MATHEMATICAL FRONTIERS

The National Academies of SCIENCES ENGINEERING MEDICINE

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Board on Mathematical Sciences & Analytics

MATHEMATICAL FRONTIERS
2018 Monthly Webinar Series, 2-3pm ET

February 13: ***Recording posted***
Mathematics of the Electric Grid

March 13: ***Recording posted***
Probability for People and Places

April 10: ***Recording posted***
Social and Biological Networks

May 8:
Mathematics of Redistricting

June 12:
Number Theory: The Riemann Hypothesis

July 10: *Topology*

August 14:
Algorithms for Threat Detection

September 11:
Mathematical Analysis

October 9: *Combinatorics*

November 13:
Why Machine Learning Works

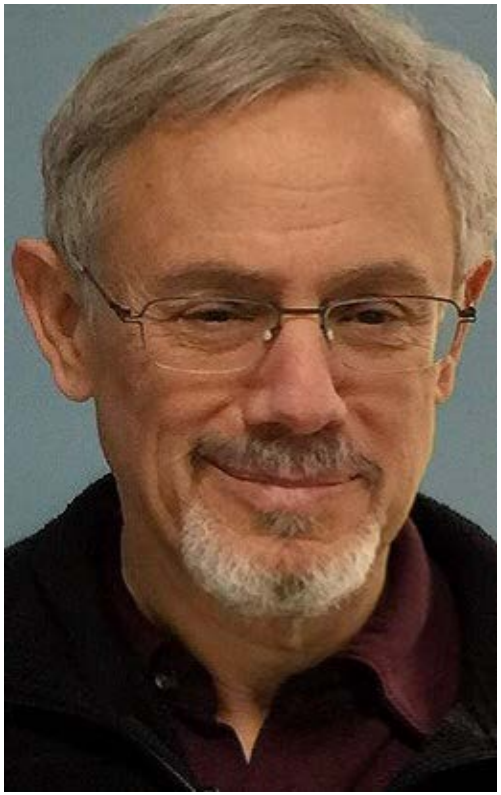
December 11:
Mathematics of Epidemics

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers

MATHEMATICAL FRONTIERS
Mathematics of Redistricting

Karen Saxe,
American Mathematical Society

Jonathan Mattingly,
Duke University



Mark Green,
UCLA (moderator)

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MATHEMATICAL FRONTIERS
Mathematics of Redistricting

Karen Saxe,
American Mathematical Society

Director
Office of Government Relations
American Mathematical Society

DeWitt Wallace Professor of Mathematics
Macalester College

The US Congress

Census ➡ Reapportionment ➡ Redistricting

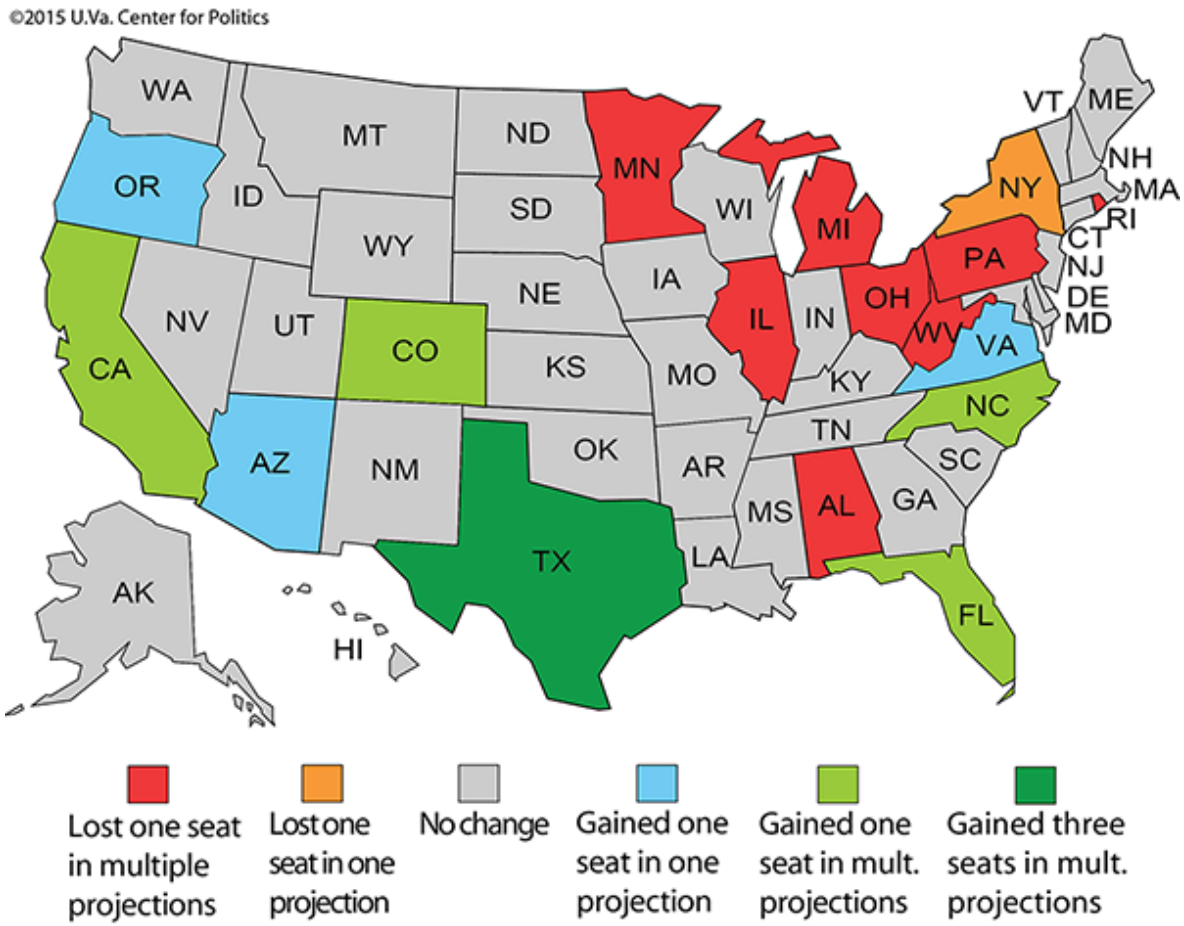
1. Perform the census to determine the population of the states.

2. Distribute the 435 House of Representatives seats to the states through reapportionment.

3. Redistrict each state, to partition into districts, one district per seat.

These 3 steps occur at least every decade, since 1790

2020 Apportionment Predictions



Redistricting Principles

1. Number – must have equal population
2. Shape – must be contiguous and **compact**
3. Race – must comply with Voting Rights Act of 1965
4. Political – attempt to keep cities, counties together

What else could be taken into consideration?

- *Communities of interest*
- *Incumbent protection*
- ***Partisan make-up of proposed districts***

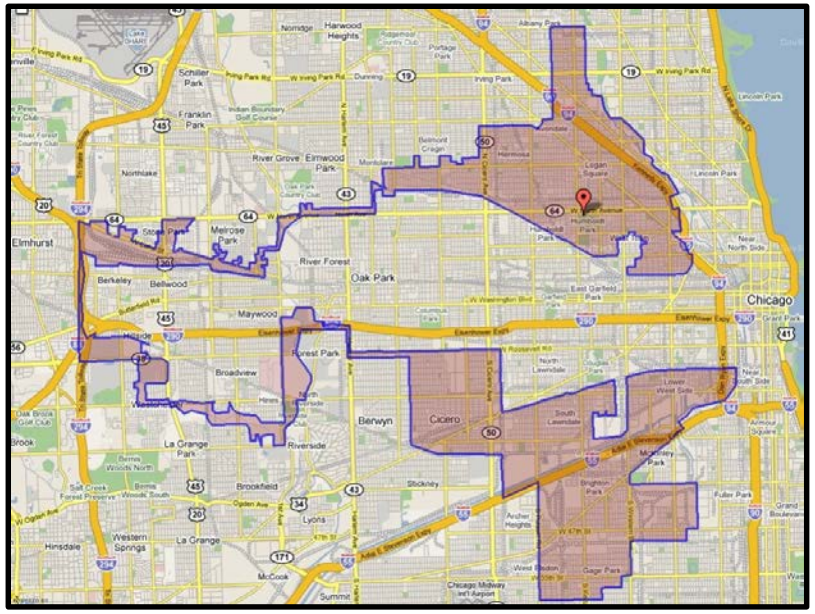
Who does the redistricting?

- 42 states give legislature primary control (includes 5 single district states)
  - 2 of these 42 (OH and RI) appoint advisory commissions to help the legislature
  - 2 of these 42 (CT and IN) have backup procedures if legislative process fails
- The other 8 states use commissions (includes 2 single district states)
  - HA and NJ use politician commissions
  - AL, AZ, CA, ID, MT and WA use independent commissions

Gerrymandering

"the intentional manipulation of territory toward some desired electoral outcome"

Example: IL 4



Measuring compactness using Polsby-Popper

$$\frac{area\ of\ district}{area\ of\ circle\ with\ same\ perimeter} = \frac{4\pi A}{P^2}$$

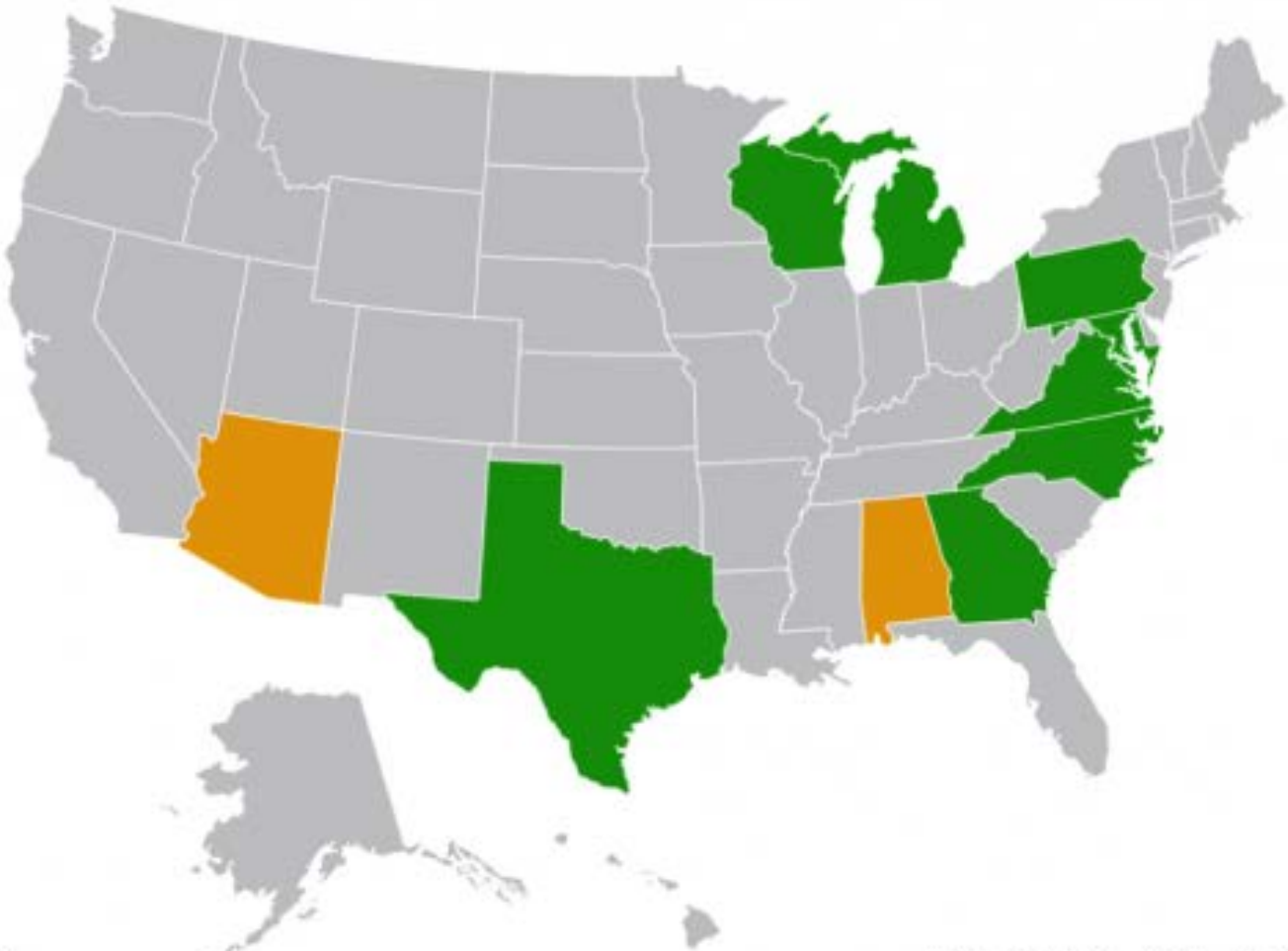
One of many measures based on the ***Isoperimetric Theorem***

Value always between 0 (bad) and 1 (good)

Representative: Luis Gutierrez (D)
Has won 13 elections; always with at least 75% of the vote & often unopposed

A = 39.43 square miles & P = 116 miles
Rearrange perimeter into circle; circle has area ~1071 square miles
Polsby-Popper = 0.037

Where Redistricting Cases Are Still Pending or Recently Resolved



Pending
Resolved

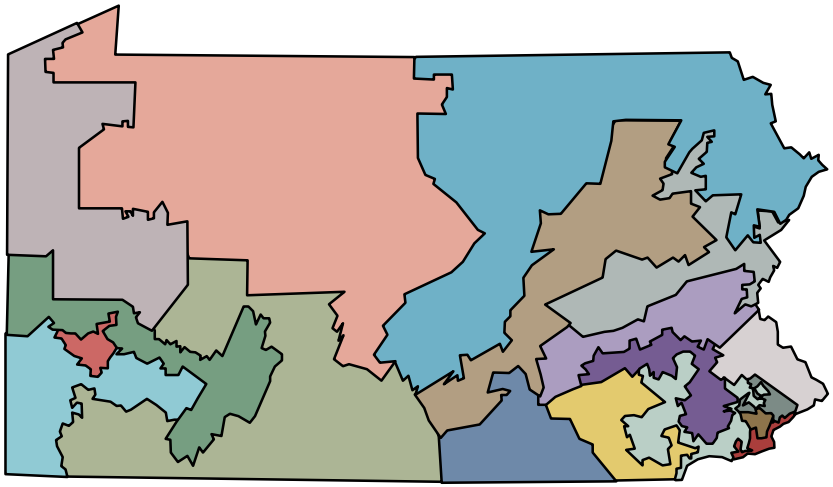
February 2018

BRENNAN CENTER
FOR JUSTICE
TWENTY YEARS

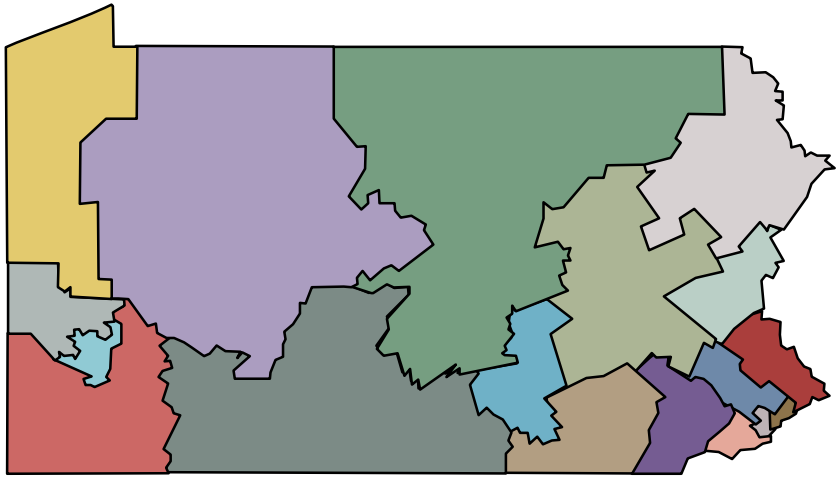
Changes in Pennsylvania congressional districts to address charge of *partisan* gerrymandering

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Congressional districts, 2013-2017



Congressional districts, mandated by PA Supreme Court in 2018



MATHEMATICAL FRONTIERS
Mathematics of Redistricting

Jonathan Mattingly,
Duke University

Professor of Mathematics
Chair of the Department of Mathematics
Professor of Statistical Science
Duke University

Quantifying Gerrymandering

Revealing Geopolitical Structure through Sampling

Impact of Duke Team's work

Common Cause v. Rucho *(N.C. Congressional)*:

- *3 judge conditional panel. Direct appeal to SCOTUS. Nov 2017*
- *Provided expert testimony and report in lawsuit*
- *Heavily cited in court judgment*

2014 - present
(arXiv:1410.8796 - arXiv:1801.03783)

