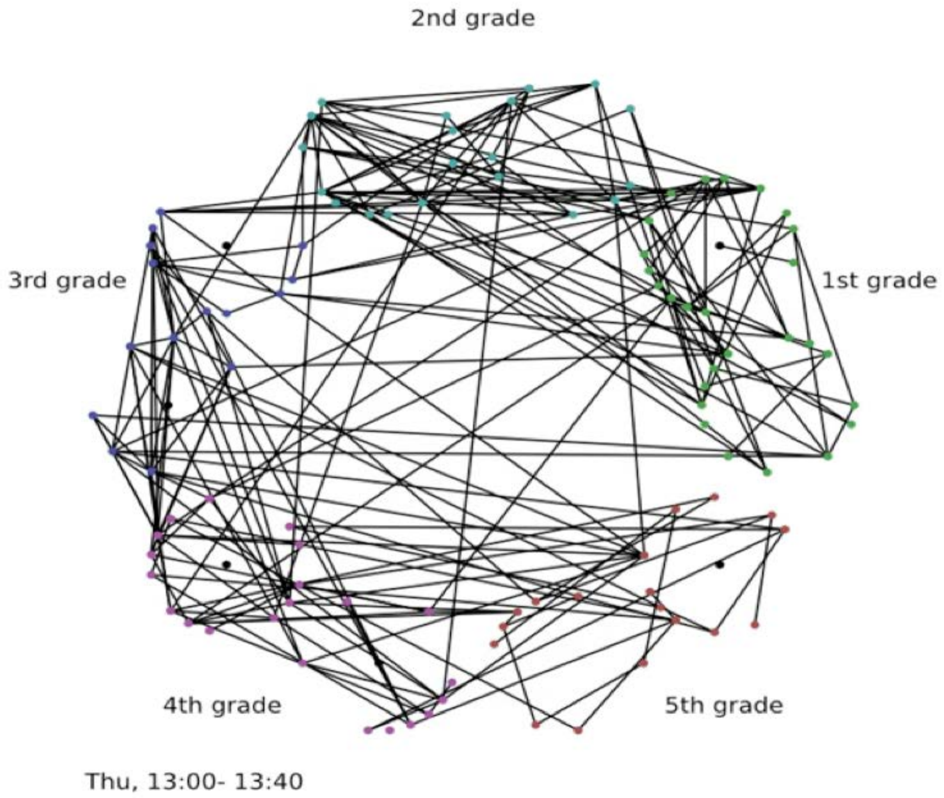


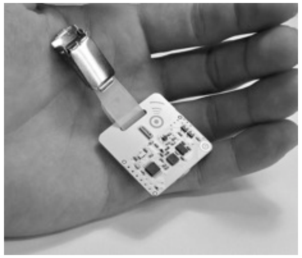
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



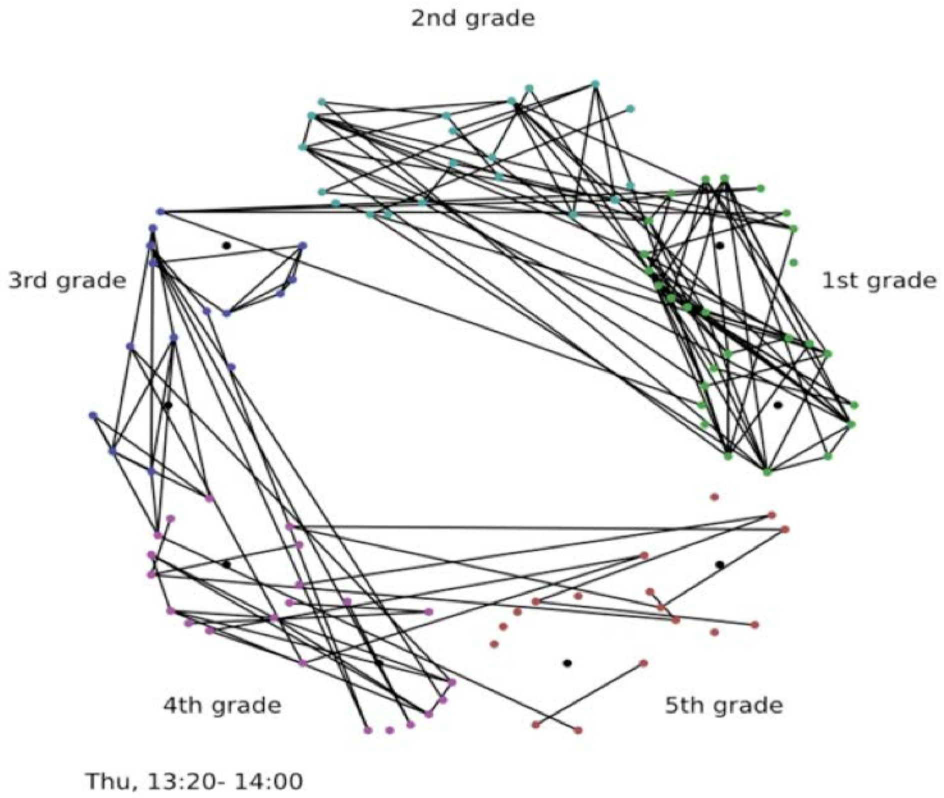


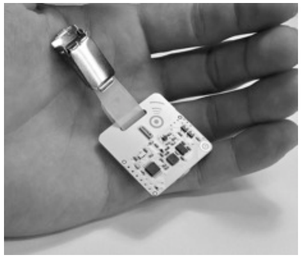
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



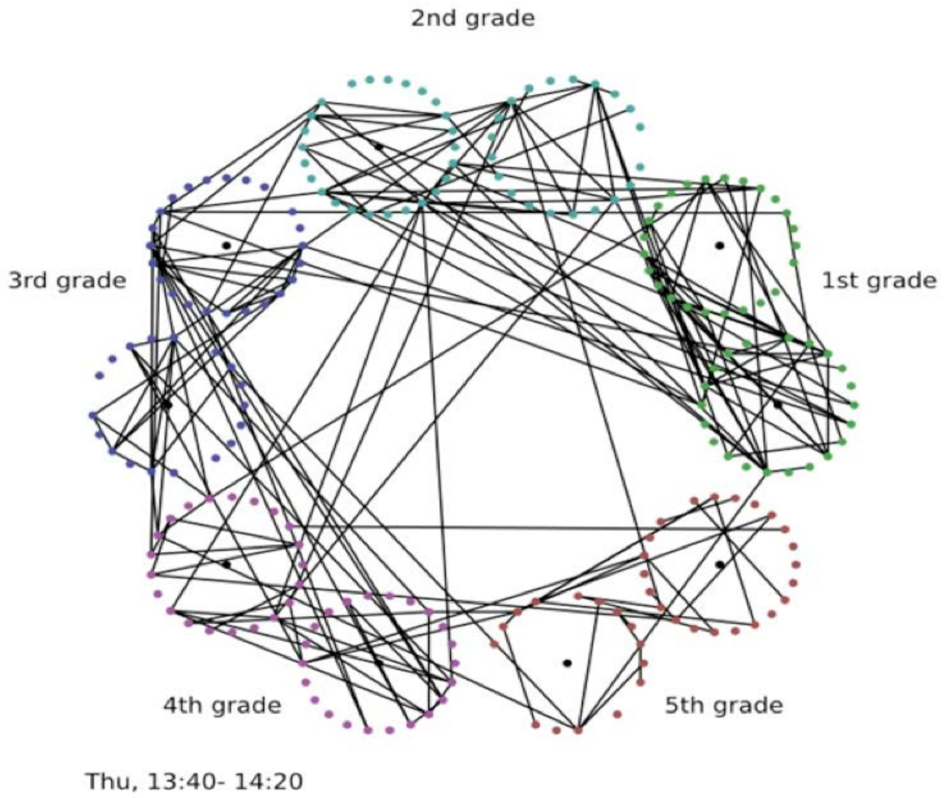


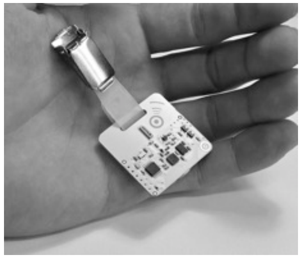
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



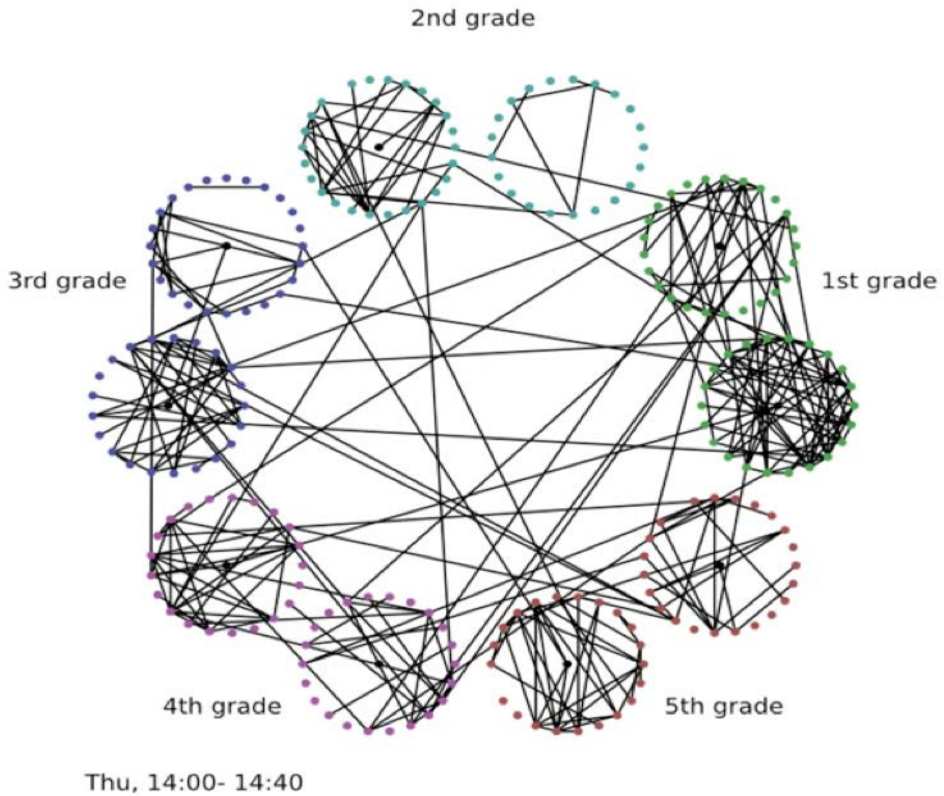


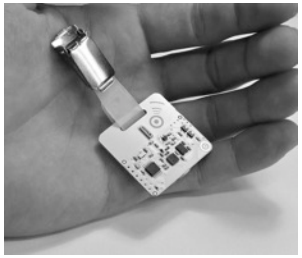
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



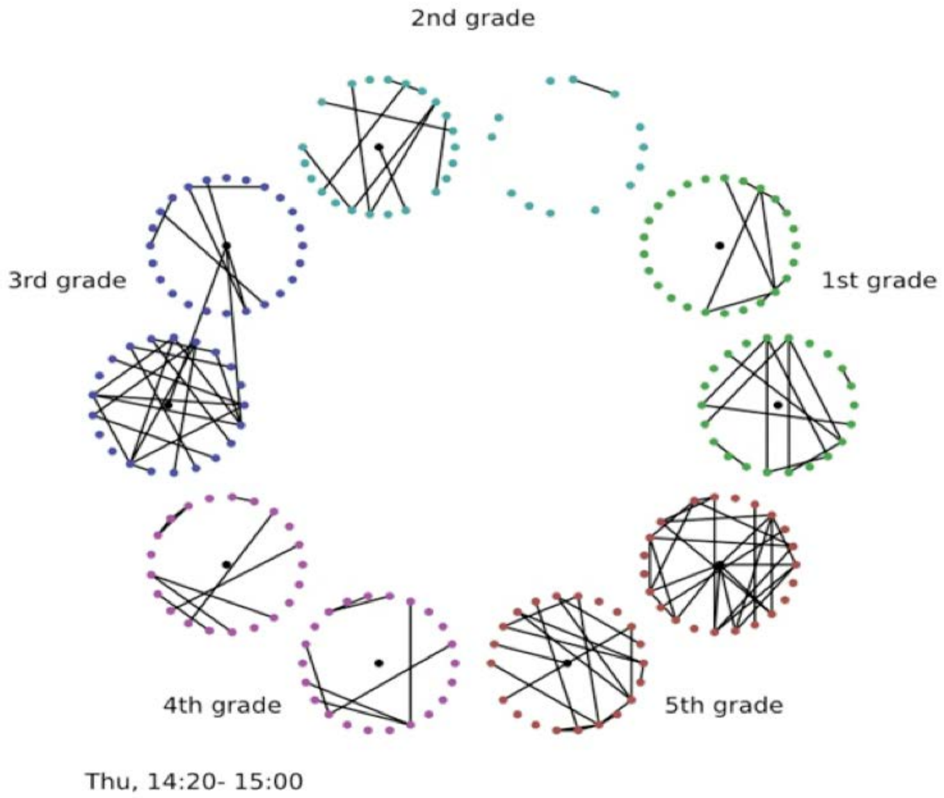


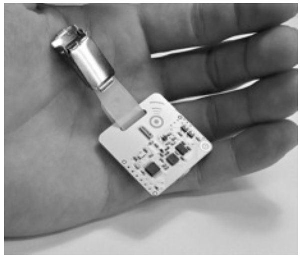
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



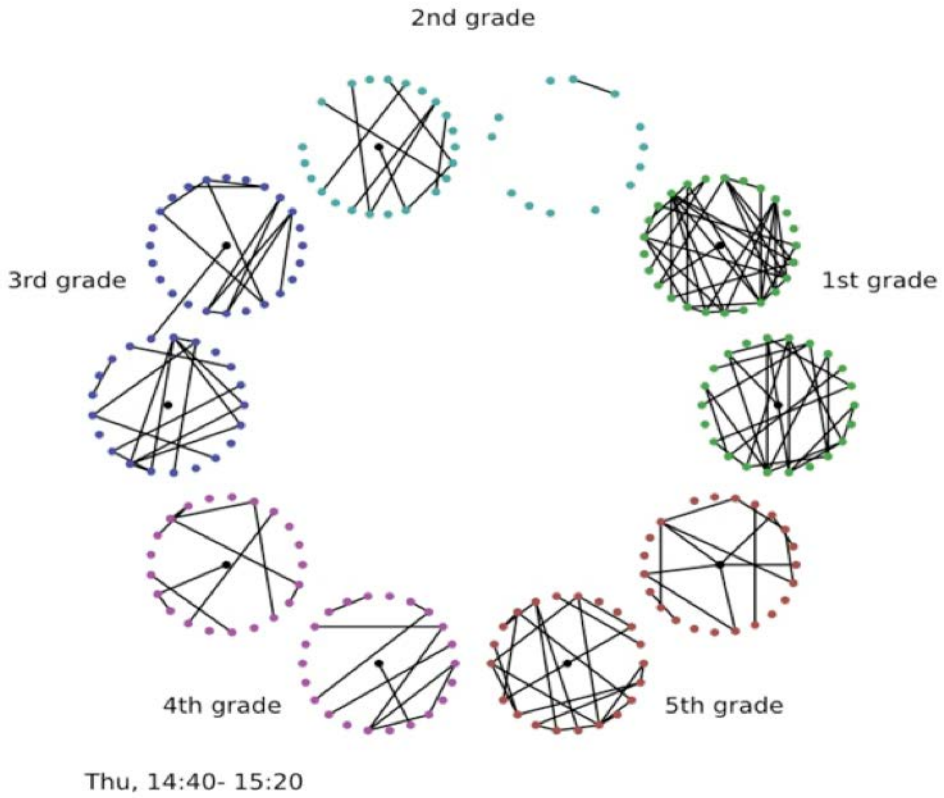


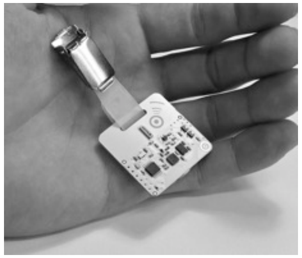
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010))