Gill v. Whitford *(WI State Assembly) :*

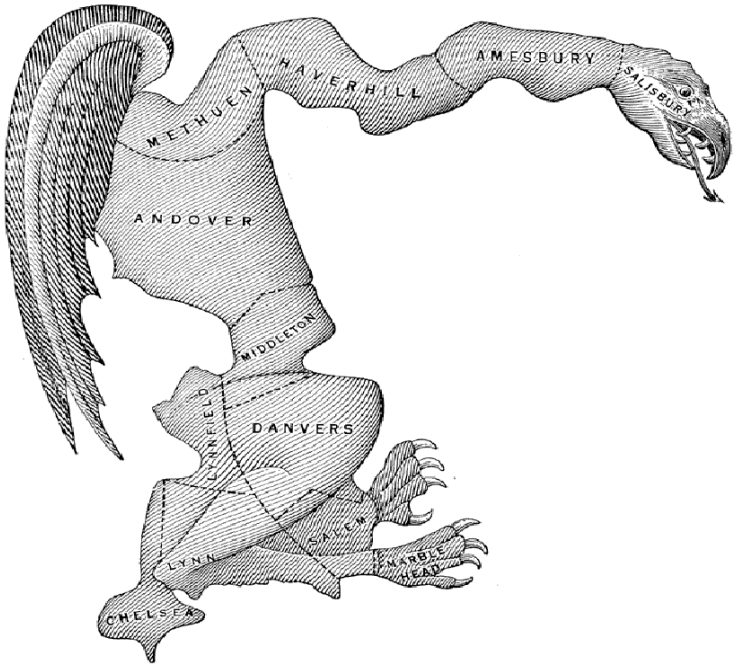
- *Oral argument held in Supreme Court (SCOTUS) October 3, 2017*
- *Provide report supporting Amicus Brief by Eric S. Lander*

North Carolina v. Covington *(N.C. State Assembly)*:

- *3 judge panel rule racial gerrymander. Affirmed by SCOTUS in June*
- *Provide expert testimony on new maps produces at courts order*

sites.duke.edu/quantifyinggerrymandering

Definition of Gerrymandering



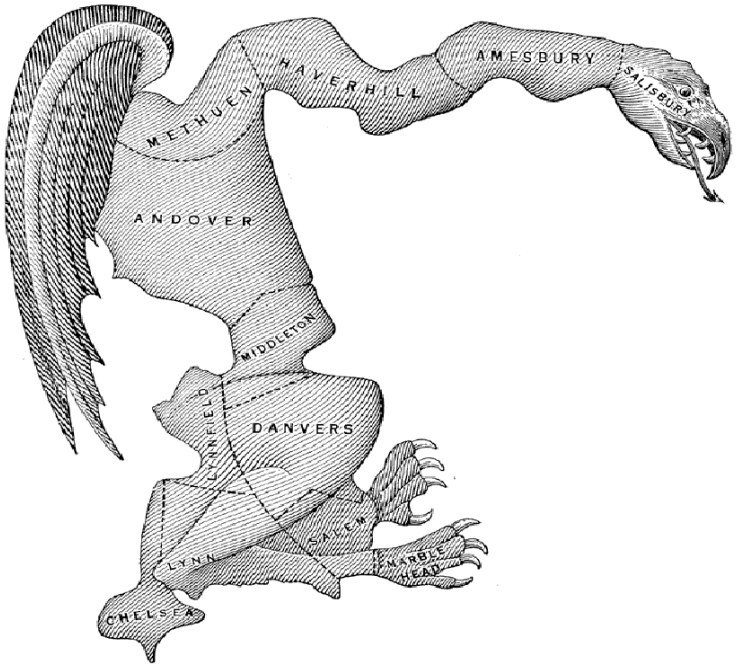
Gerrymander

- **Manipulate** district boundaries to favor one party (partisan) or class (racial)
- **Change** the outcome of an election

"gerrymander the results"

Implies the existence of an expected election result

Definition of Gerrymandering



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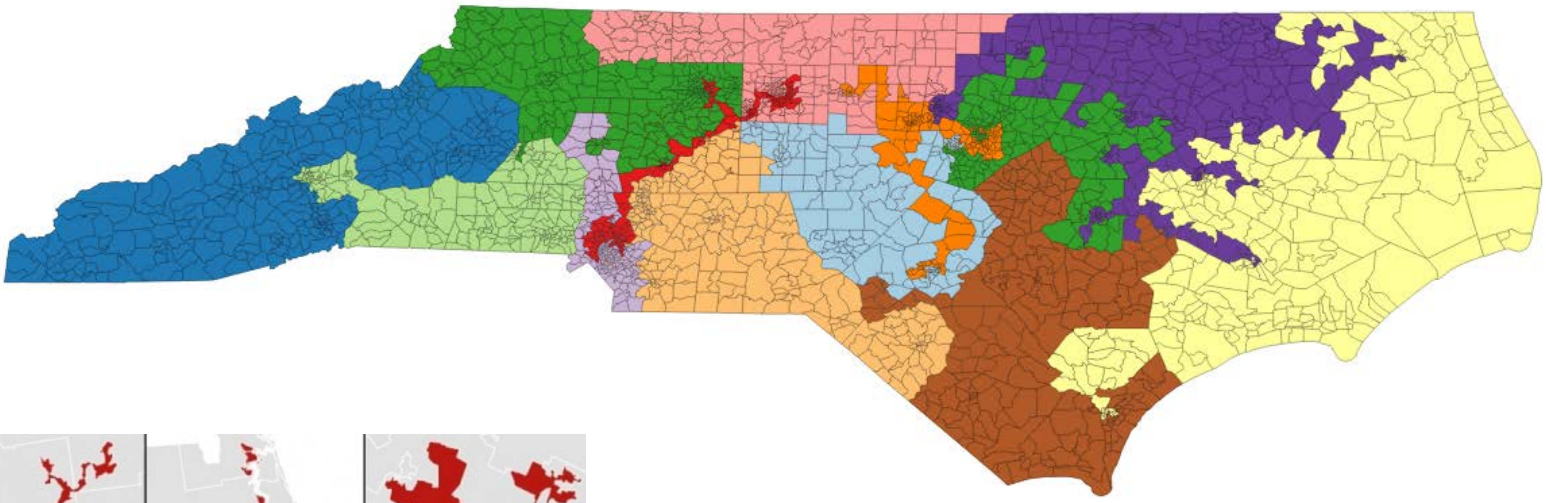
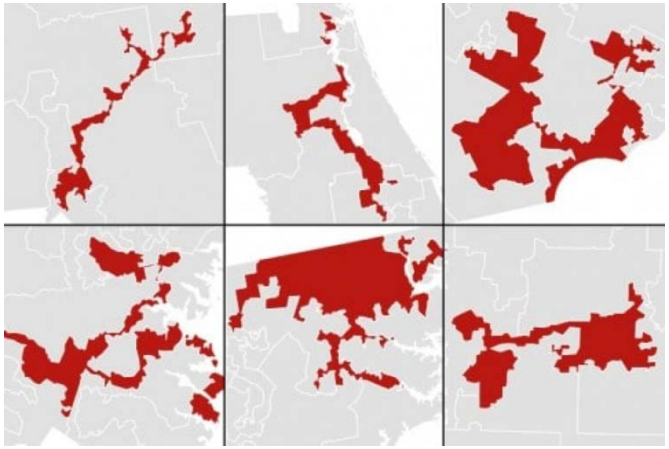
racial vs *partisan* *gerrymander*

Implies the existence of an expected election result

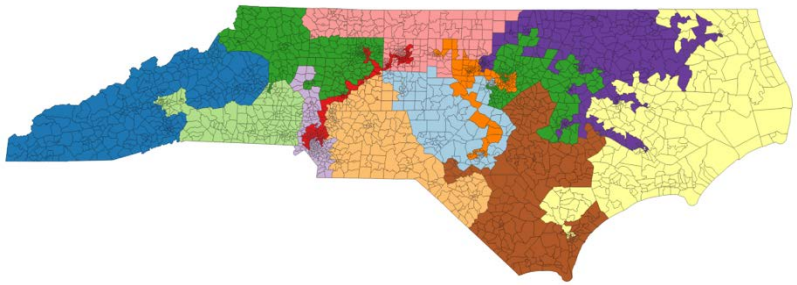
NC 2012
US Congressional Districts



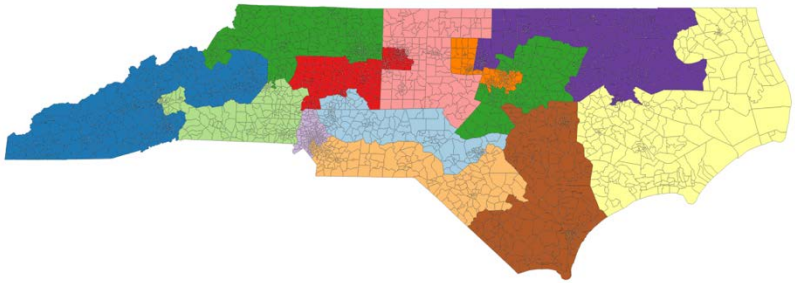


Is Gerrymandering Oddly Shape Districts ?

Which Doesn't Belong ?



Same



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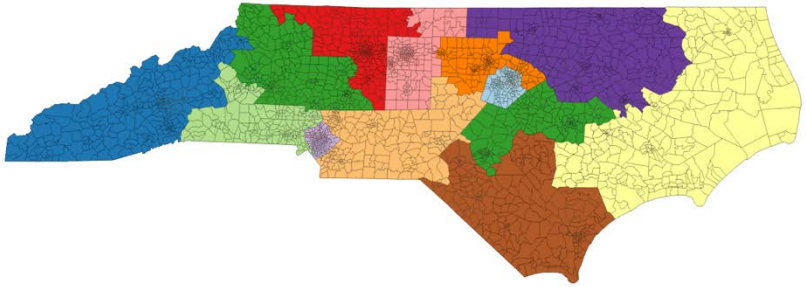

Different

Gerrymandering as "Startling" Election Results

NC : US House 2012

	Vote	Seats
Democratic	50.65%	4 (31%)
Republican	48.80%	9 (69%)

WI : Gen Assembly 2014

	Vote	Seats
Democratic	51.28%	36 (36%)
Republican	48.72%	63 (64%)

MD : US House 2012

	Vote	Seats
Democratic	63%	7 (87.5%)
Republican	33%	1 (12.5%)

USA : US House 2012

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Deviation from some expectation of symmetry

- The most Democratic district had 79.63% Democratic votes
- The most Republican district had 63.11% Republican votes.

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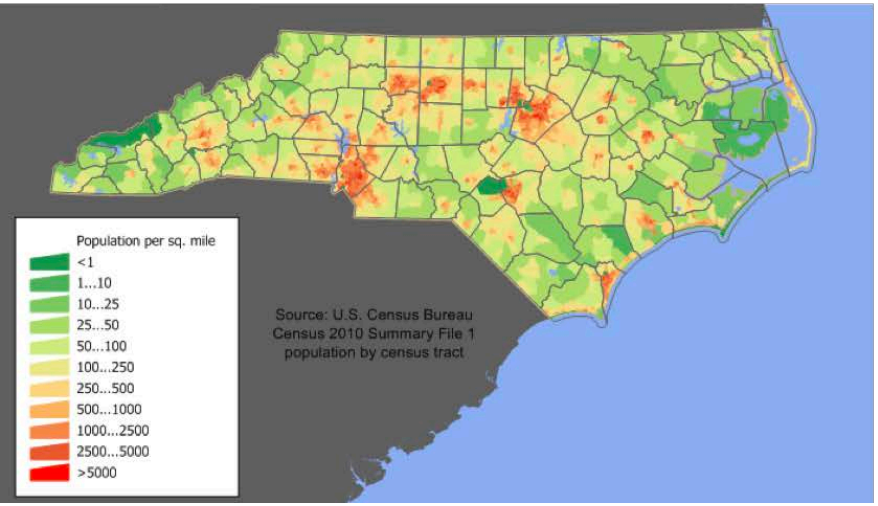
What is fair or correct ?

U.S. Not a Proportional Representation System

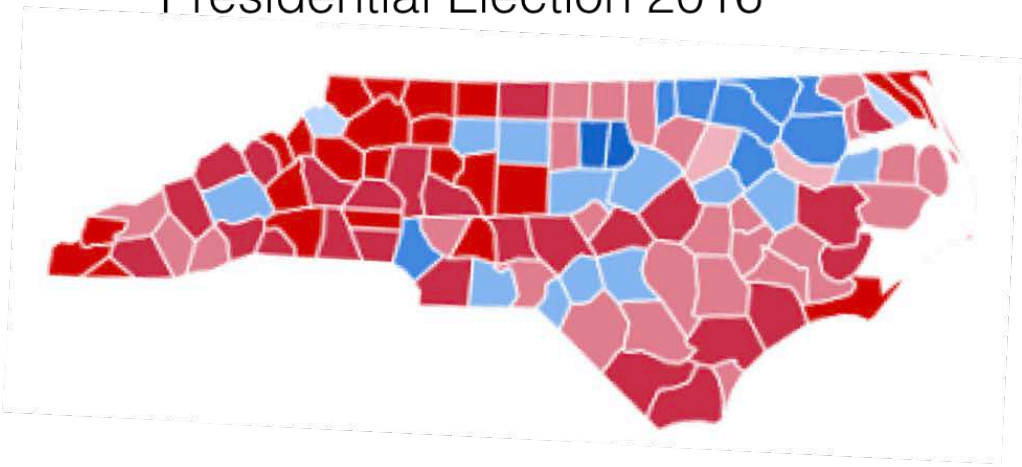
Geographically Localize Representation

Geographically Diverse

Population Density



Presidential Election 2016



States are inherently inhomogeneous and not symmetric

How to reveal a state's natural geopolitical structure?

What if we drew the districts randomly?

with no regard for party registration or most demographics

Look for the “likely” behavior of an ensemble of districting plans

create a null-hypothesis without partisan bias

A number of Groups using algorithmic generated maps to benchmark

- Jowei Chen (Michigan), Jonathan Rodden (Stanford)
- Wendy Cho (UIUC)
- Kosuke Imai, Benjamin Fifield (Princeton)
- Alan Frieze, Wesley Pegden, Maria Chikina (CMU,Pitt)

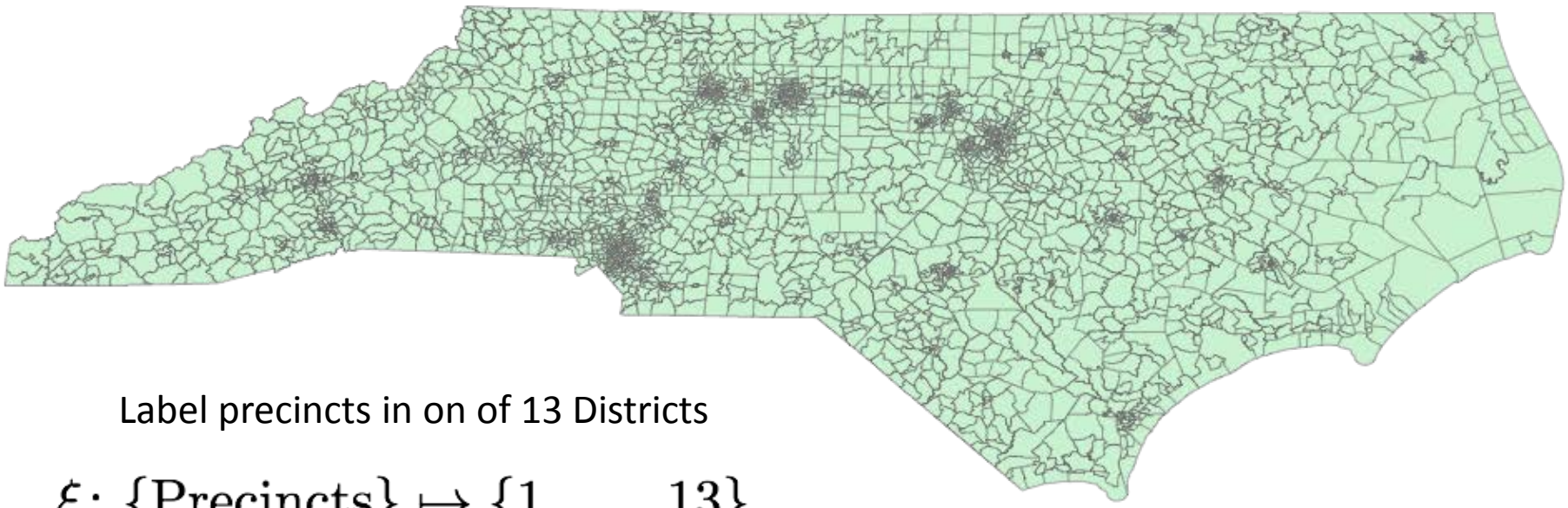
All generating alternative maps.
Some sampling a defined distribution.
Some using actual surrogate districts.

Focus on our group at Duke & N.C.
based on principled, explicit distribution on redistricting plans

N.C. HOUSE BILL 92 REDISTRICTING STANDARDS

- Districts within 0.1% of equal population (we get close)
- Districts shall be reasonably compact
- Contiguous territory, attempting not to split cities or counties
- Comply with the Voting Rights Act of 1965
- Ignore: Incumbency, party affiliation, demographics

around 3,000 in Precincts N.C.