MOBS LAB

Micro-scale: proximity interactions in confined environments



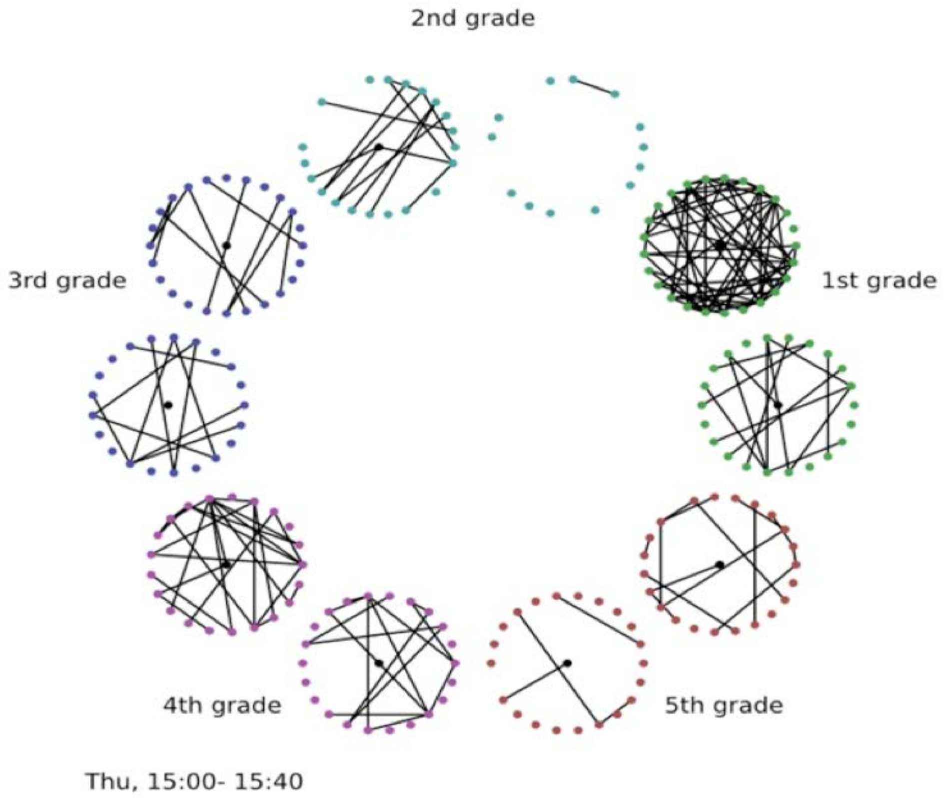


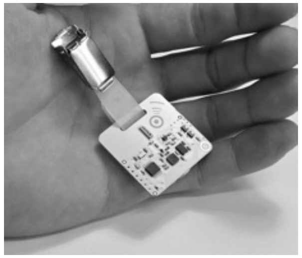
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



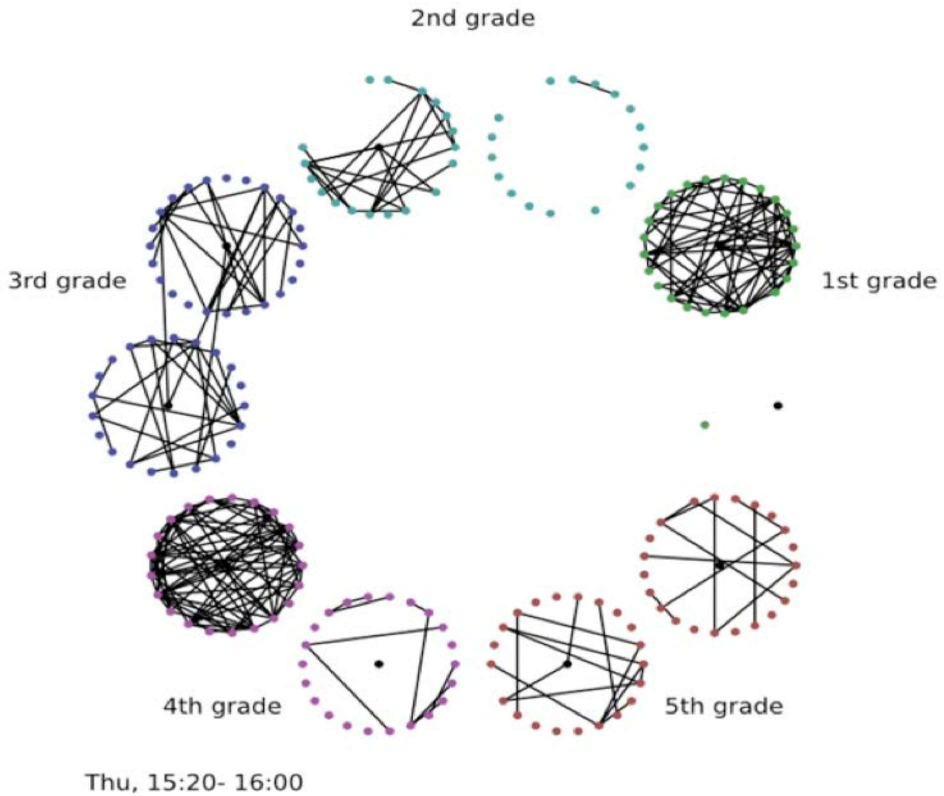


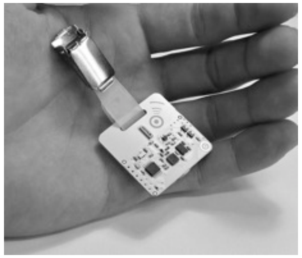
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



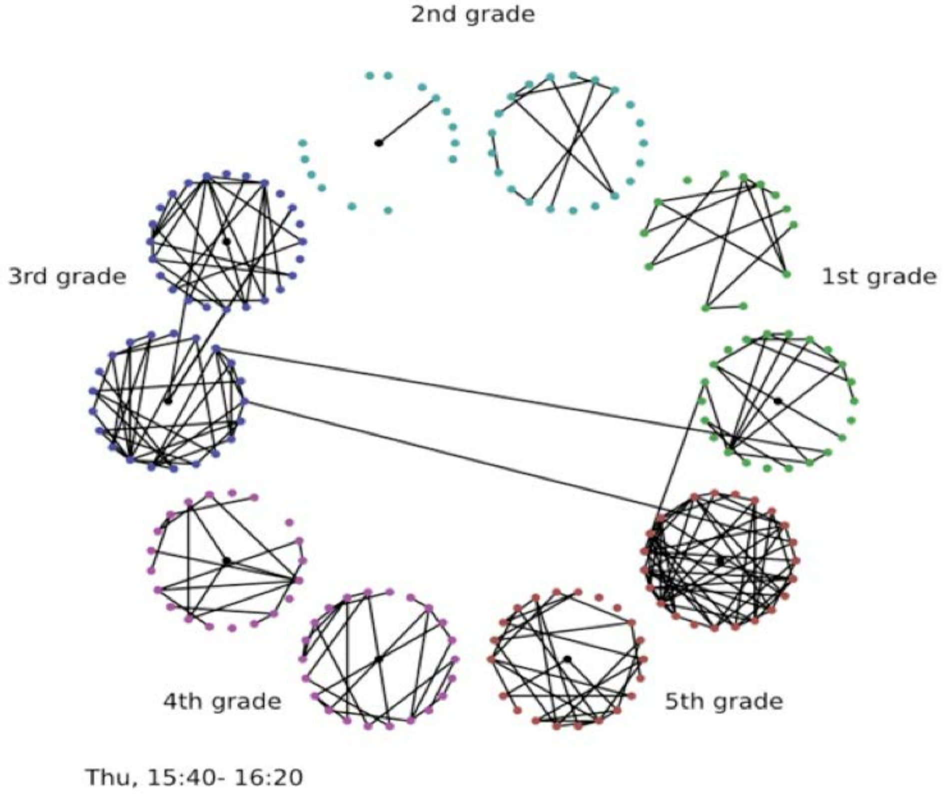


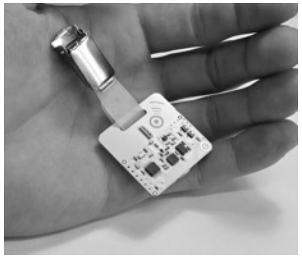
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



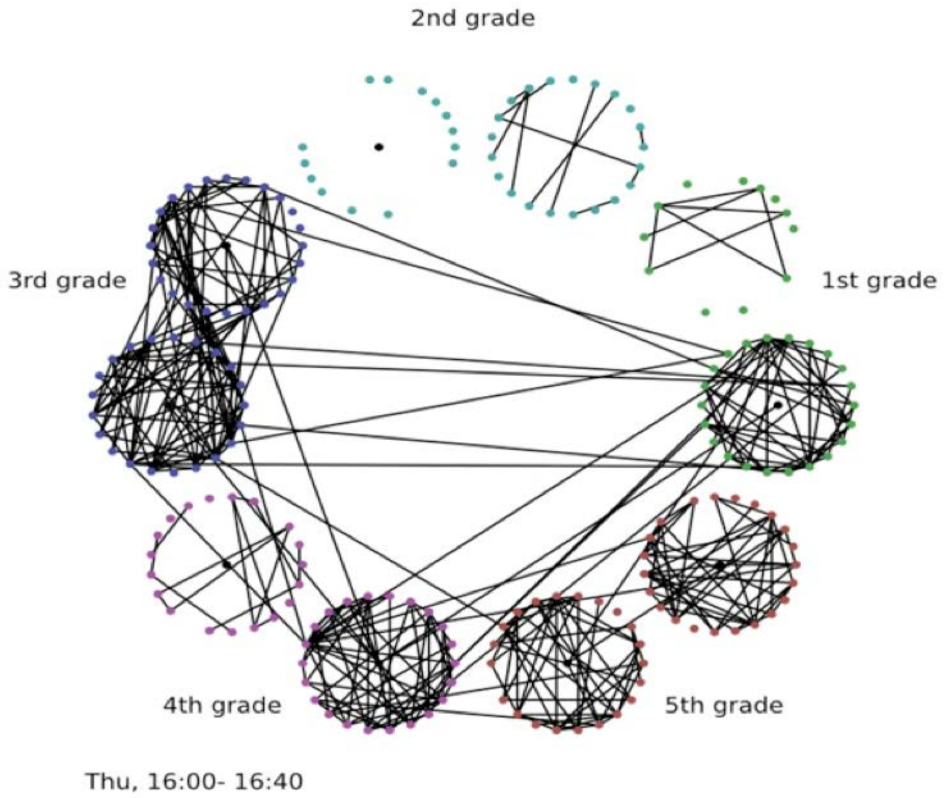


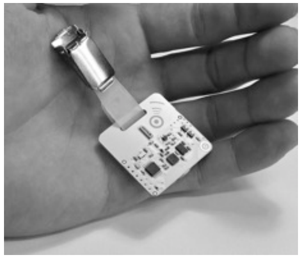
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



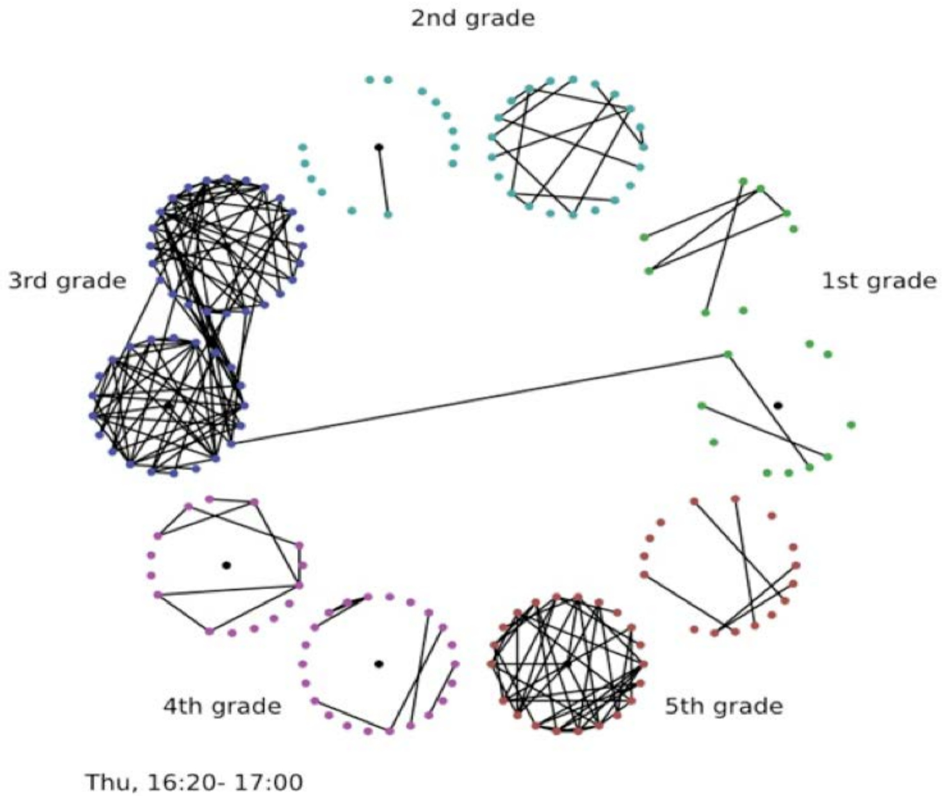


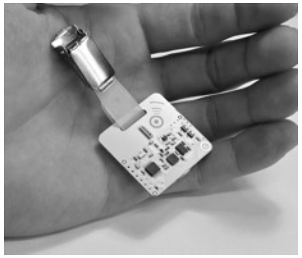
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

Micro-scale: proximity interactions in confined environments



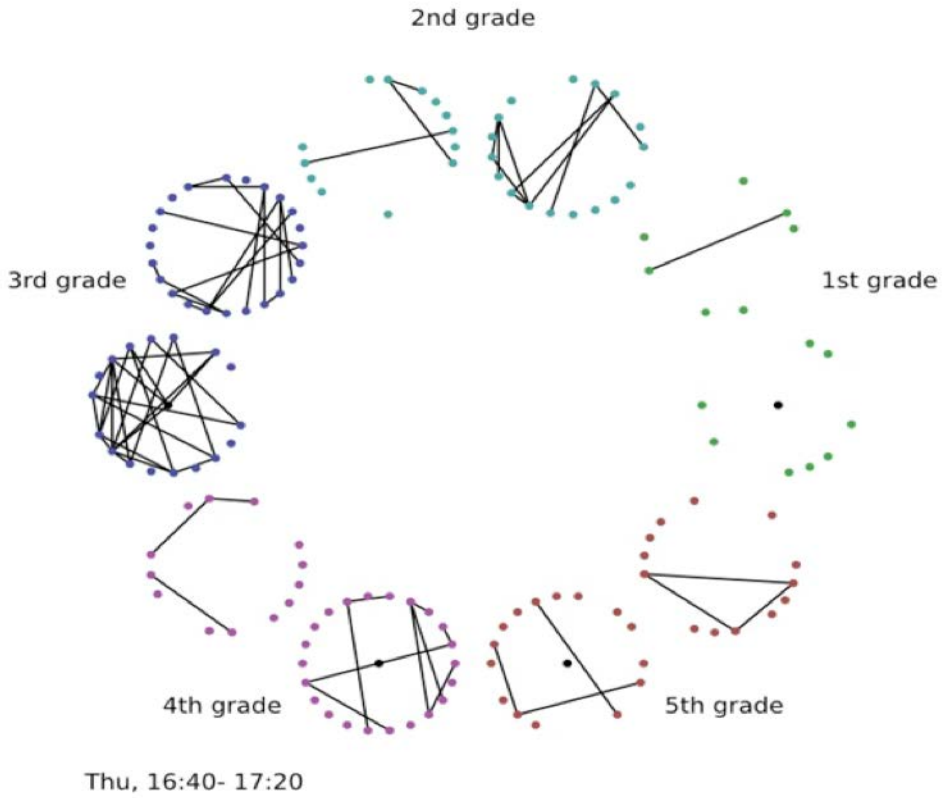


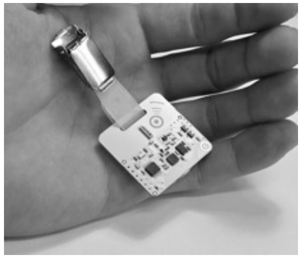
Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010))