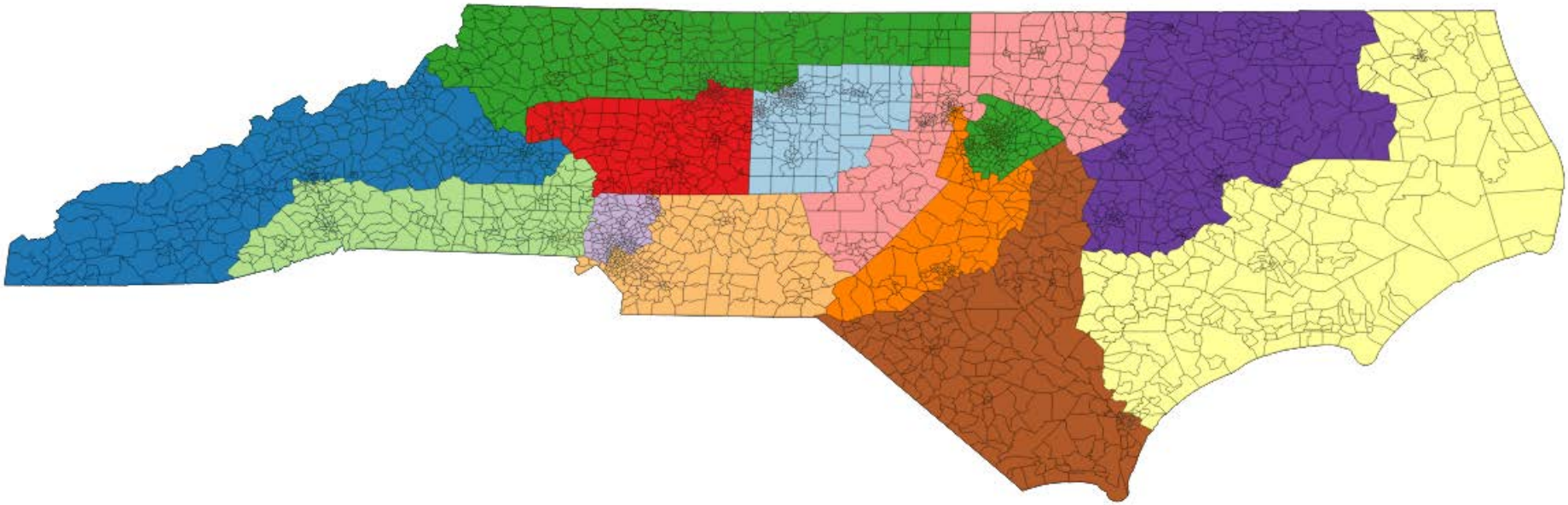


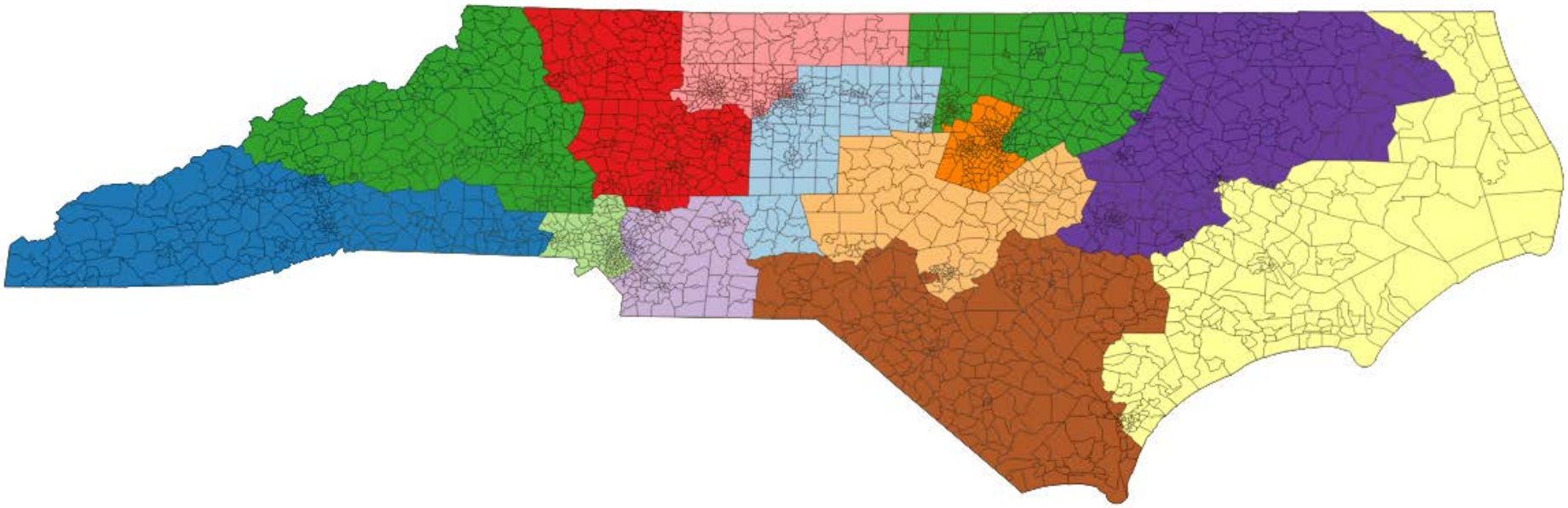
Label precincts in on of 13 Districts

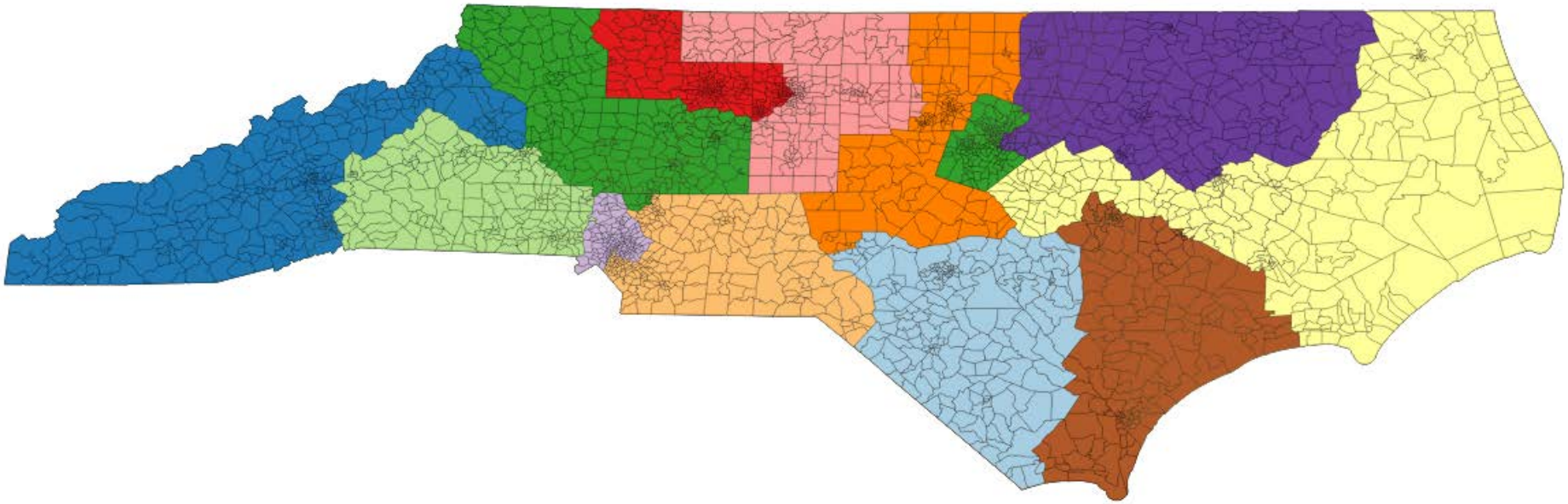
$$\xi\colon \{\text{Precincts}\} \mapsto \{1, \ldots, 13\}$$

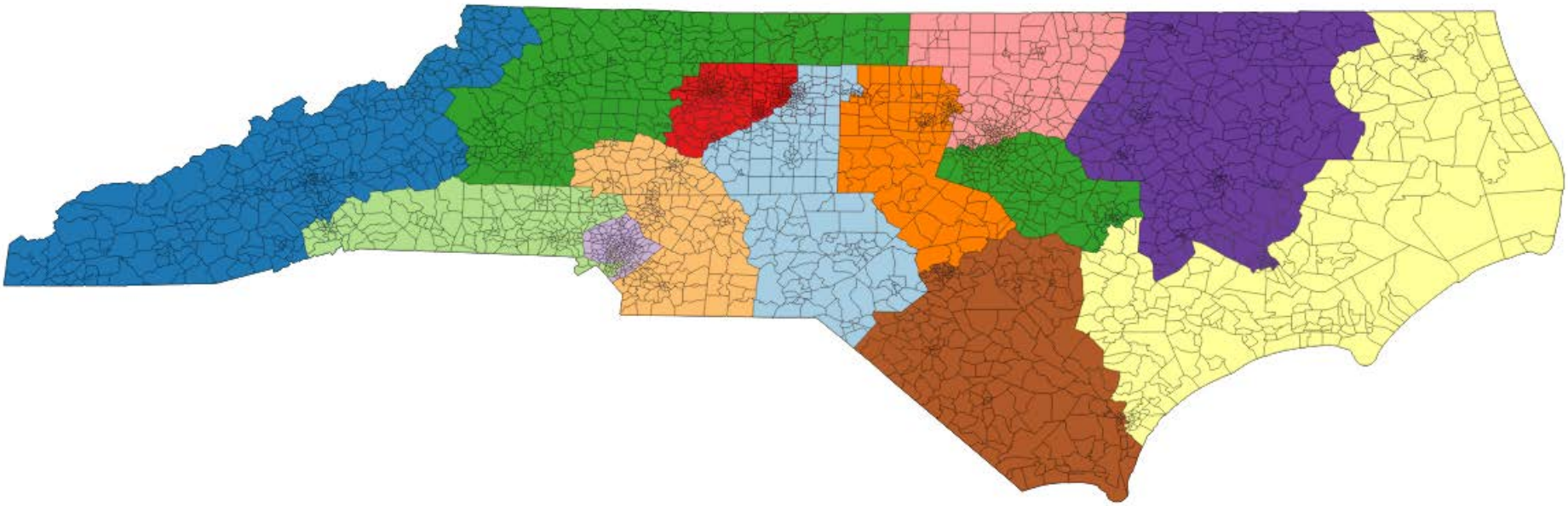
Place Distribution on admissible redistrictings: $P(\xi) = \frac{1}{Z} e^{-\beta J(\xi)}$

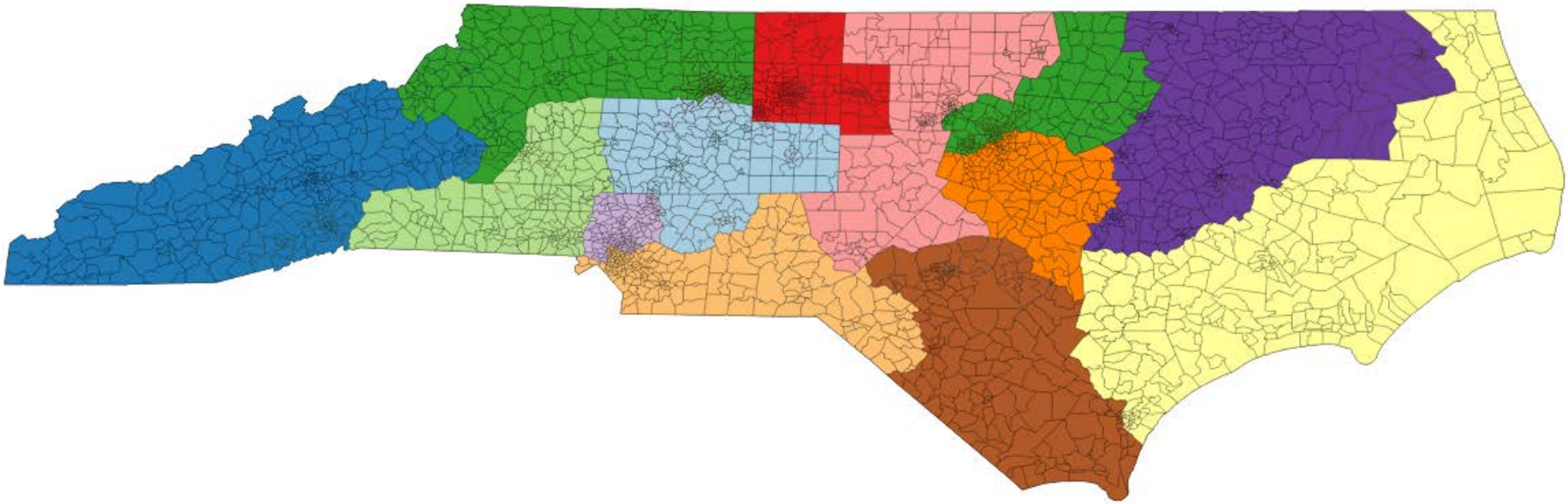
$$J(\xi) = w_p J_{pop}(\xi) + w_I J_{compact}(\xi) + w_c J_{county}(\xi) + w_m J_{mino}(\xi)$$

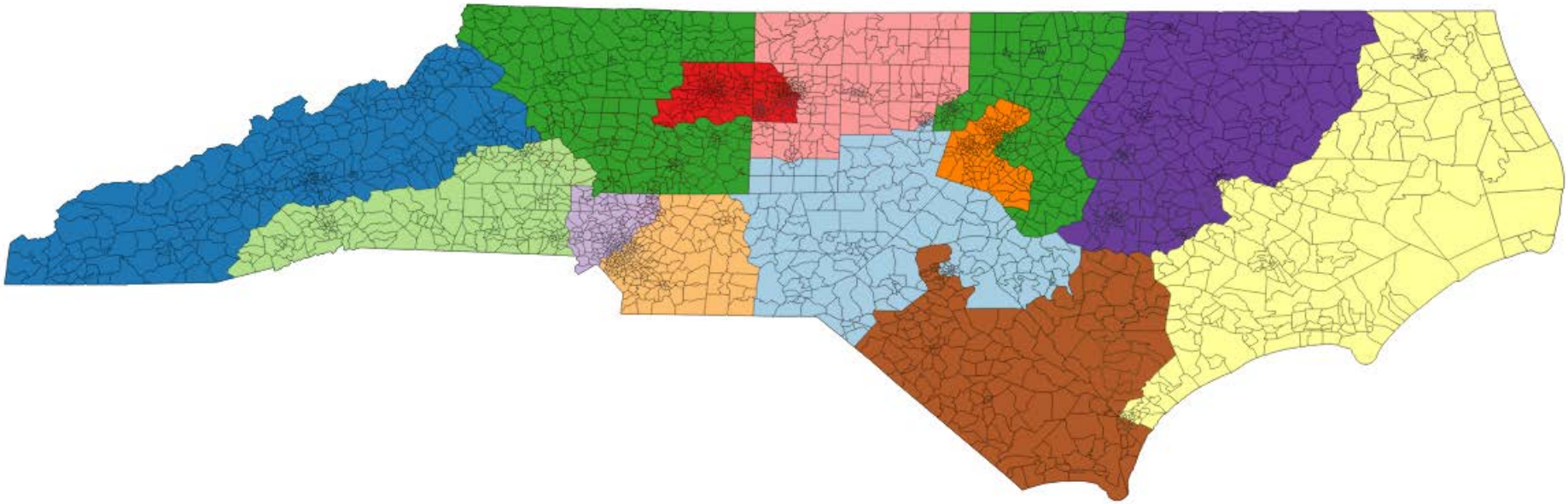


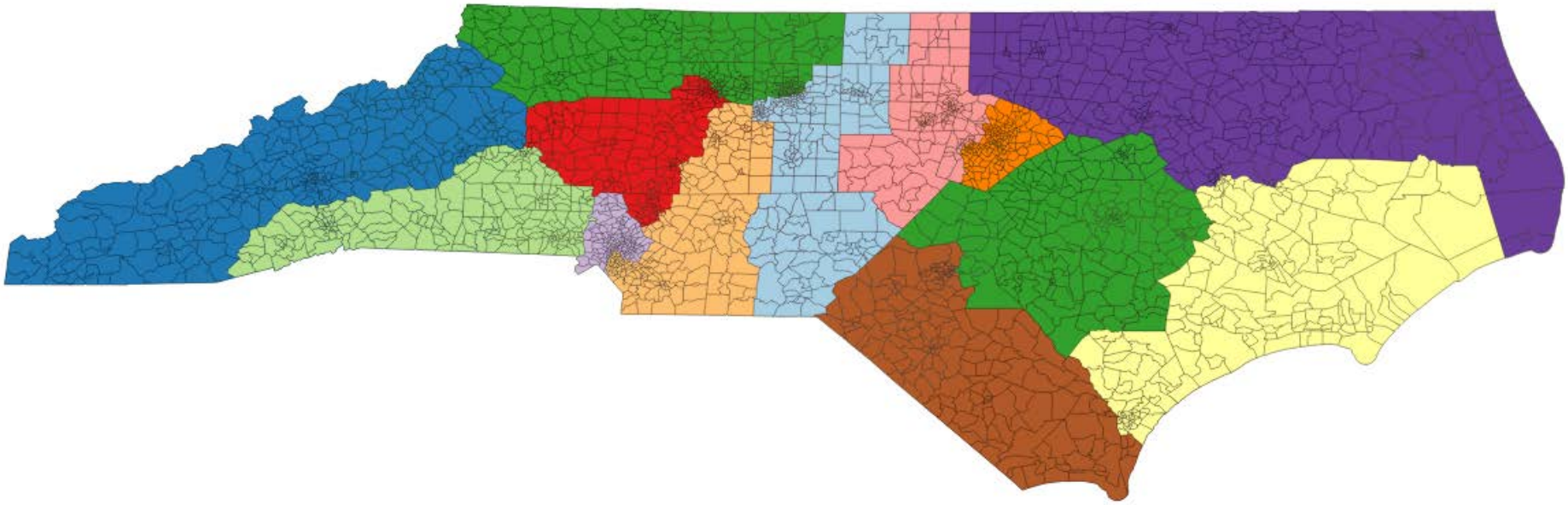


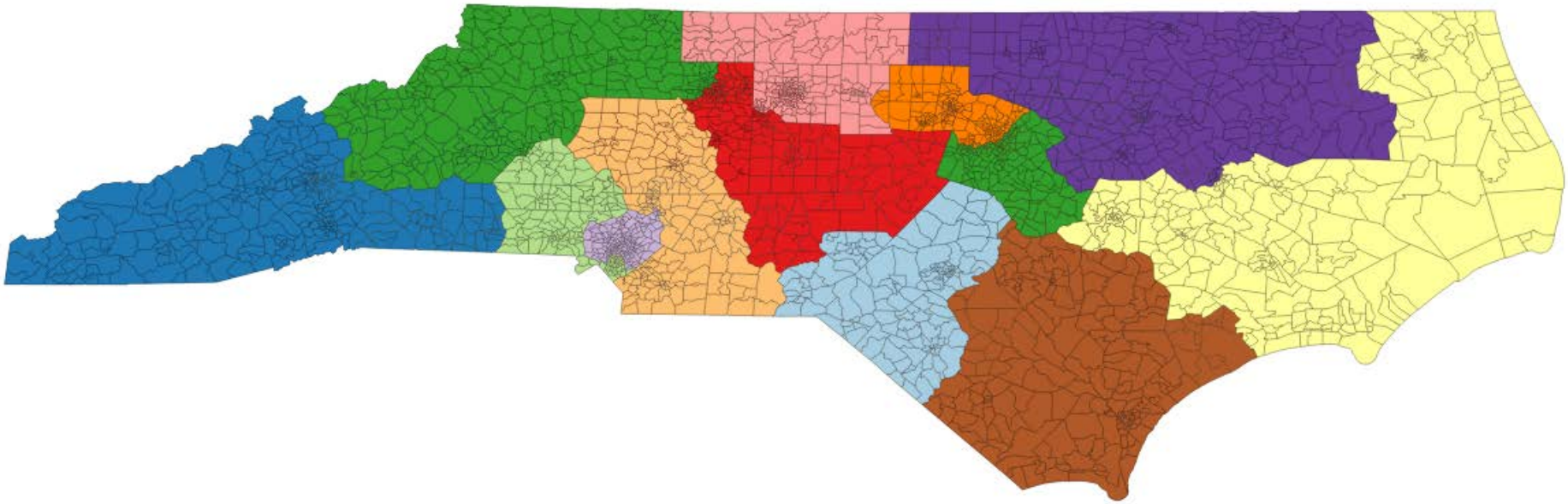


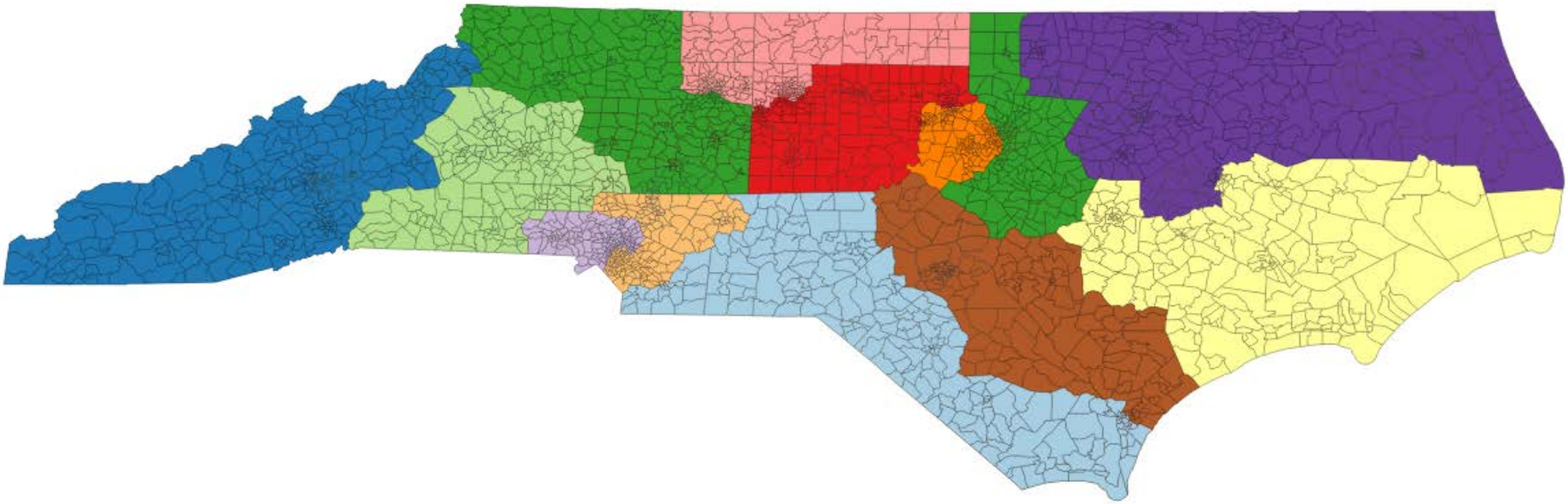




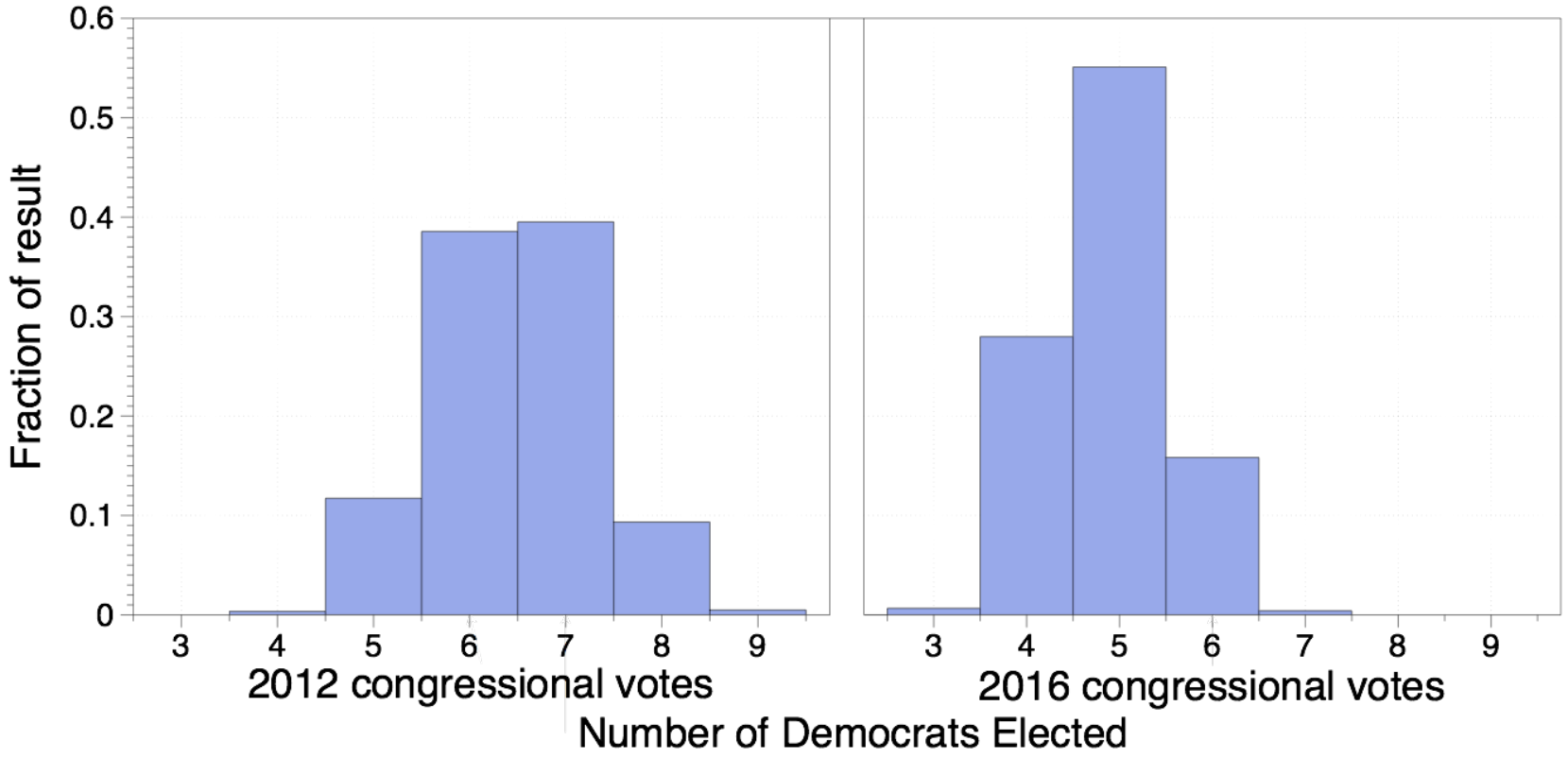




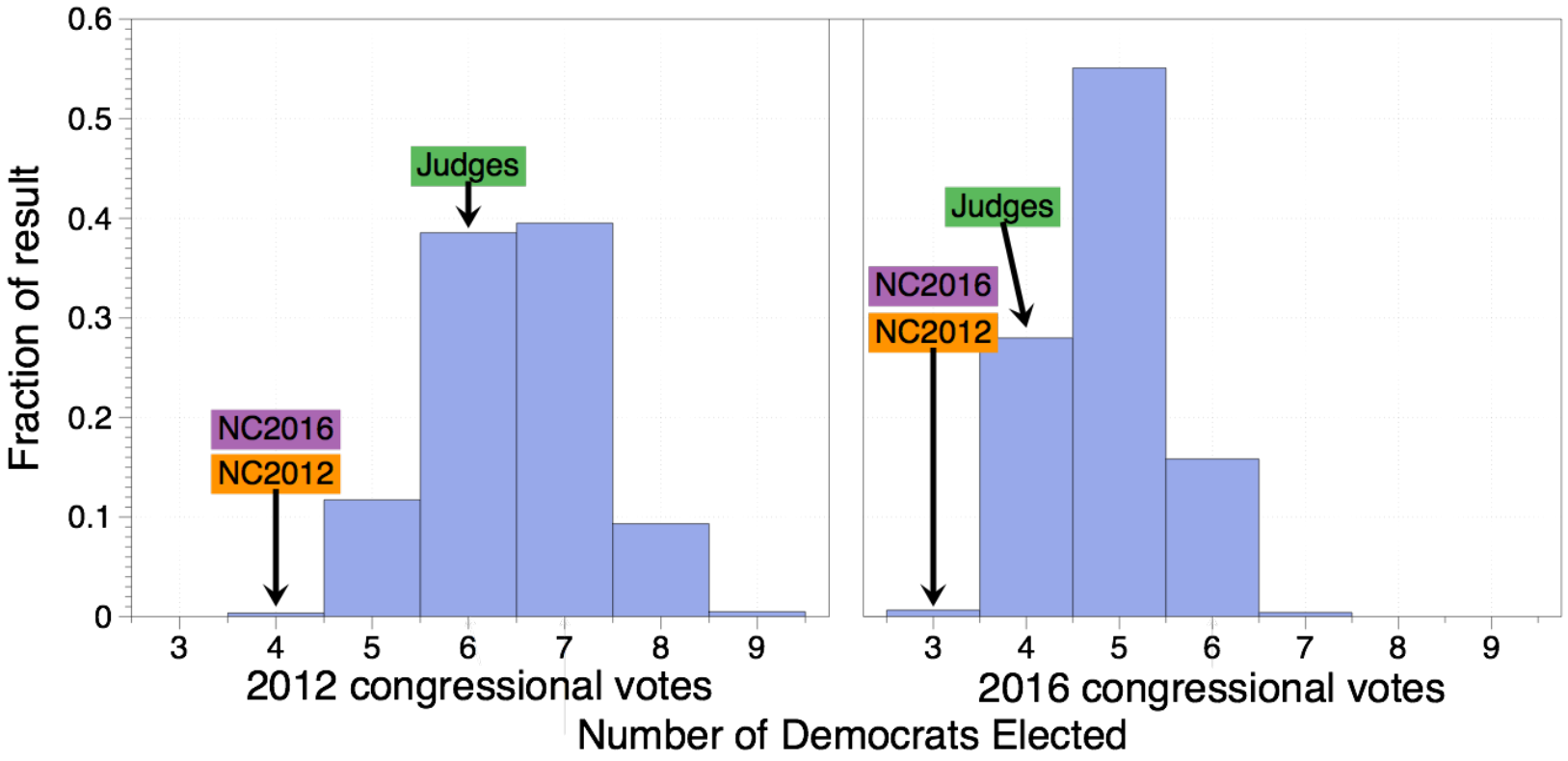




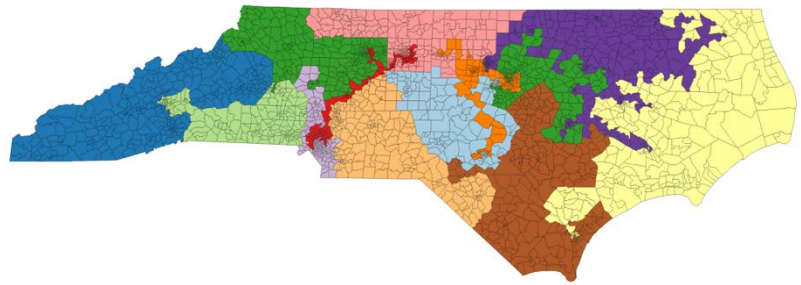
Ensemble of ~24,000 NC redistricting plans



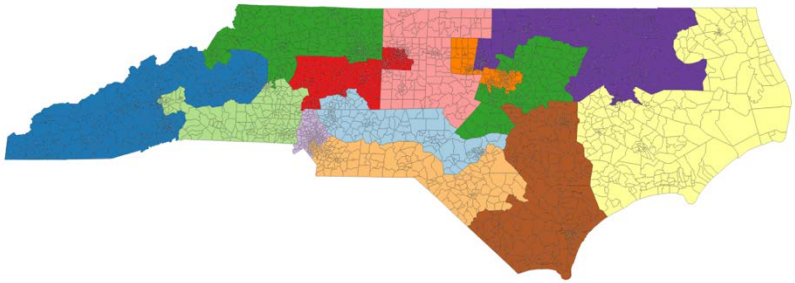
Situate maps in ensemble of 24,000 redistricting plans