MOBS LAB

Micro-scale: proximity interactions in confined environments



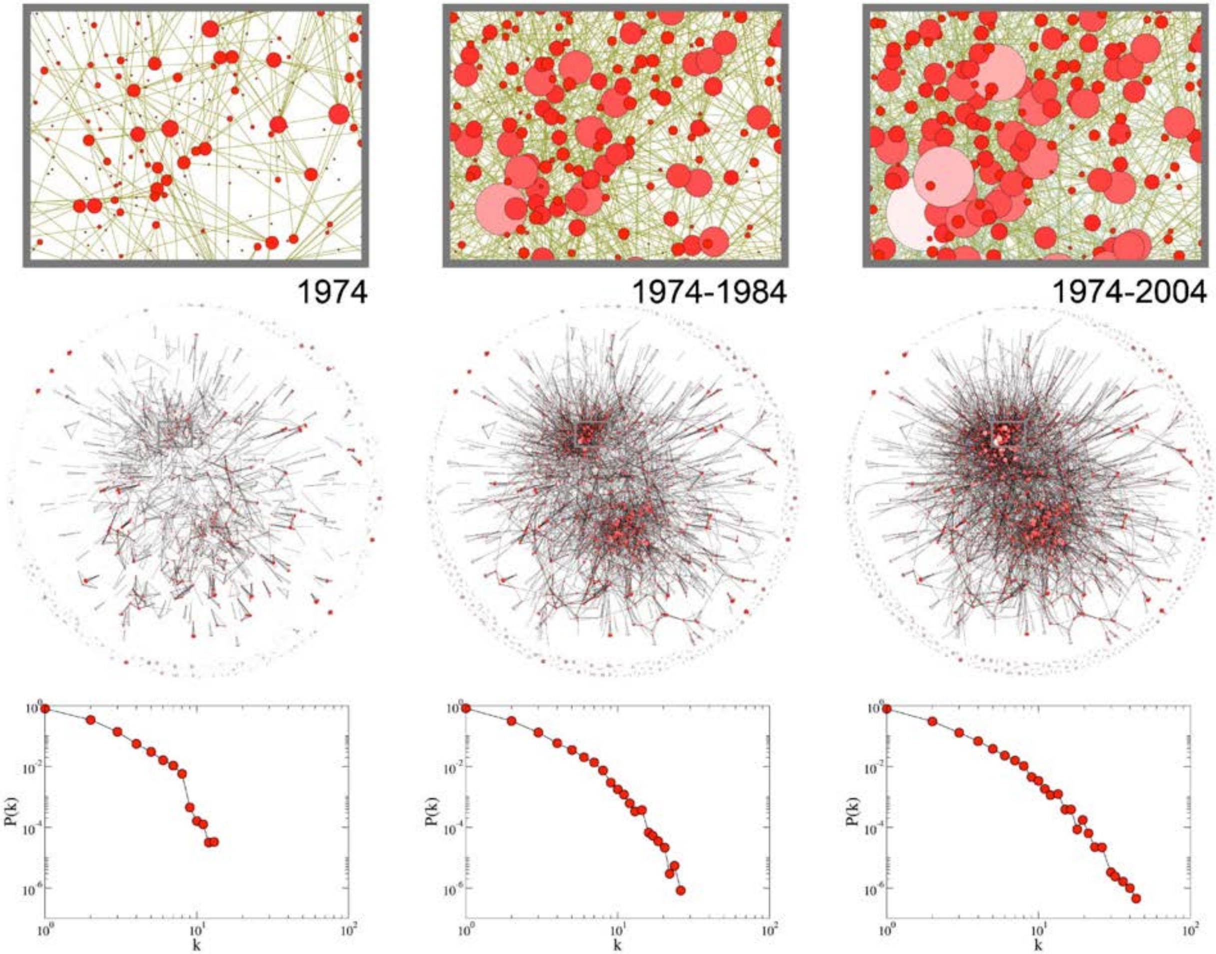


Sociopatterns experiment in school

(Cattuto, PLoS ONE 5(7): e11596 (2010)

MOBS LAB

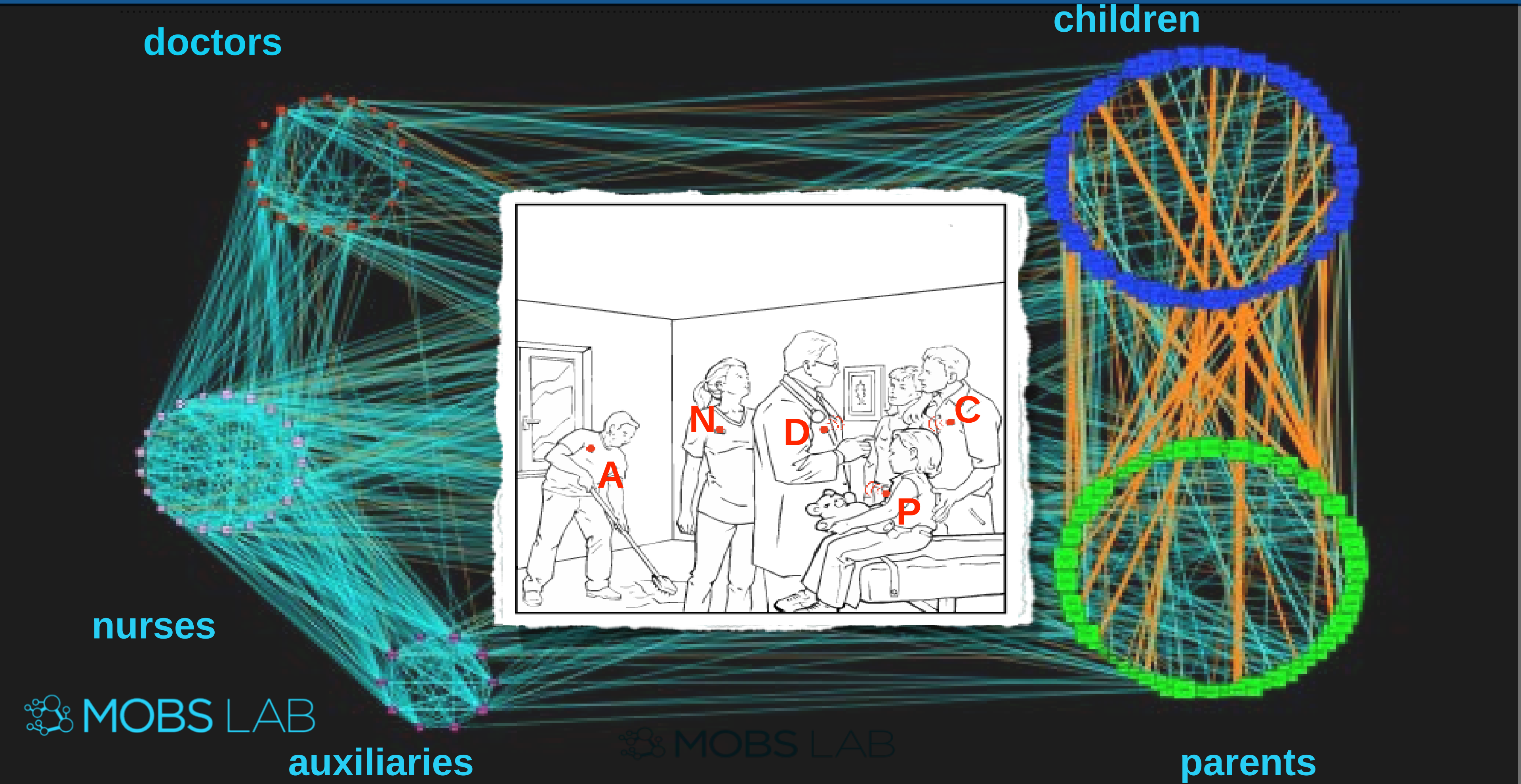
Scientific collaboration network



MOBS LAB

Interaction in hospital wards

Vanhems PLoS ONE 8(9), e73970 (2013)