Gerrymandering can occur in the absence of oddly shaped districts



Atypical



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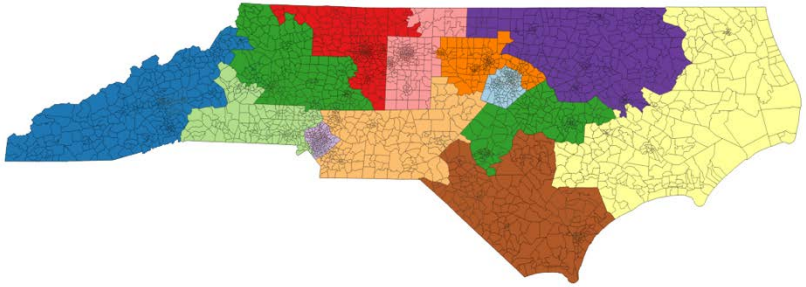
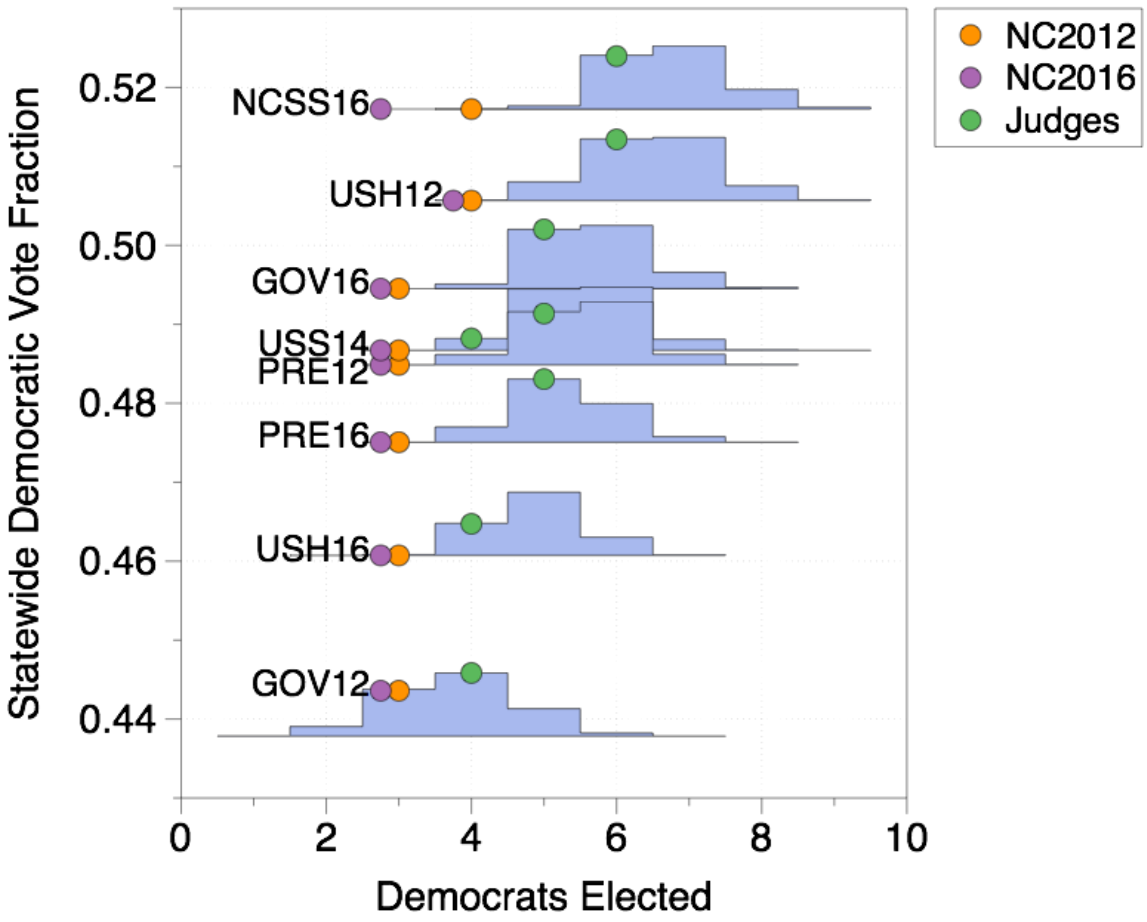
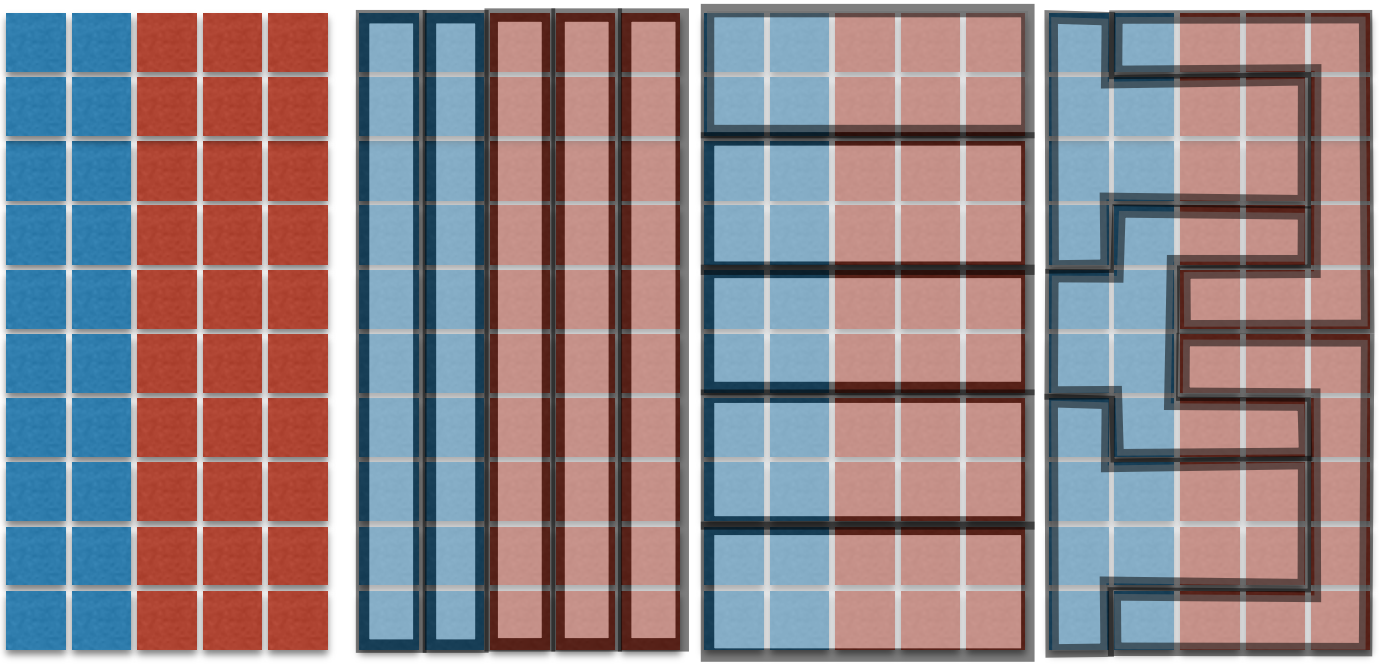


Typical

Unresponsiveness
Across many elections



Explore the structure through and example



60% Red
40% Blue

Red wins 3
Blue wins 2

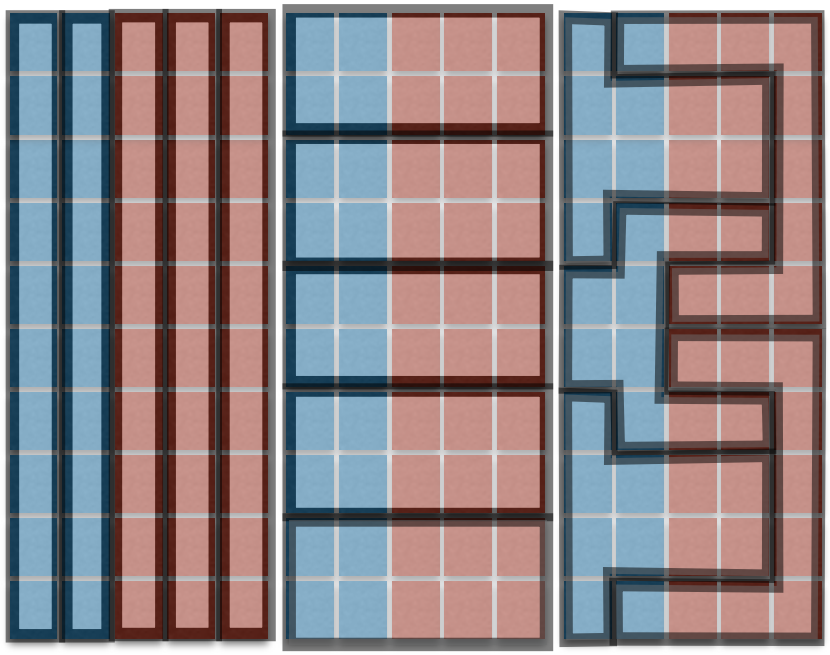
Red wins 5
Blue wins 0

Red wins 2
Blue wins 3

Wikipedia; image by Steven Nass

Wikipedia; image by Steven Nass

Packing and Cracking



Red wins 3
Blue wins 2

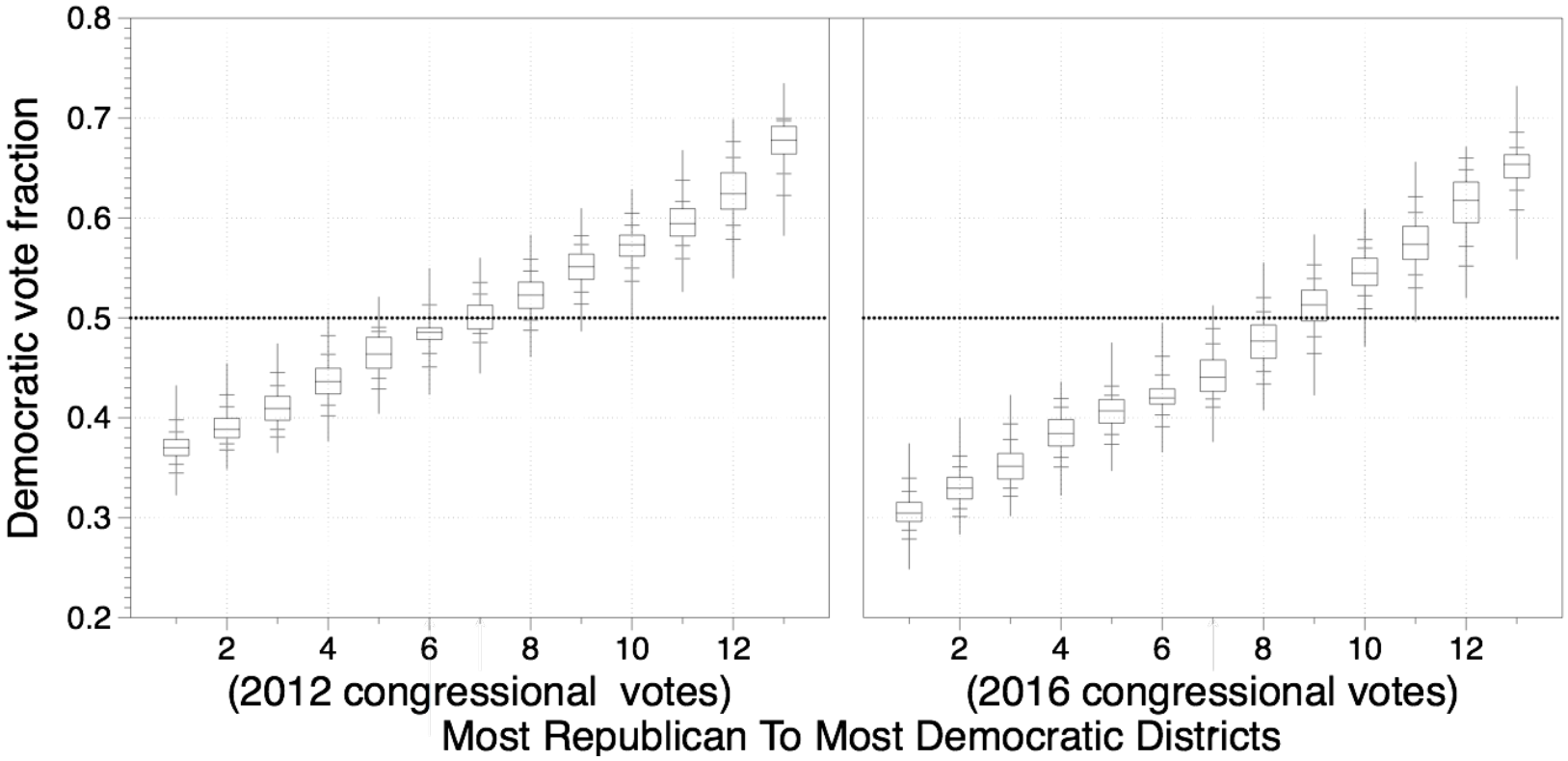
Red wins 5
Blue wins 0

Red wins 2
Blue wins 3

Percentage of Democrats from lowest to highest

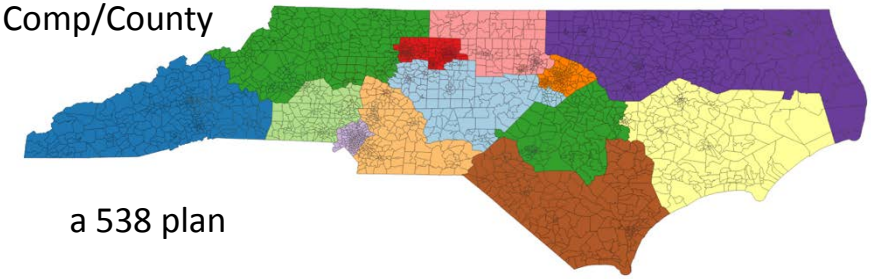
Most Red				Most Blue
10%	10%	60%	60%	60%
0%	0%	0%	100%	100%
40%	40%	40%	40%	40%

NC Congressional Delegation



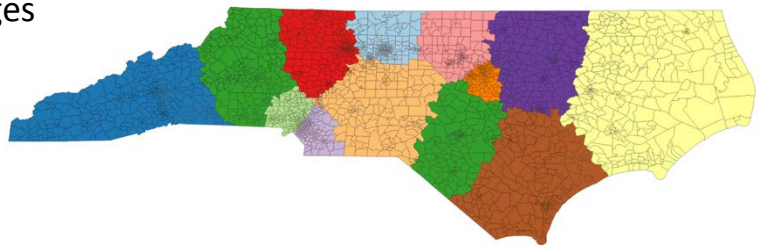
Baseline Maps

Comp/County

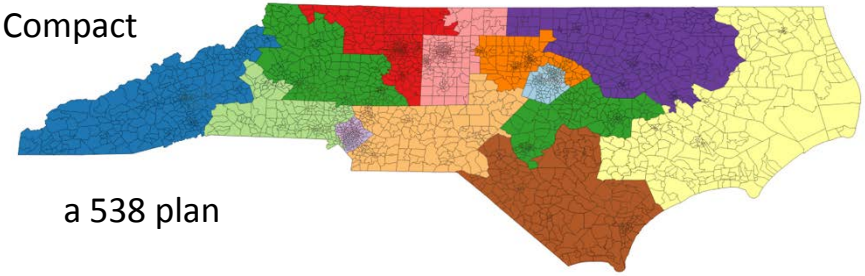


a 538 plan

Judges

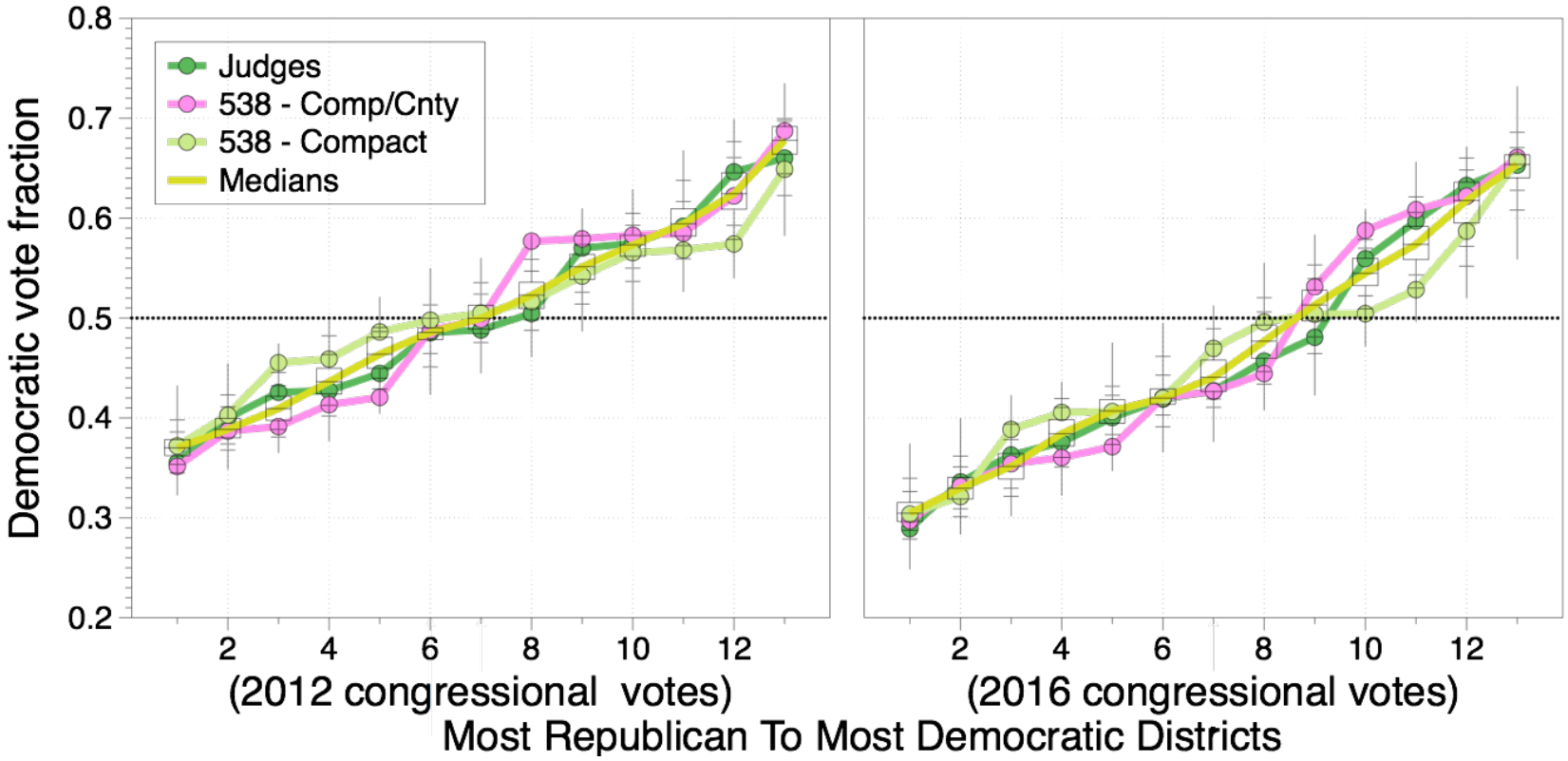


Compact

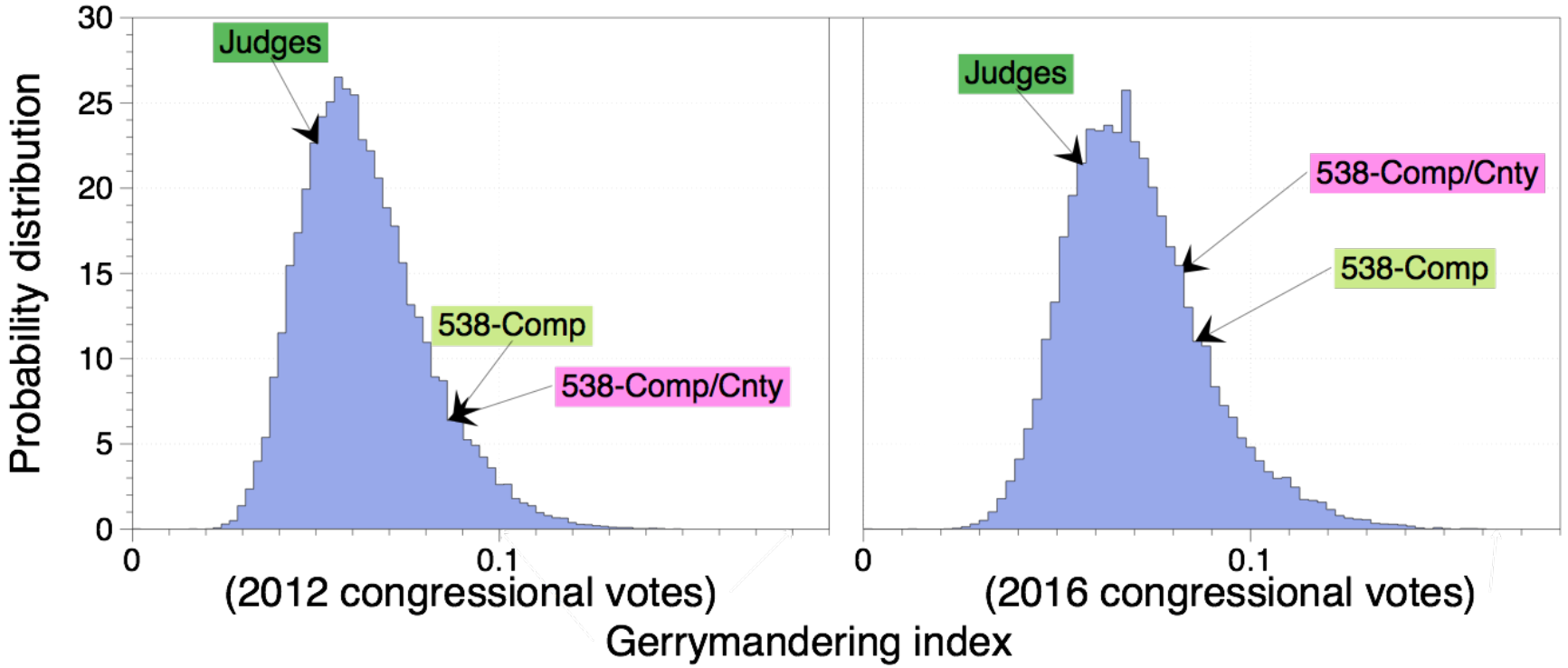


a 538 plan

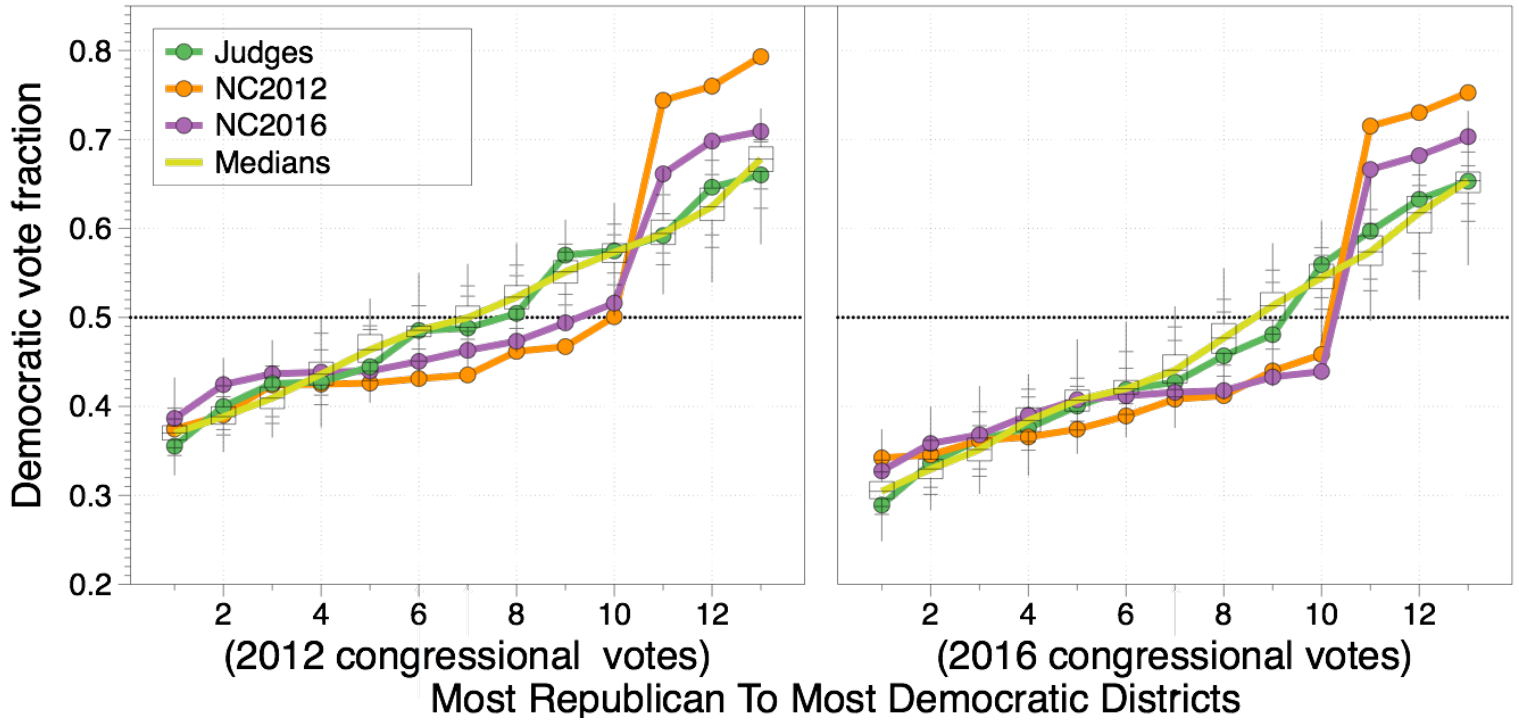
Similar to the median curve



Gerrymandering Index



NC Congressional Delegation



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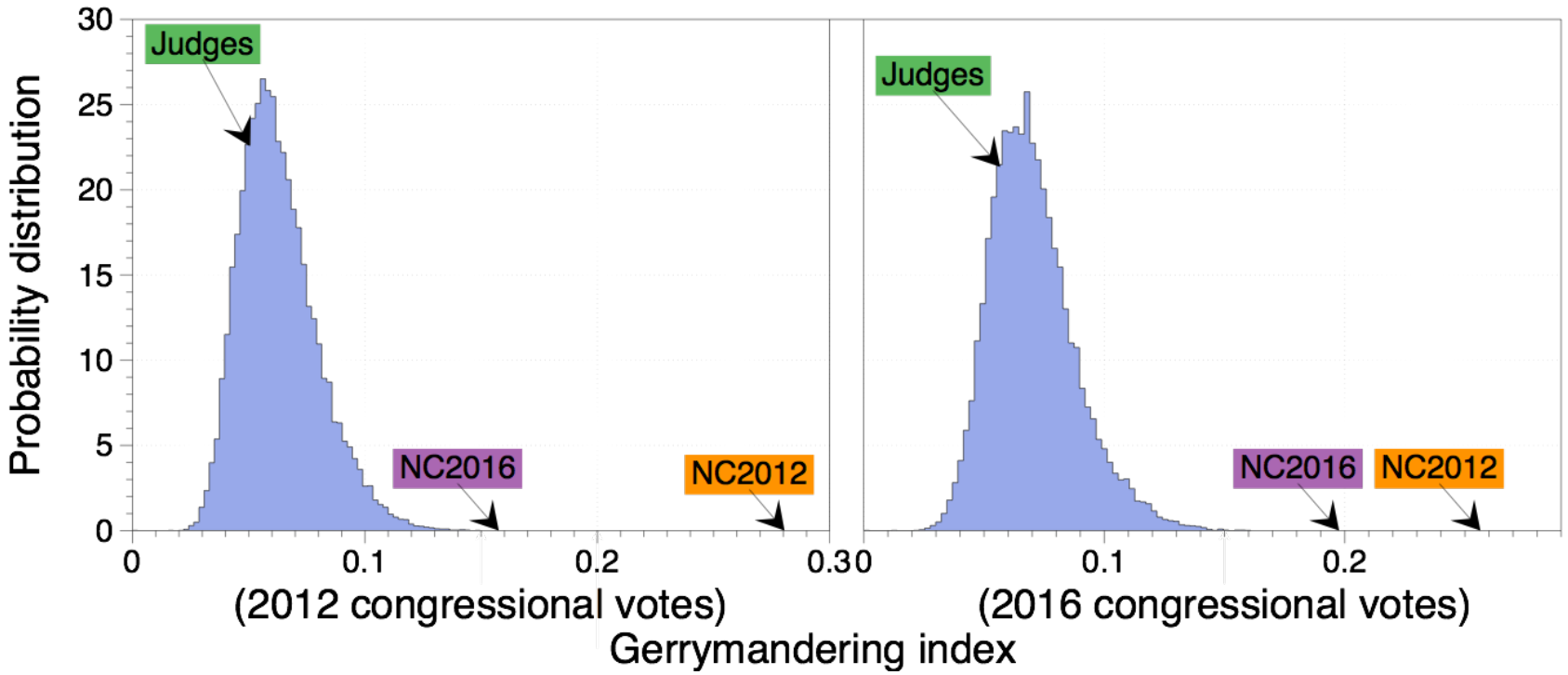
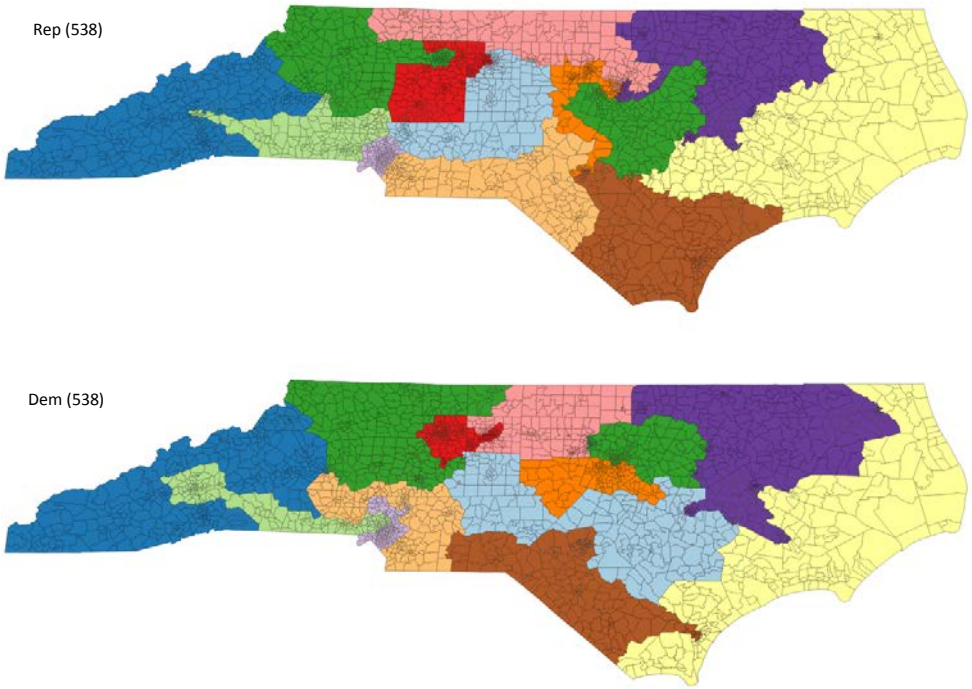
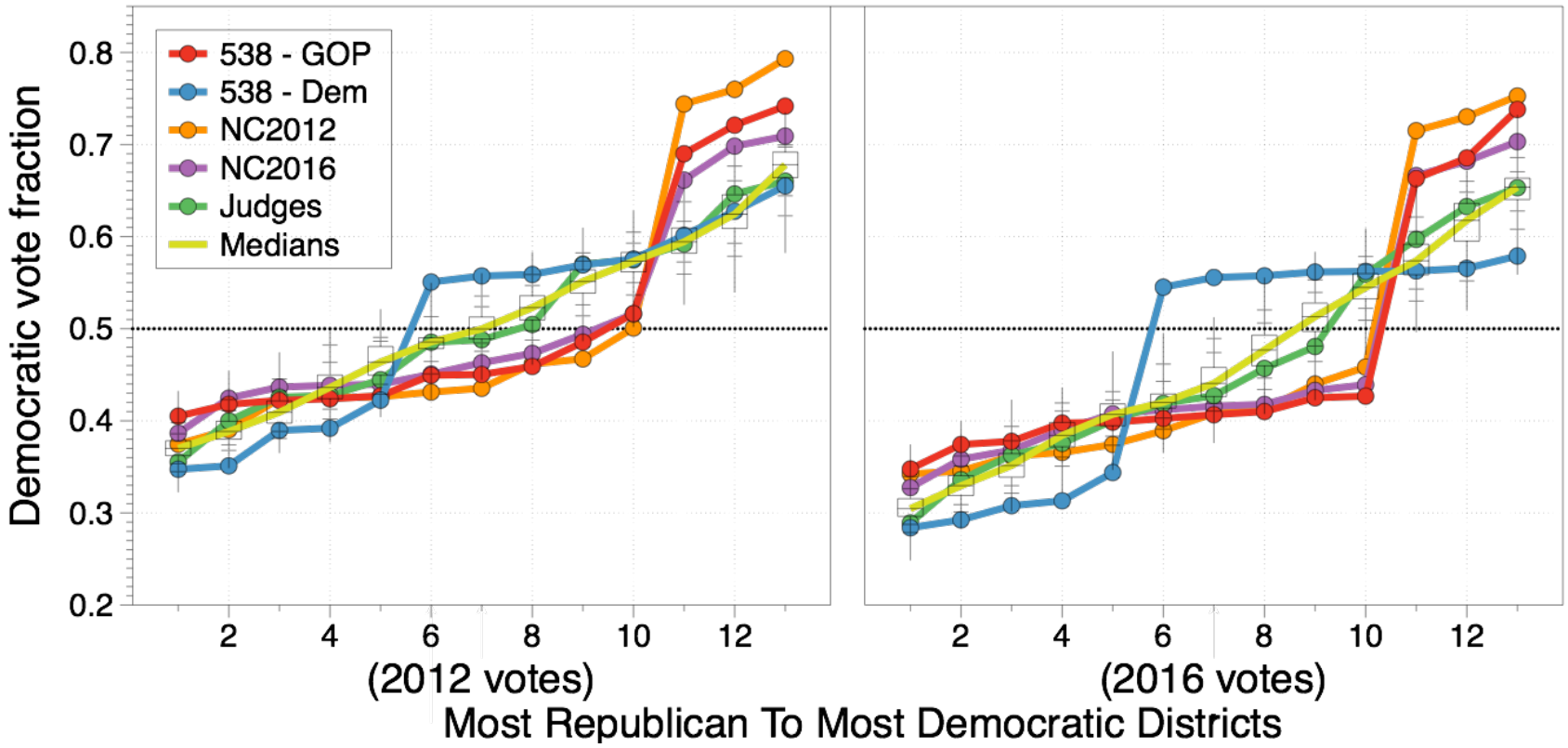