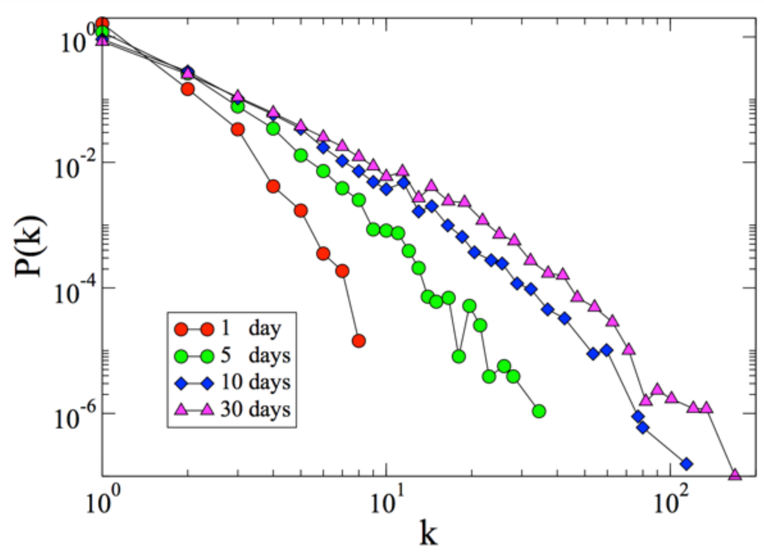


Heterogeneity and structure

Statistical distribution for the # of contacts (degree k)

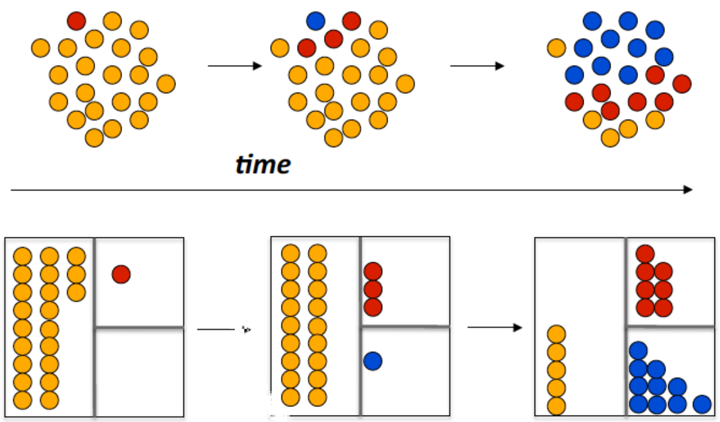
- Skewed
- Heterogeneity and high variability
- Very large fluctuations (variance>>average)