"Outlier analysis" Eric Lander's Amicus Brief in *Gill v. Whitford*

Extreme Maps



Signature of Gerrymandering



Wisconsin General Assembly

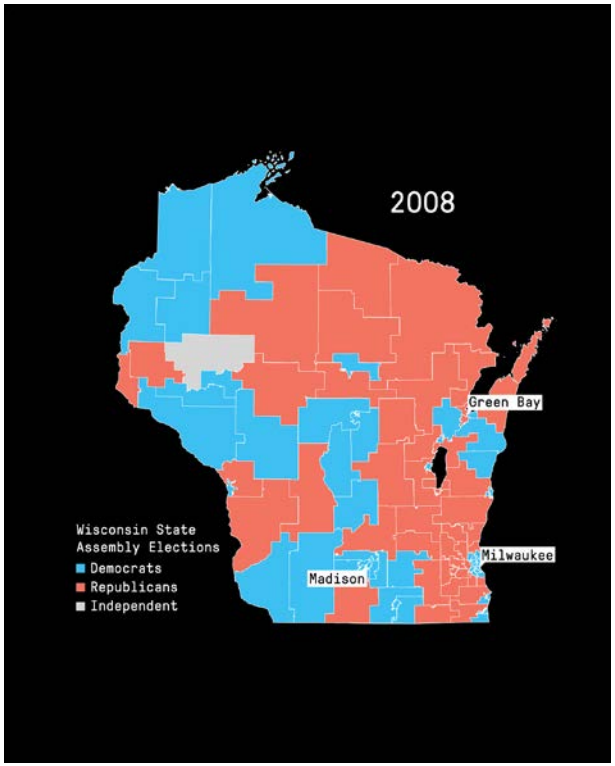
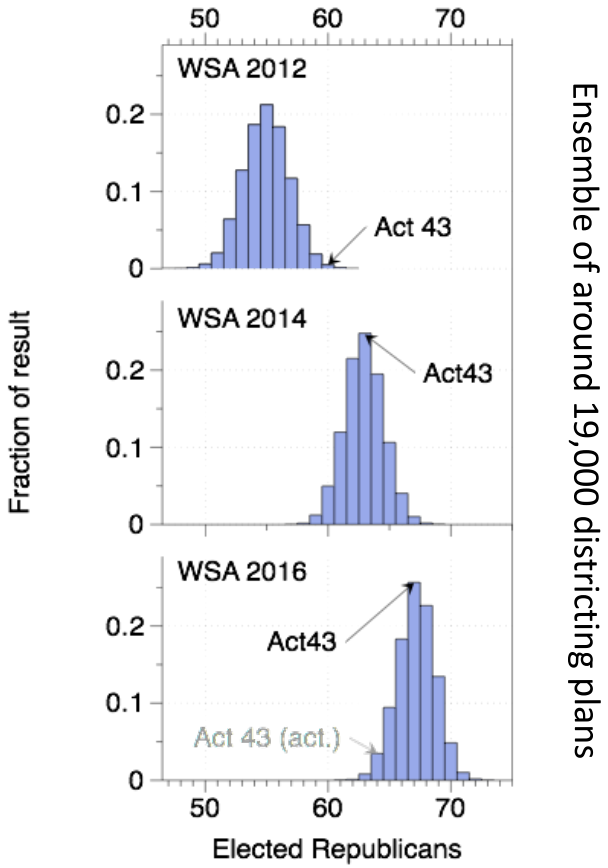
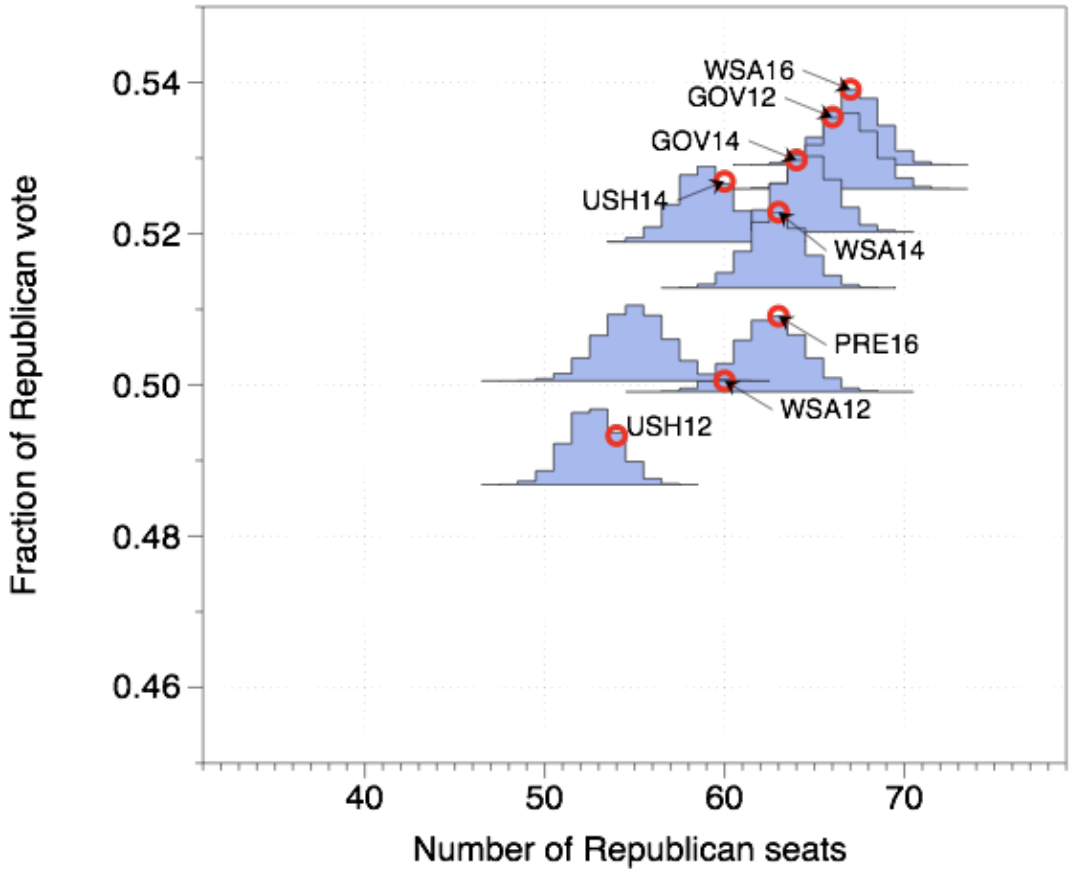


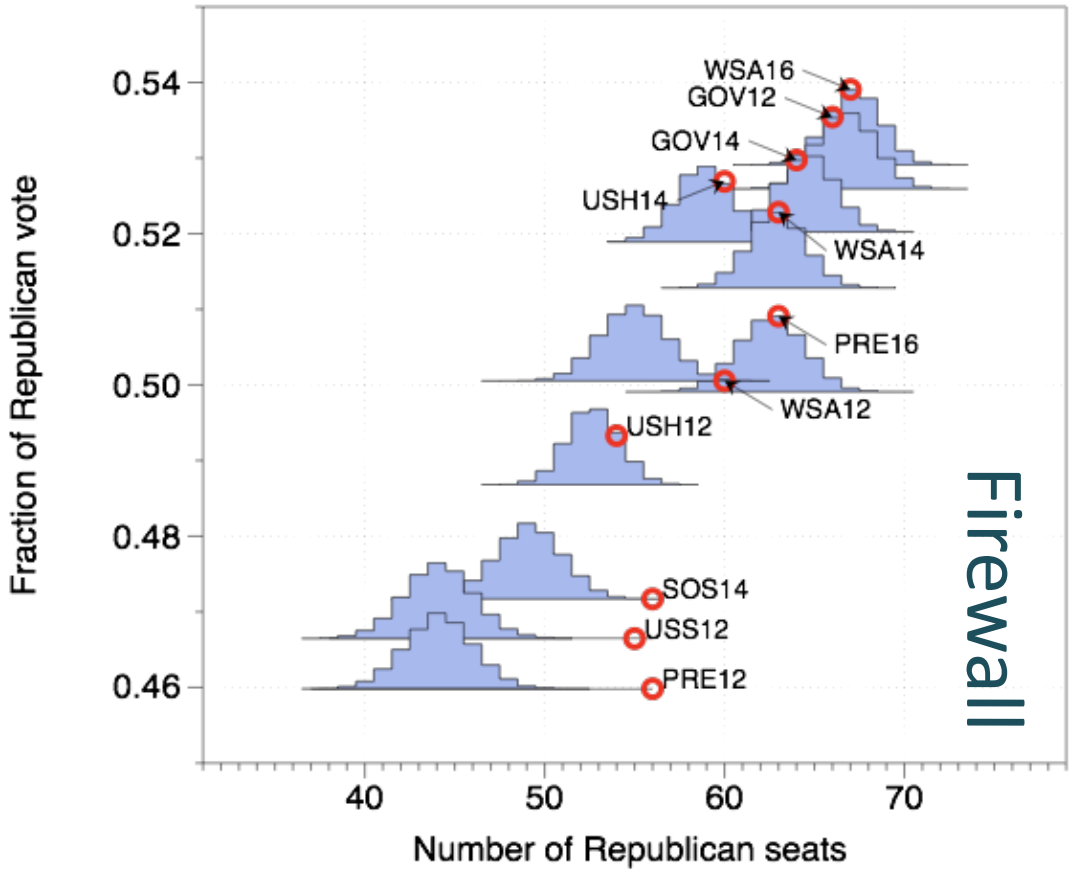
Image: NY Times



Wisconsin historical elections



Wisconsin historical elections

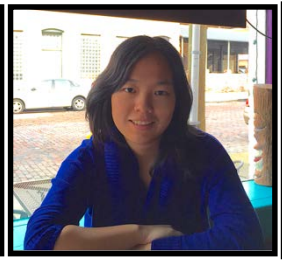


The Team


Jonathan Mattingly

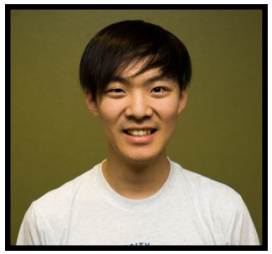

Christy Graves


Sachet Bangia


Sophie Guo


Bridget Dou


Justin Luo


Hansung Kang


Robert Ravier


Greg Herschlag


Michael Bell

2013-Present

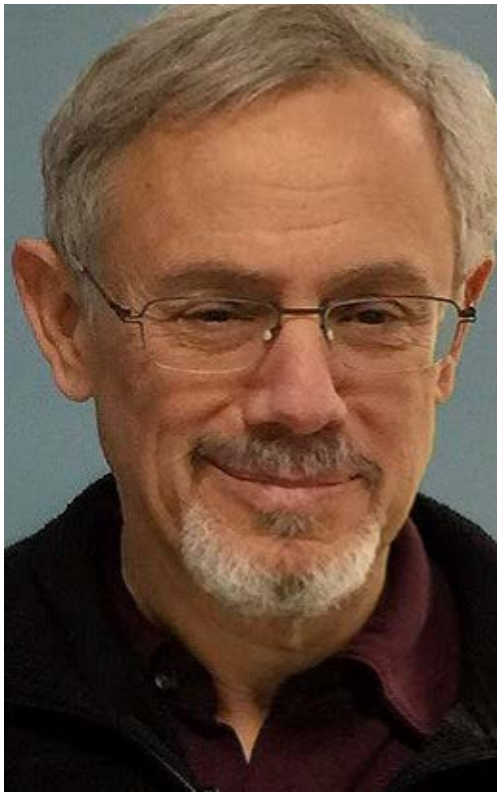
Duke UNIVERSITY MATH · Data+ · iiD · Duke UNIVERSITY | POLIS

https://sites.duke.edu/quantifyinggerrymandering/

MATHEMATICAL FRONTIERS
Mathematics of Redistricting – Q&A

Karen Saxe,
American Mathematical Society

Jonathan Mattingly,
Duke University



Mark Green,
UCLA (moderator)

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers

MATHEMATICAL FRONTIERS
2018 Monthly Webinar Series, 2-3pm ET

February 13: ***Recording posted***
Mathematics of the Electric Grid

March 13: ***Recording posted***
Probability for People and Places

April 10: ***Recording posted***
Social and Biological Networks

May 8:
Mathematics of Redistricting

June 12:
Number Theory: The Riemann Hypothesis

July 10: *Topology*

August 14:
Algorithms for Threat Detection

September 11:
Mathematical Analysis

October 9: *Combinatorics*

November 13:
Why Machine Learning Works

December 11:
Mathematics of Epidemics

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers

Common Metrics

- Efficiency Gap: (McGhee & Stephanopoulos)

$$\text{Waste} = \sum_{\text{districts won}} (\text{vote fraction} - 0.5) + \sum_{\text{districts lost}} \text{vote fraction}$$

$$\text{EG} = \text{Waste(Democrat)} - \text{Waste(Republican)}$$

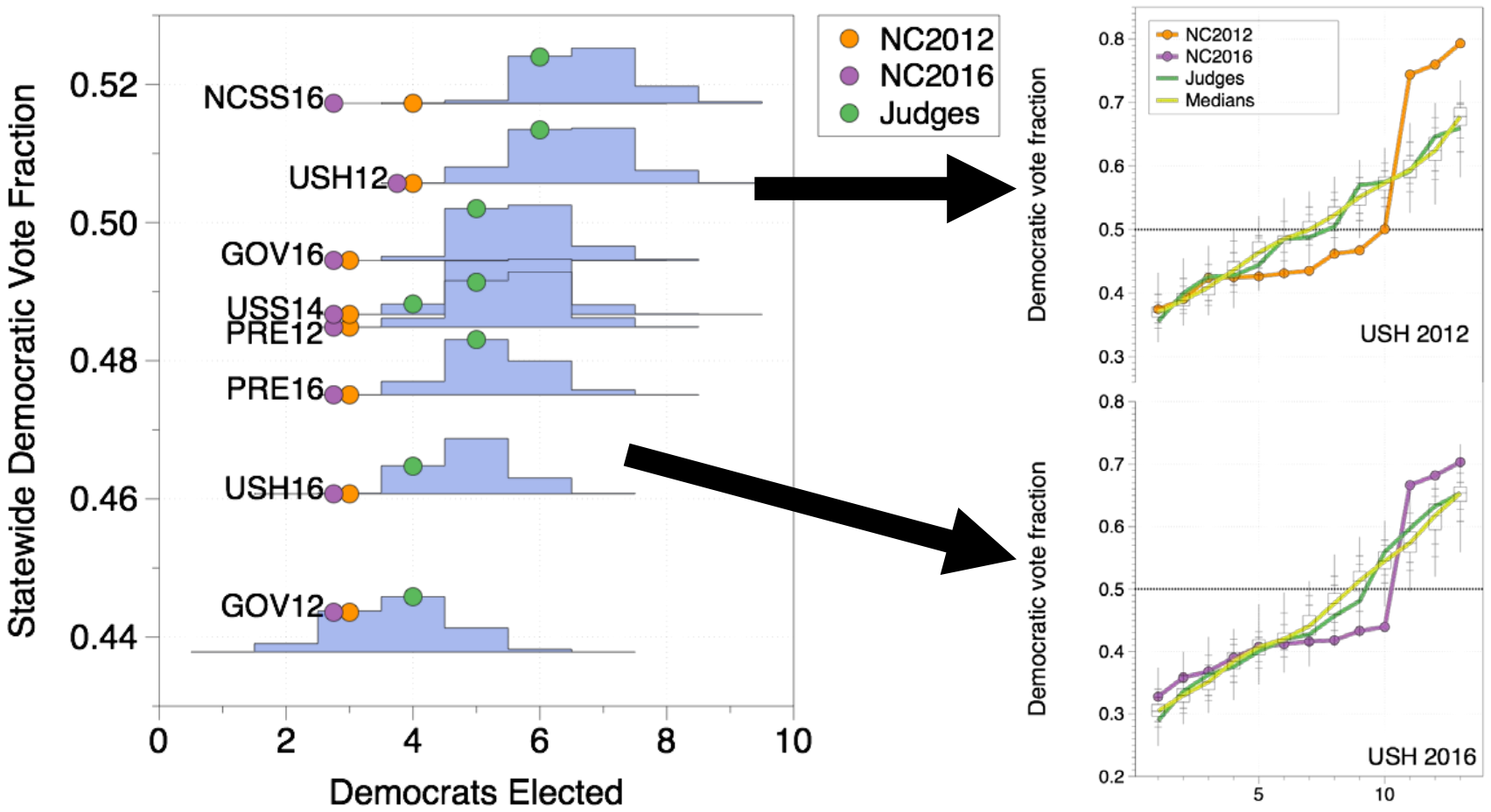
$$\approx [\text{Vote(Dem)} - \text{Vote(Rep)}] - \tfrac{1}{2}[\text{Seats(Dem)} - \text{Seats(Rep)}]$$

Bernstein & Duchin '16

- Partisan Bias: (Gelman and King)

  difference in seat fraction won by the Republicans if they receive 55% of the vote and the seat fraction won by the Democrats if they receive 55% of the vote (under partisan swing assumption).

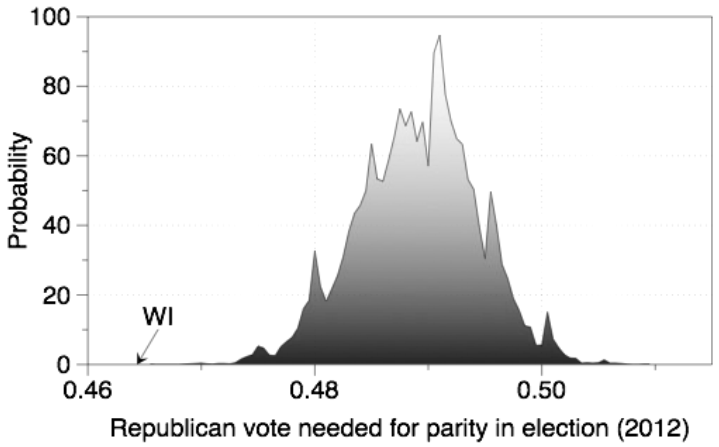
Stagnating NC election results due to Gerrymandering

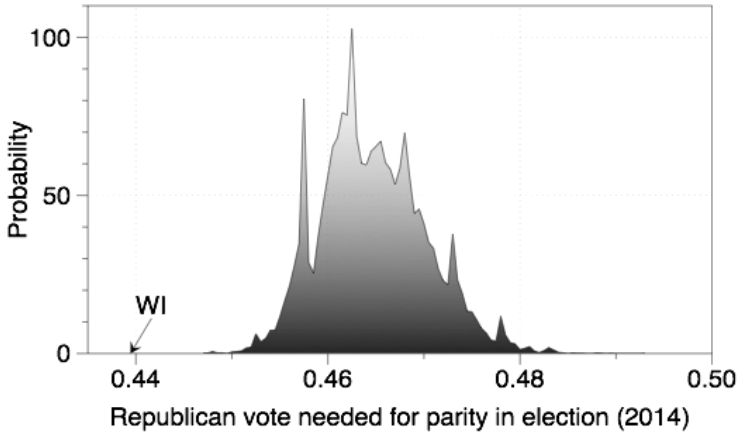


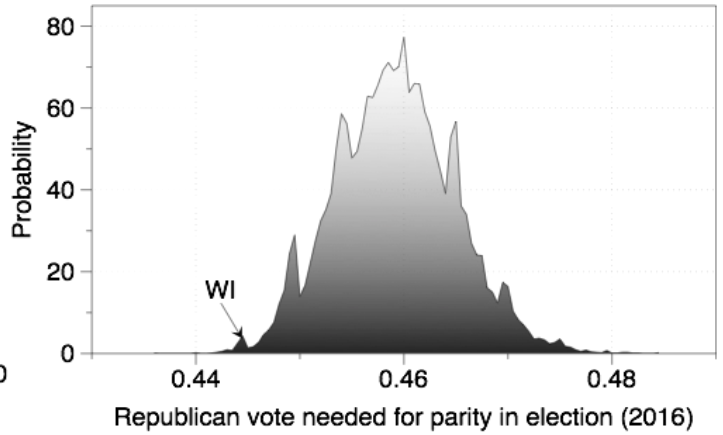
Structural advantage exists

Back to WI…

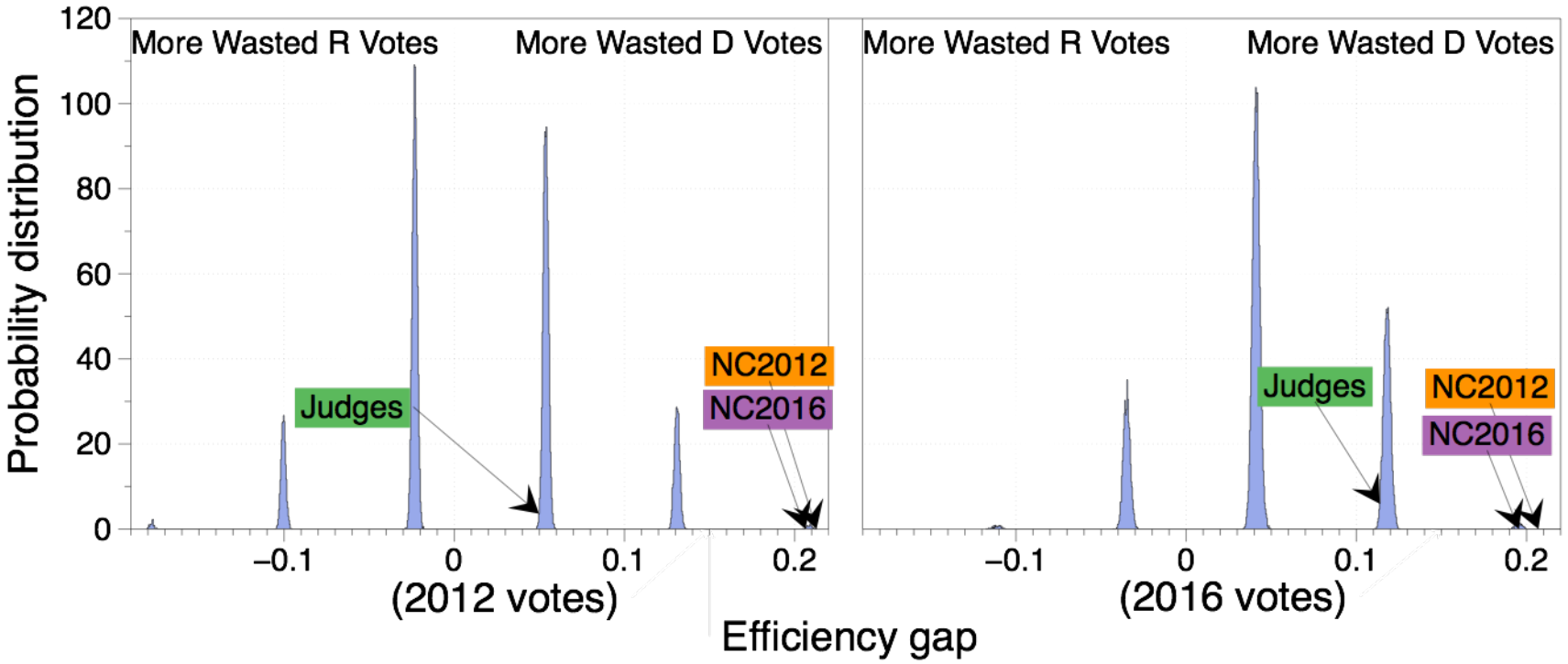
sampling decouples geopolitical effects from Gerrymandered effects



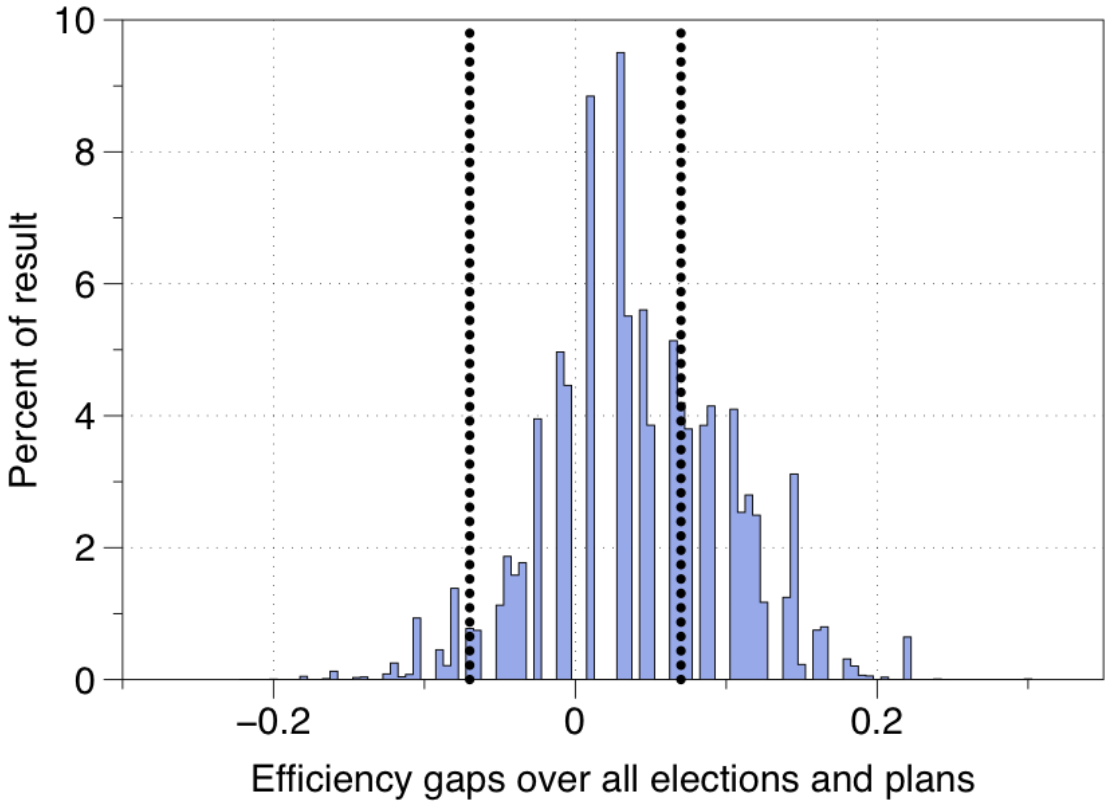


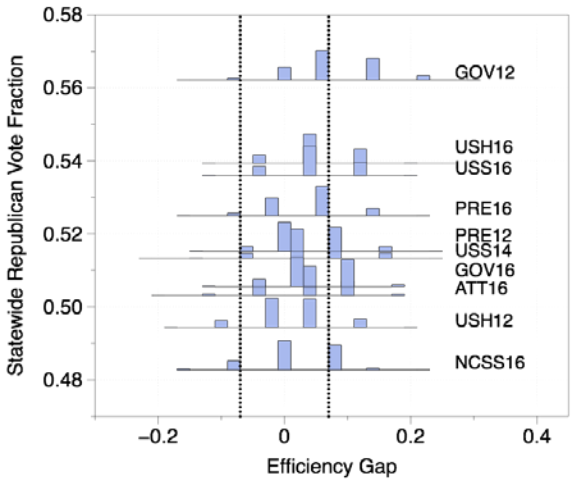


Efficiency Gap over Ensemble

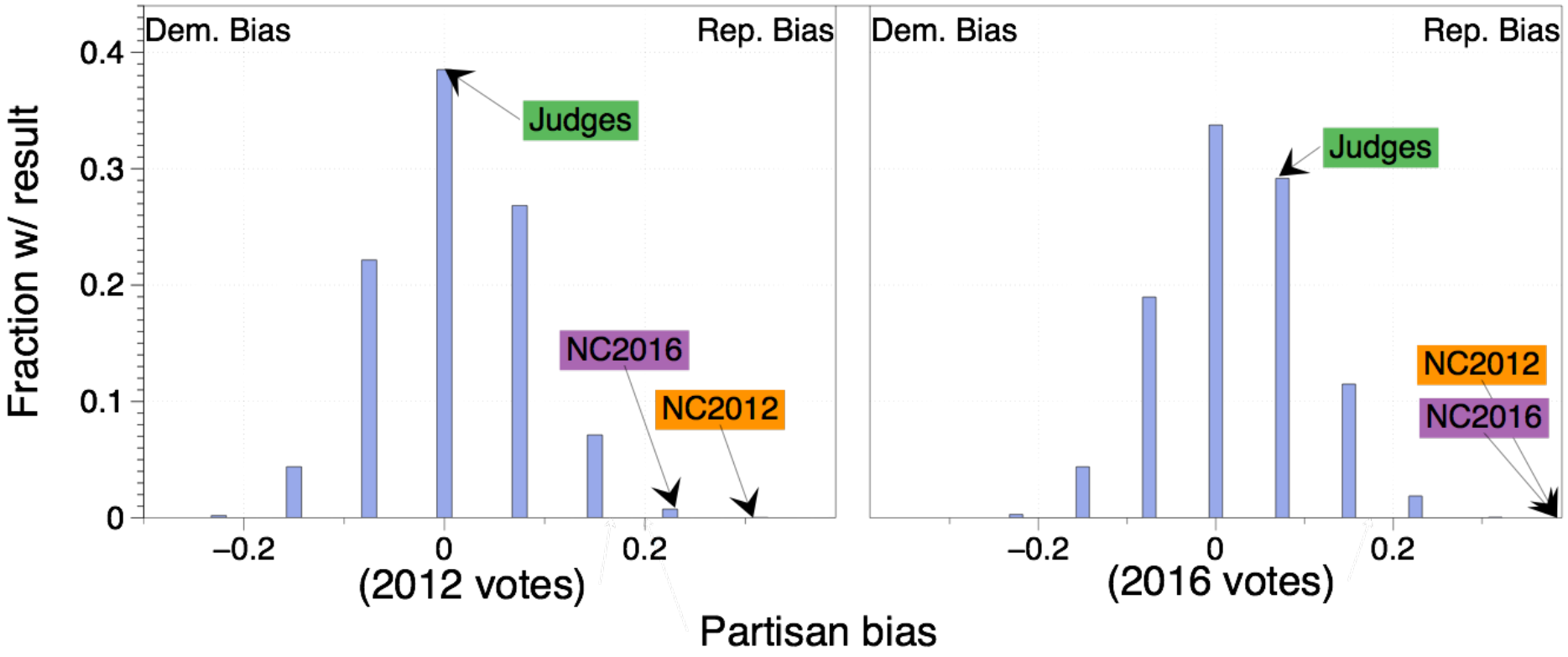


Efficiency Gap over Many Ensembles

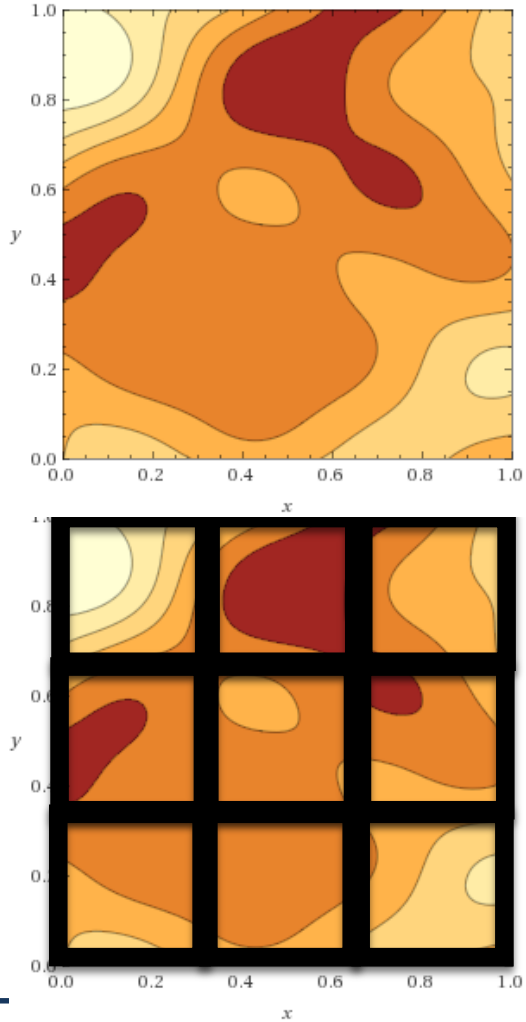




Partisan Bias over Ensembles

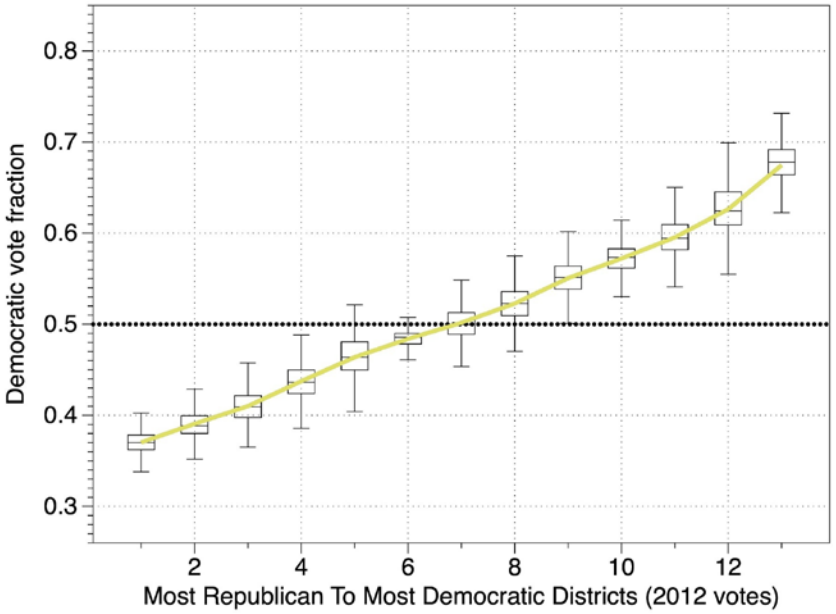


Q: Find null distribution of order statistics for district make up



Assume the population is uniform

model a random distribution of political parties



Q: Give some form of stability of plots over a class of energy functions which have certain marginal statistics.