- Small World
- Clustering
- Assortativity

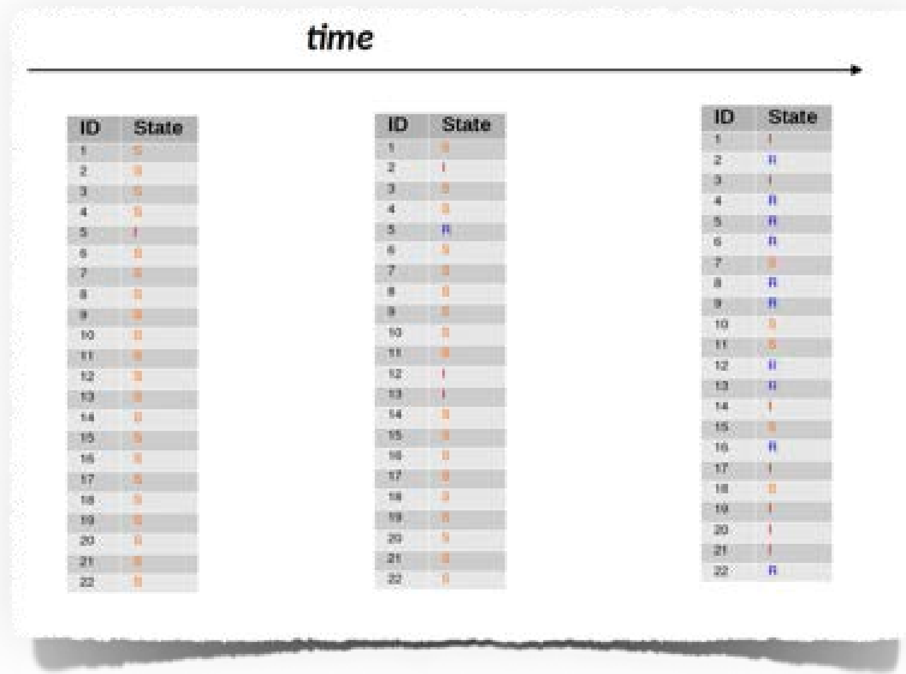


MOBS LAB

Individual based Models



Discrete individuals tracked one by one

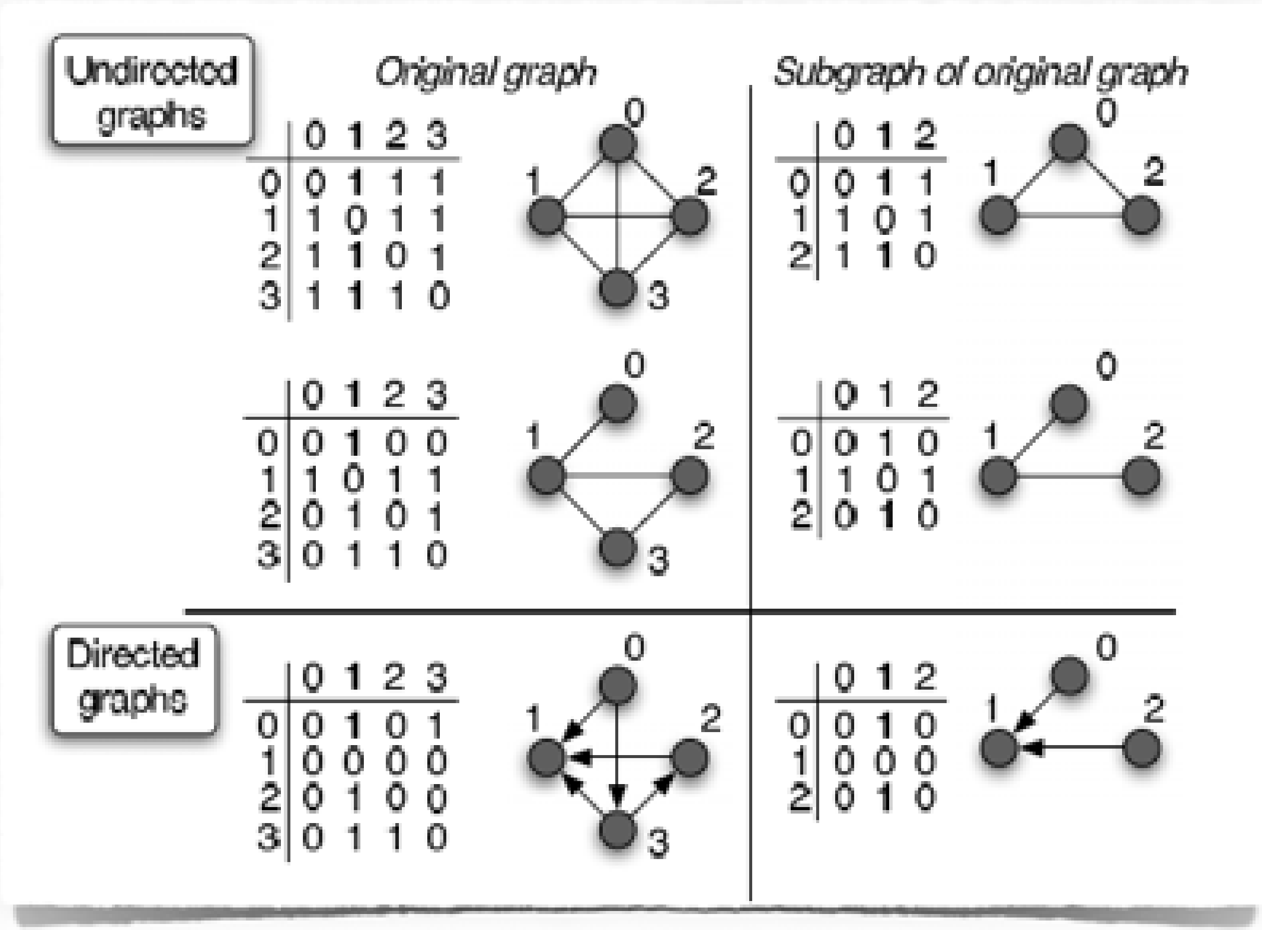


State vector
$X = (x_1, x_2, \ldots x_N)$

Random process for each specific variable

MOBS LAB

Contact Networks

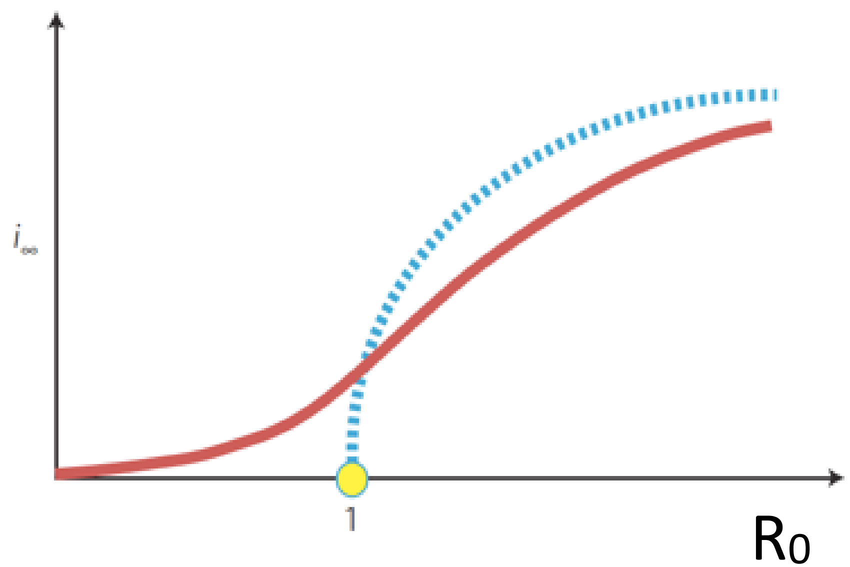


Network topology affects epidemic threshold and dynamical behavior of epidemics

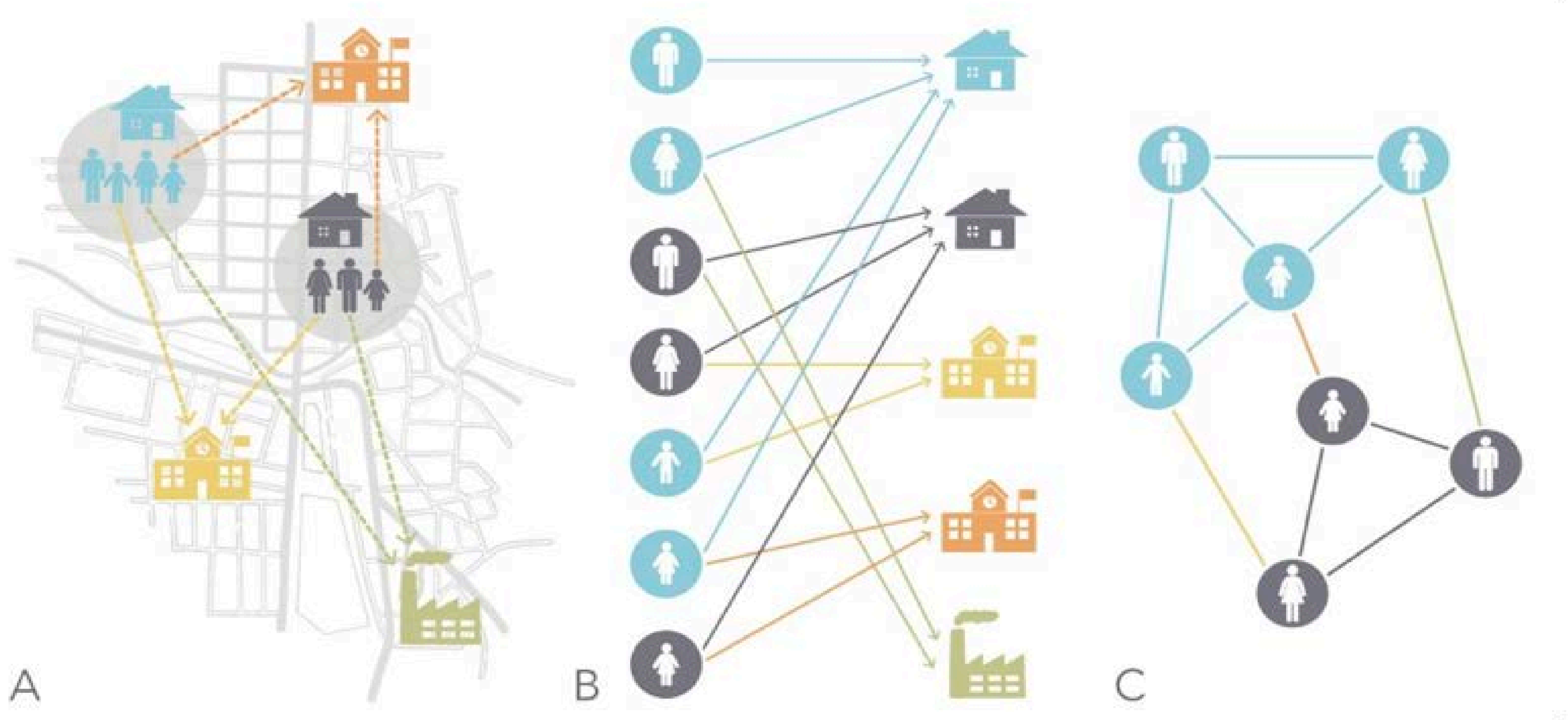
- Network heterogeneity tend to suppress the spreading threshold
- **Random-annealed network patterns have a vanishing epidemic threshold**

$$R_0 \text{ -> } R_0 \langle k^2 \rangle / \langle k \rangle^2$$

- Quenched networks have an epidemic threshold vanishing as the inverse of the largest eigenvalue of the connectivity matrix.
- Structure may change the results, however, assortativity/disassortativity do not impact the qualitative behavior concerning threshold.

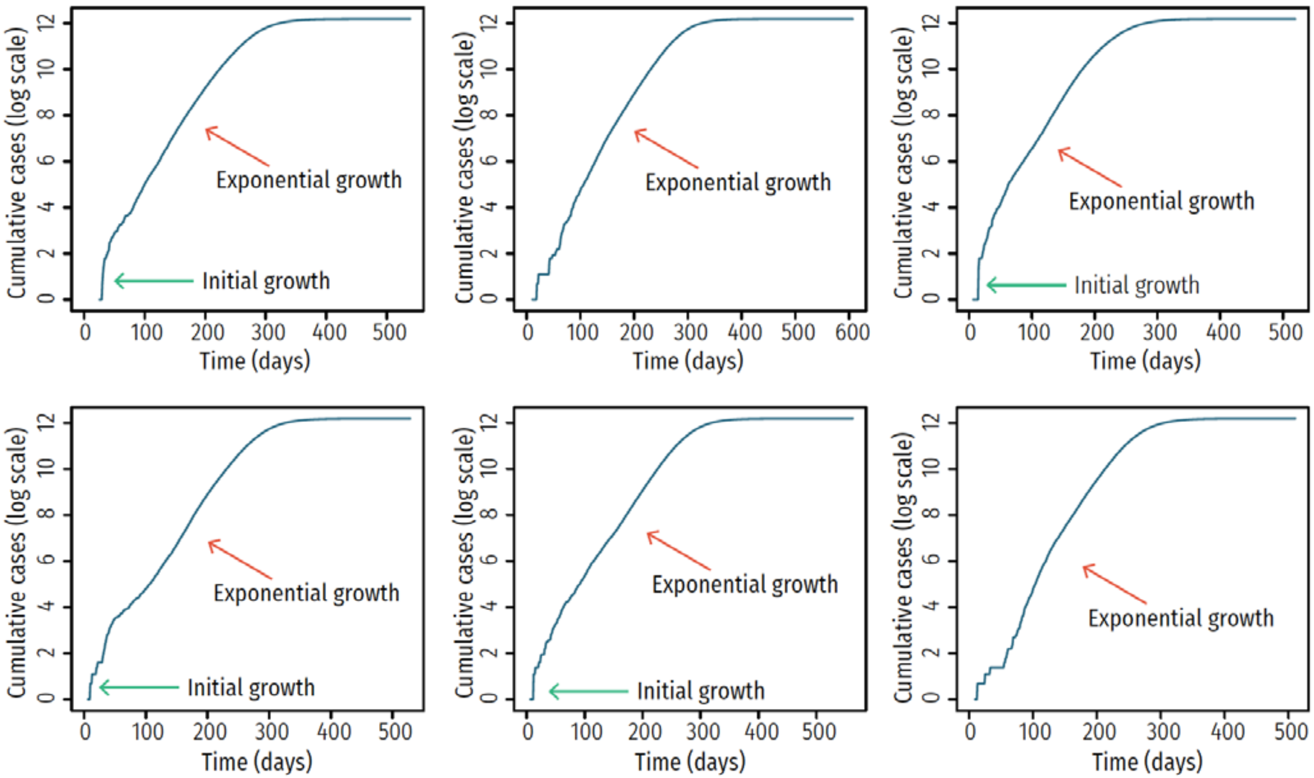


Large scale population mapping and Networks



MOBS LAB

Reproduction number... what?

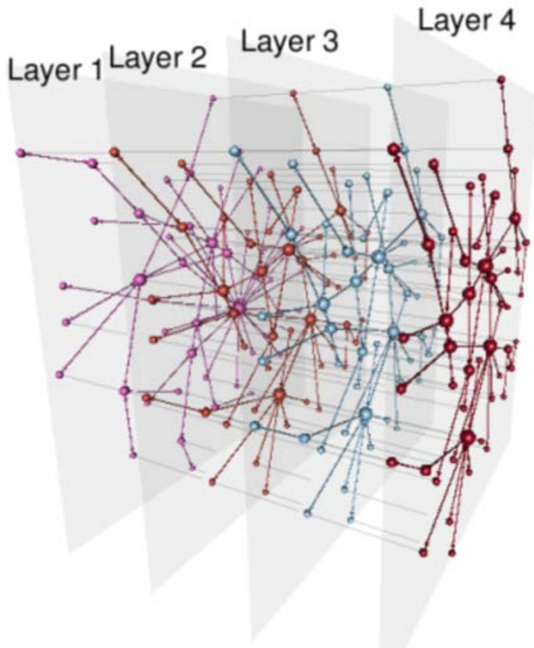


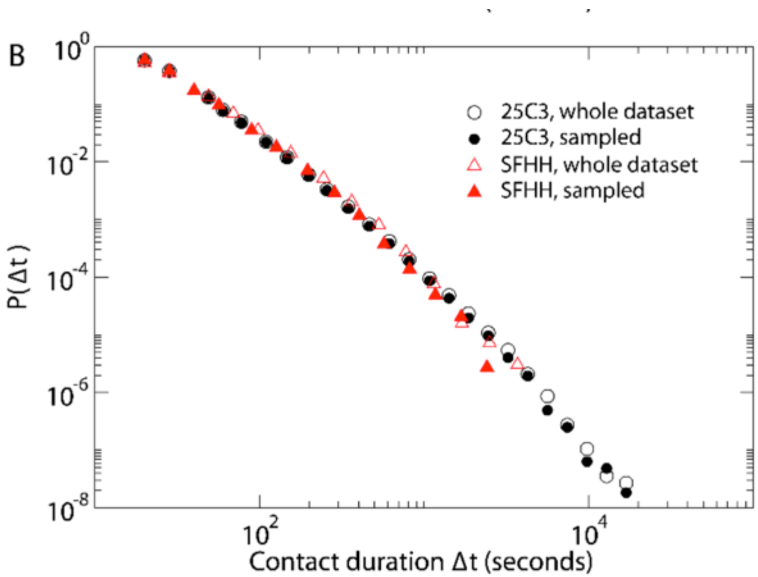
- **Clustering effects and heterogeneous transmission interplay to create a complicate picture for which we do not yet have analytic understanding.**
- **Major barrier in modeling alignment and forecasts.**

MOBS LAB

The frontier

Multiplexity



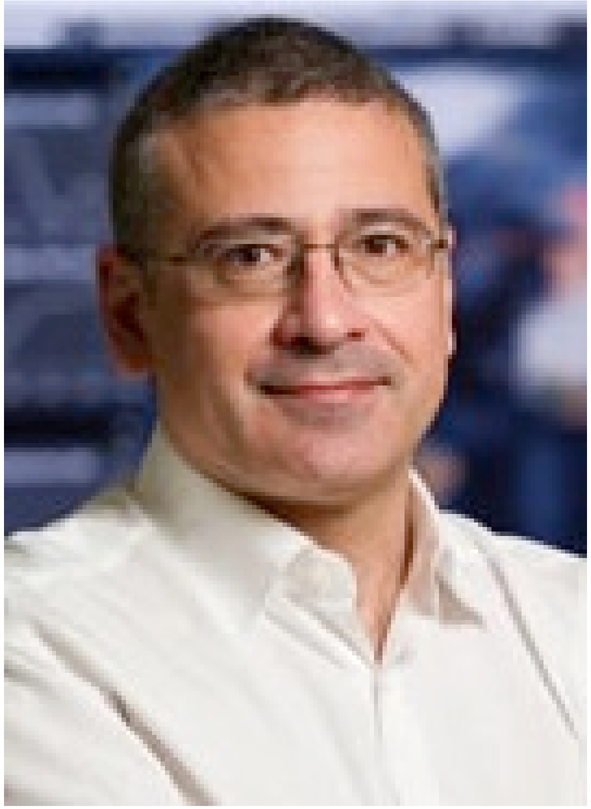


Time scale: From minutes to years.

MOBS LAB

MATHEMATICAL FRONTIERS
Social and Biological Networks – Q&A

Nina H. Fefferman,
University of Tennessee, Knoxville



Alessandro Vespignani,
Northeastern University

James H. Curry,
University of Colorado, Boulder

MATHEMATICAL FRONTIERS
2018 Monthly Webinar Series, 2-3pm ET

February 13: ***Recording posted***
Mathematics of the Electric Grid

March 13: ***Recording posted***
Probability for People and Places

April 10:
Social and Biological Networks

May 8:
Mathematics of Redistricting

June 12:
Number Theory: The Riemann Hypothesis

July 10: *Topology*

August 14:
Algorithms for Threat Detection

September 11:
Mathematical Analysis

October 9: *Combinatorics*

November 13:
Why Machine Learning Works

December 11:
Mathematics of Epidemics

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers