# MATHEMATICAL FRONTIERS

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**Mathematical Sciences & Analytics**

# MATHEMATICAL FRONTIERS
## 2019 Monthly Webinar Series, 2-3pm ET

**February 12:** *Machine Learning for Materials Science*

**March 12:** *Mathematics of Privacy*

**April 9:** *Mathematics of Gravitational Waves*

**May 14:** *Algebraic Geometry*

**June 11:** *Mathematics of Transportation*

**July 9:** *Cryptography & Cybersecurity*

**August 13:** *Machine Learning in Medicine*

**September 10:** *Logic and Foundations*

**October 8:** *Mathematics of Quantum Physics*

**November 12:** *Quantum Encryption*

**December 10:** *Machine Learning for Text*

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# MATHEMATICAL FRONTIERS
## Mathematics of Transportation

**Pascal Van Hentenryck, Georgia Institute of Technology**

**Alain Kornhauser, Princeton University**

**Mark Green, UCLA (moderator)**

Georgia Tech


# Mobilizing Accessibility


Pascal Van Hentenryck

and the ΣOM Team

H. Milton Stewart School of
Industrial and Systems Engineering

Machine Learning Center

Supply Chain and Logistics Institute

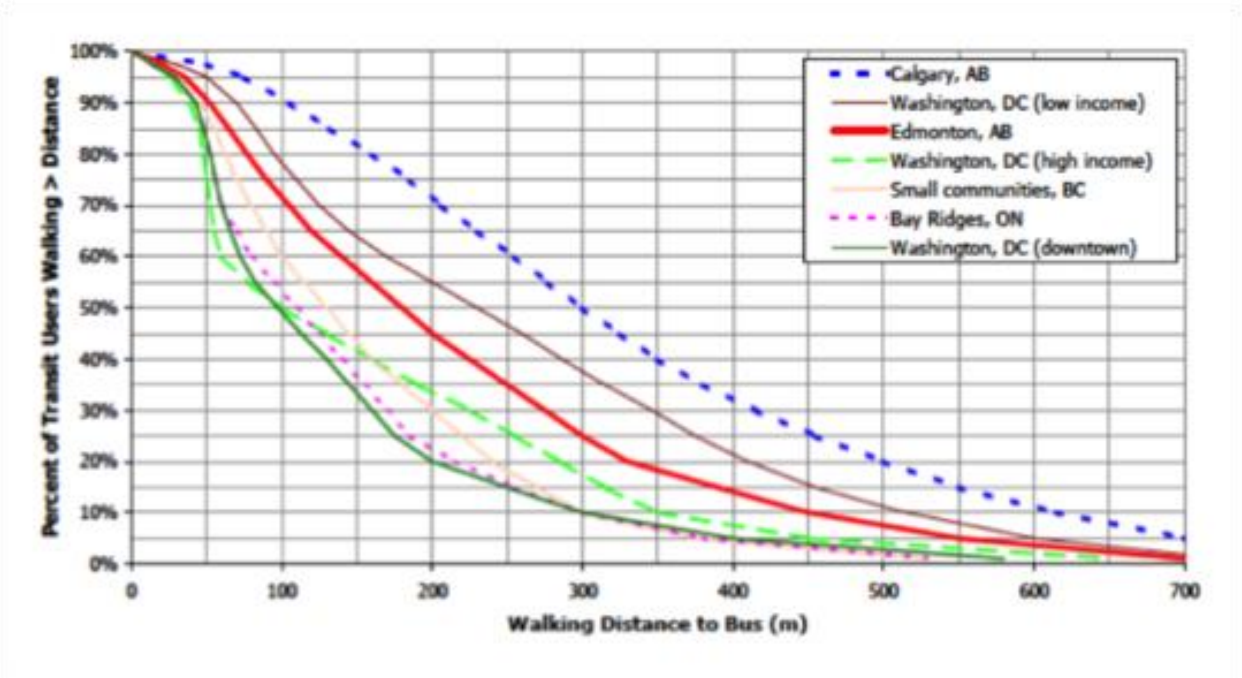
# Outline

- **Background and Motivation**
  - mobilizing accessing
  - challenges in mobility
  - opportunities in mobility
- On-demand multimodal public transit

# Mobilizing Accessibility

- Transform accessibility
  - jobs
  - health care
  - groceries
  - education
- Increasing inequalities in mobility
  - decline in public transportation

# The First/Last Mile Problem

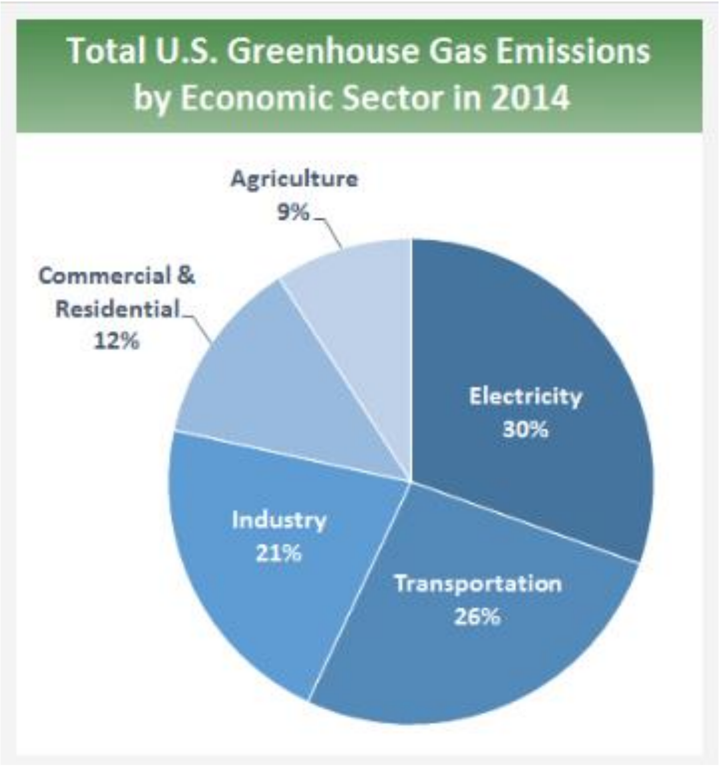


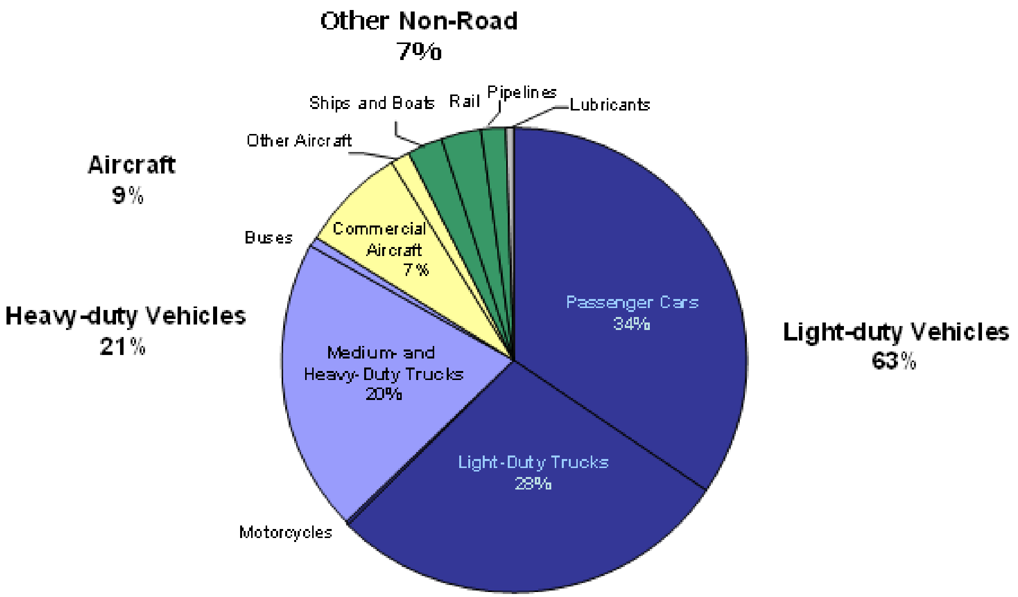

*Source: TCQSM Chapter 3, Appendix A, p. 3-93. Discussion and version in US units is on p. 3-9.*

# Congestion

- The cost of congestion
  – in 2013, 124 billions in the United States
  – predicted to be 184 billions in 2030
  – affects both large and medium-sized cities


# Greenhouse Gas Emission



# Outline

- Background and Motivation
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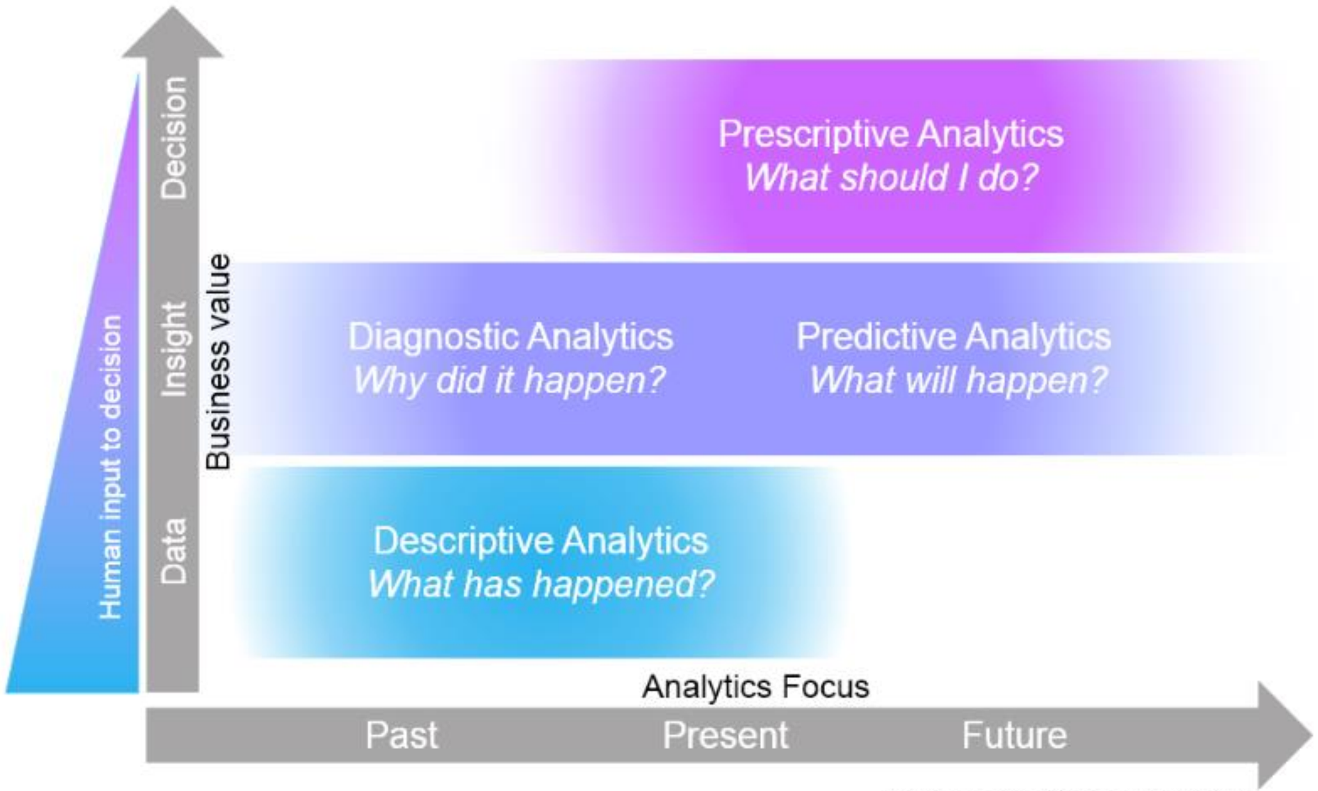
  – challenges in mobility
  – opportunities in mobility
- On-demand multimodal public transit

# Connectivity

# Autonomous Vehicles


# Analytics




Source: http://ibm.co/1gJyfl3

# Vision

Socially-aware mobility services powered by ISE technologies?

# Novel Mobility Systems

- On-Demand Multimodal Transit Systems
  – modernizing public transit
- Large-scale ride-sharing services
  – on-demand ride-shared vehicles
- Community-based car sharing
  – car-sharing for urban commuting

# Outline

- Background and Motivation
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  – opportunities in mobility
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# Transforming Public Transit

Georgia Tech

lyft

UBER

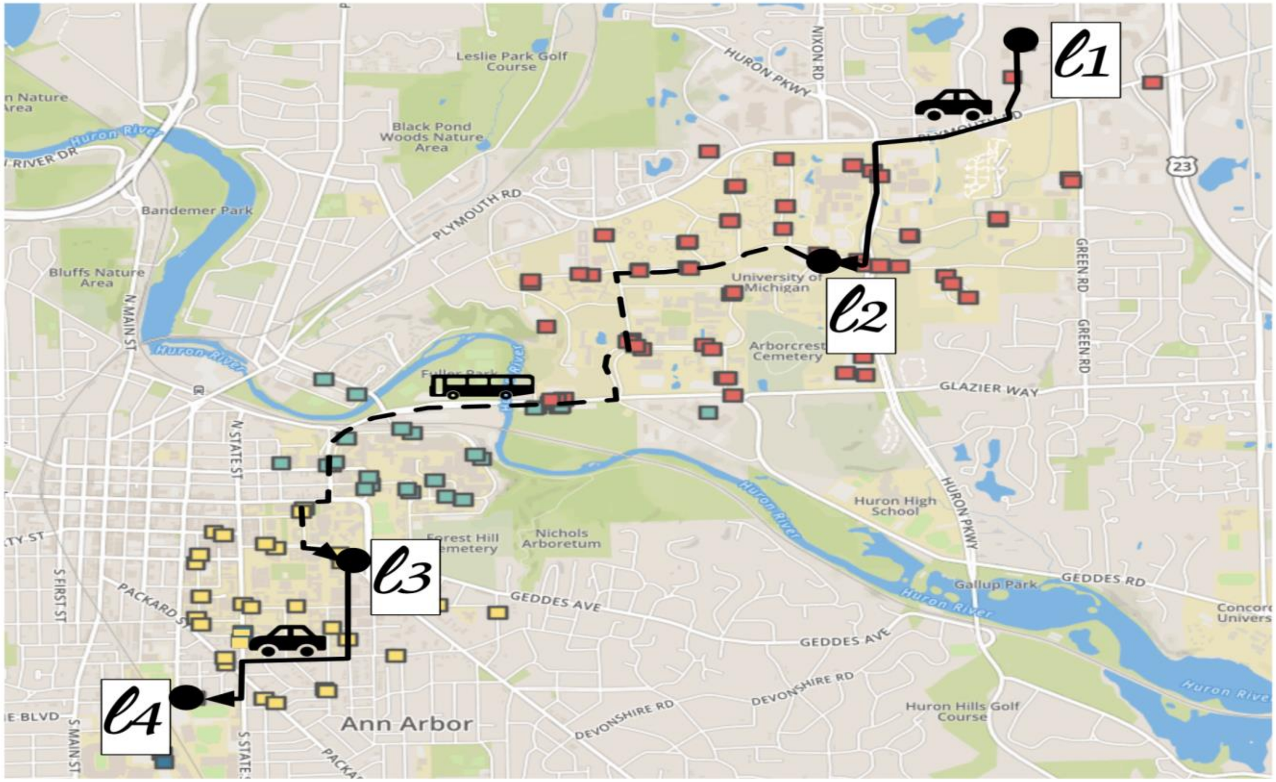
RITMO TRANSIT

# On-Demand Multimodal Public Transit

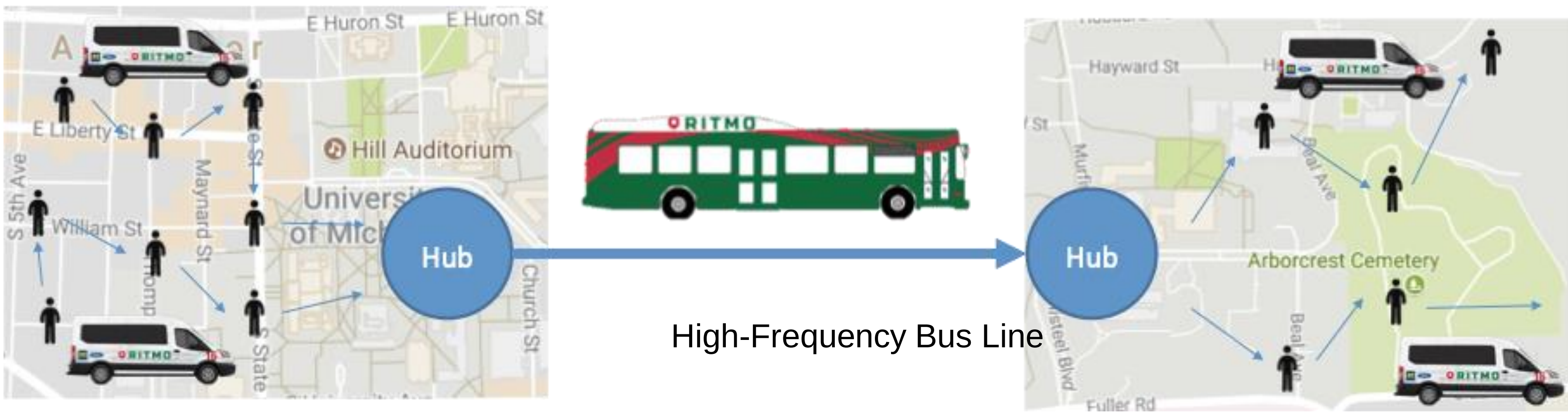
- On Demand
  – to address the first/last mile problem
- Multi-modal
  – to address congestion
  – fleets of buses, shuttles, cars, bicycles, segways, …
- Electrified
  – to address greenhouse gas emission

# On-Demand Multimodal Public Transit

Georgia Tech



# On-Demand Multimodal Public Transit




On-demand shuttles to hub

On-demand shuttles from hub

# On-Demand Multimodal Public Transit

- Booking
  - one ticket booked online / passes
- Pricing
  - same price as the transit system
- Transfers
  - completely synchronized
- Buses and light rail
  - only on high-density corridors

# Societal Impact

Georgia Tech

Annual US Personal Transit Use

Annual US Public Transit Use

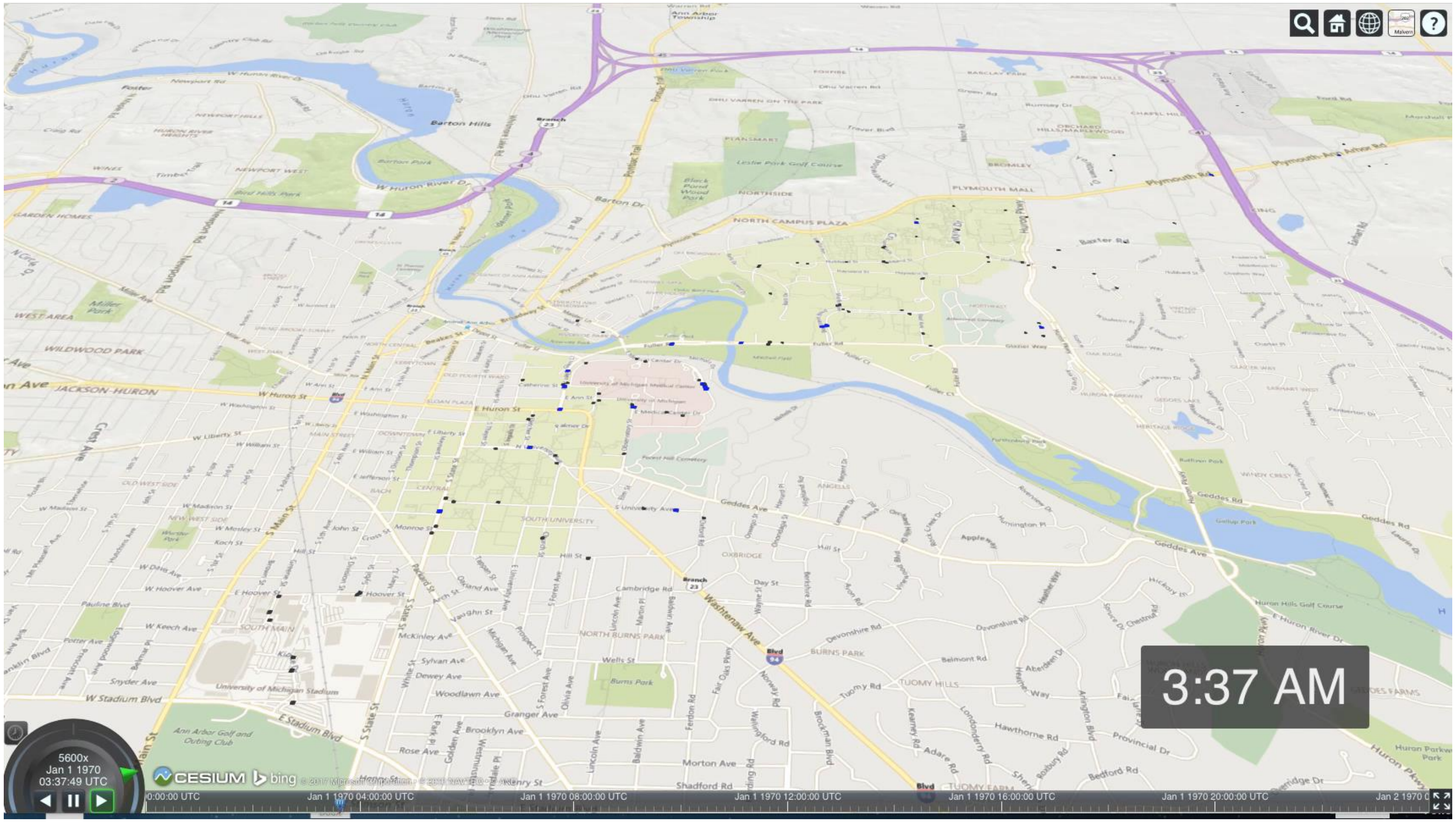
411 billion trips
3,000 billion miles

11 billion trips
59 billion miles

# Ann Arbor

# The UM Transit System

Georgia Tech



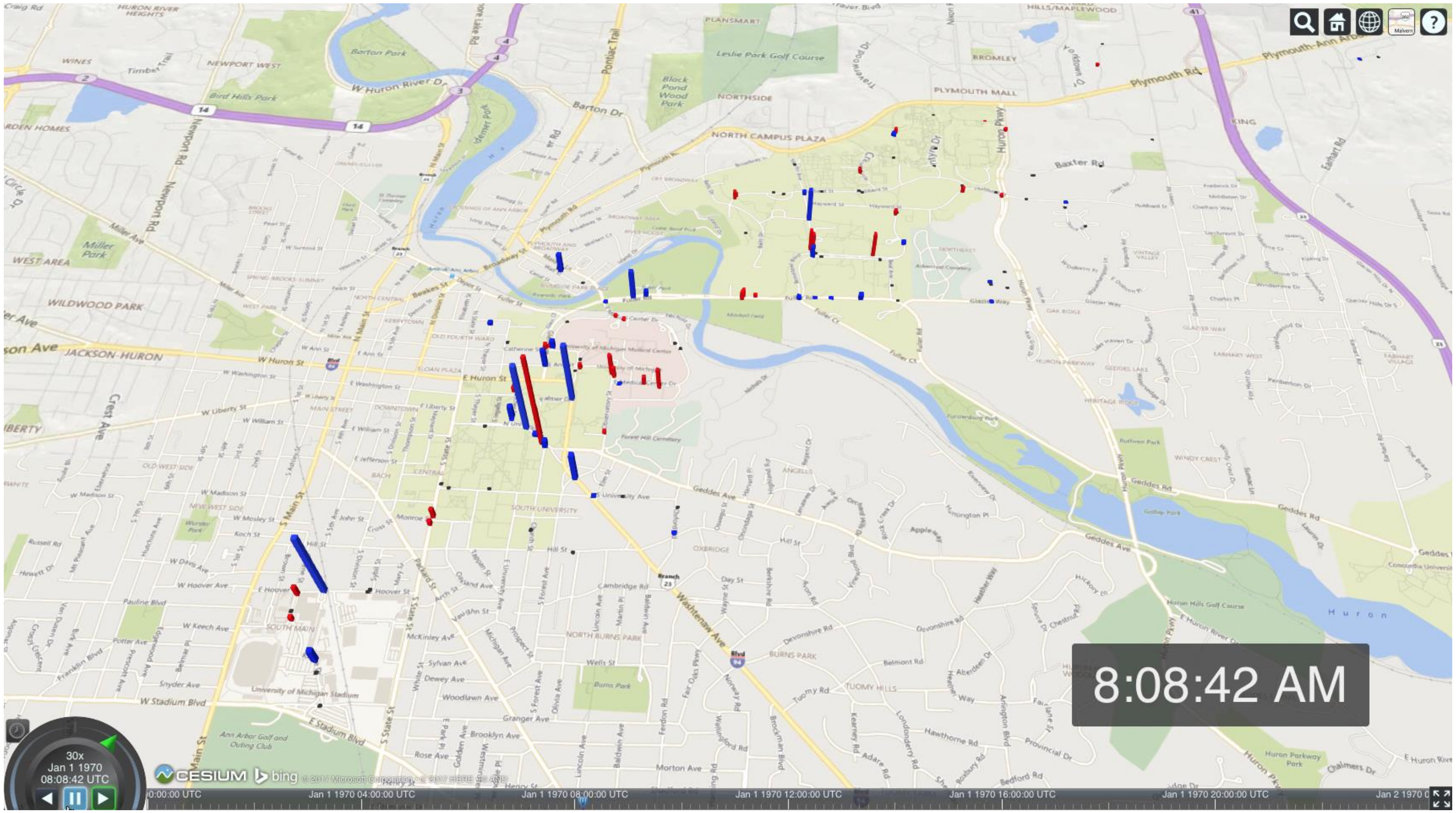
# RITMO TRANSIT

# RITMO TRANSIT

# Before/After Comparisons

# Existing Bus System

Georgia Tech



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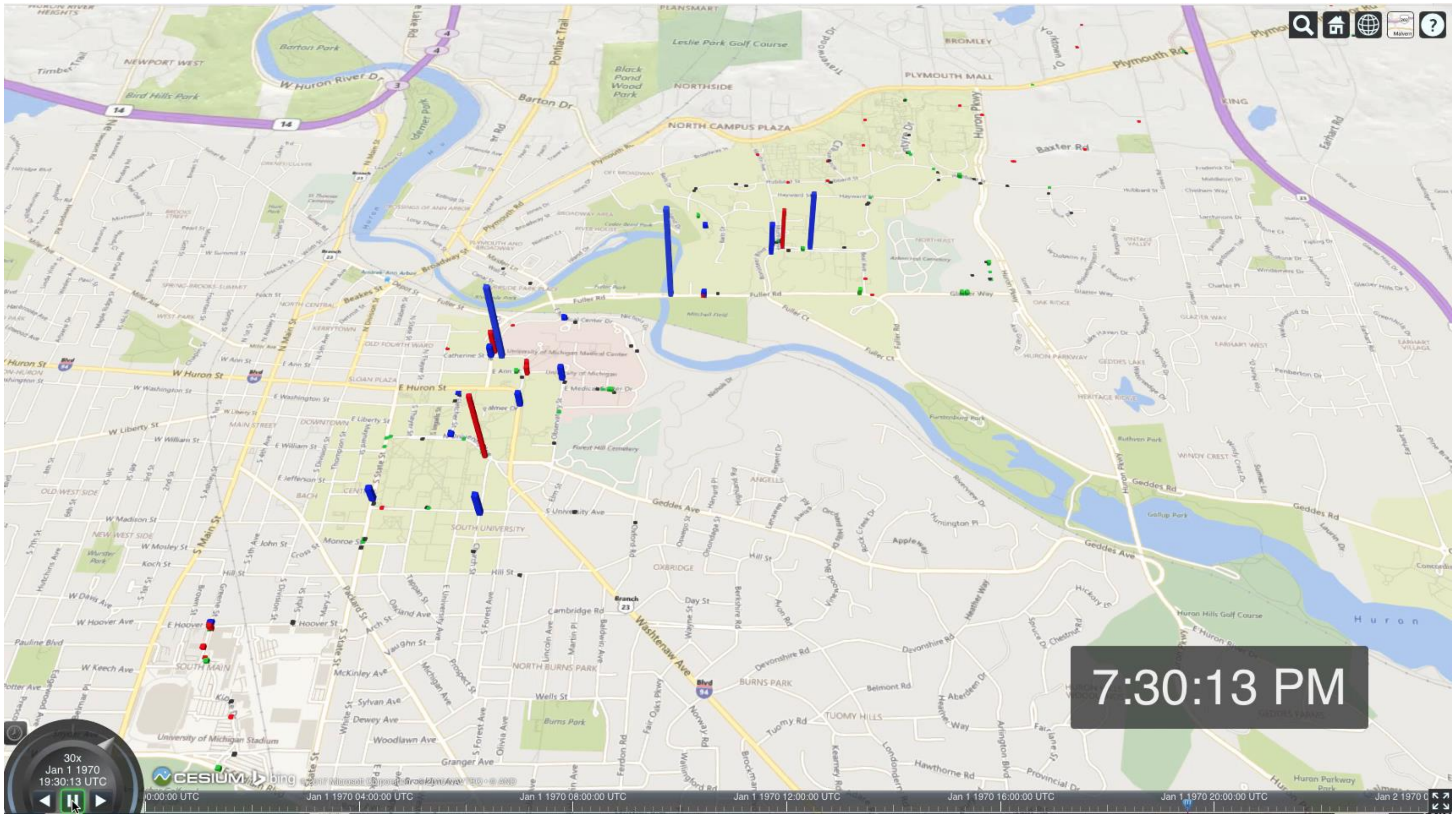

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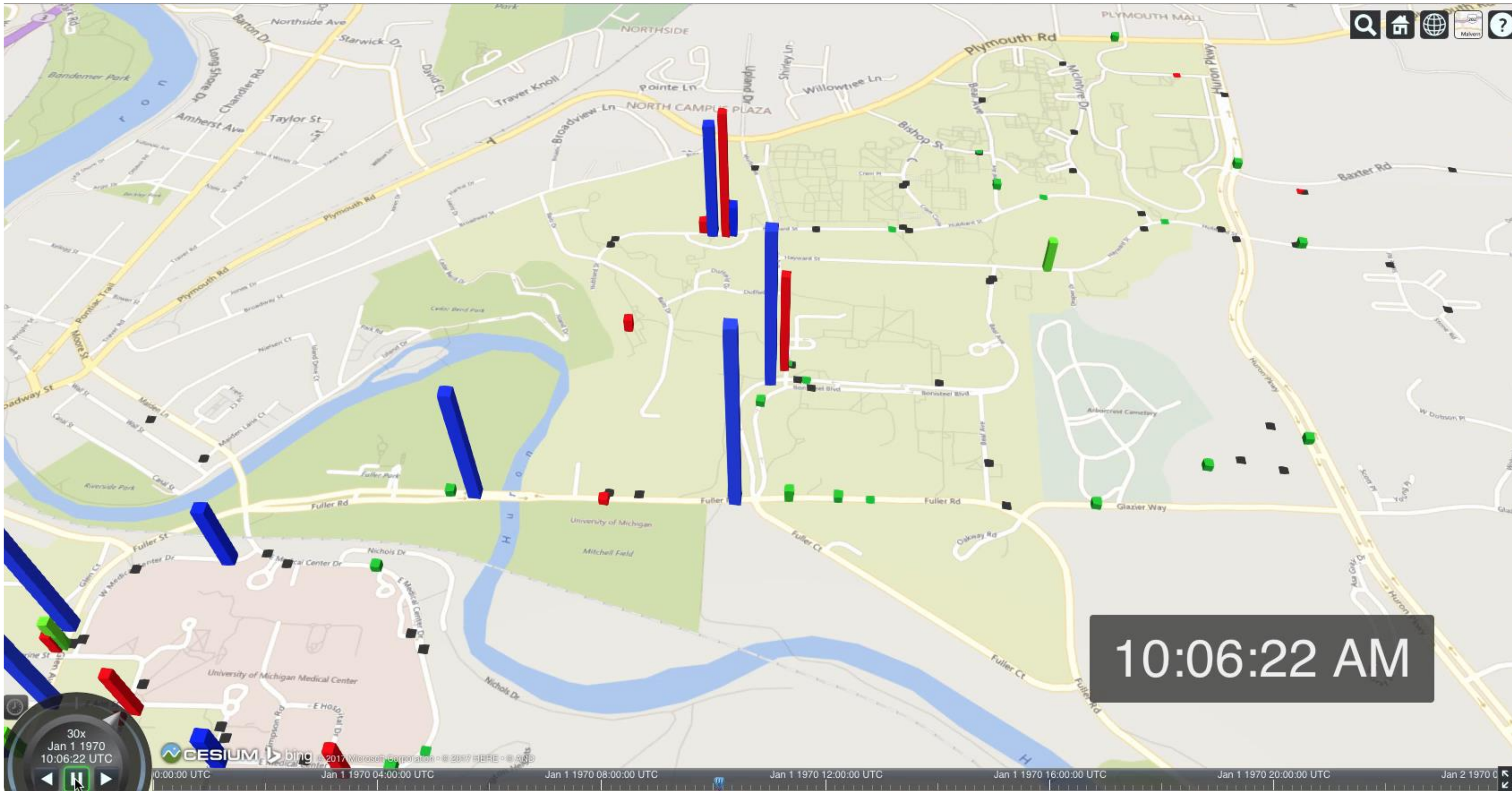

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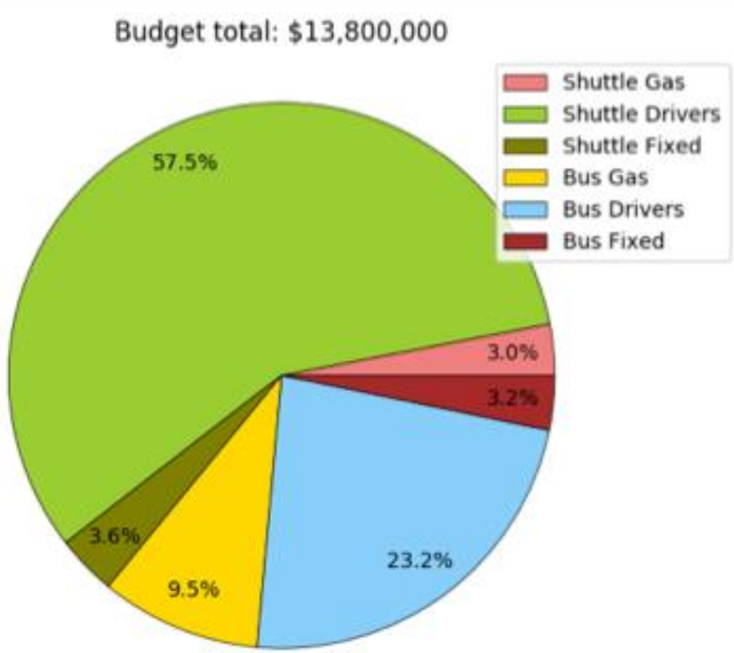
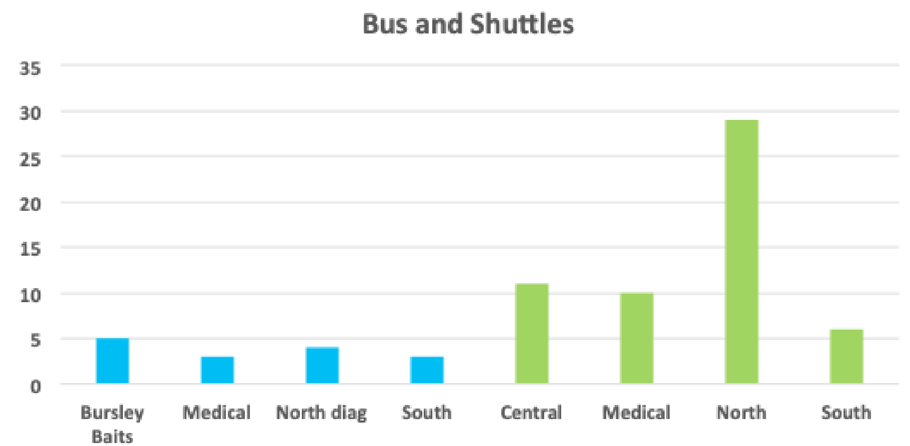
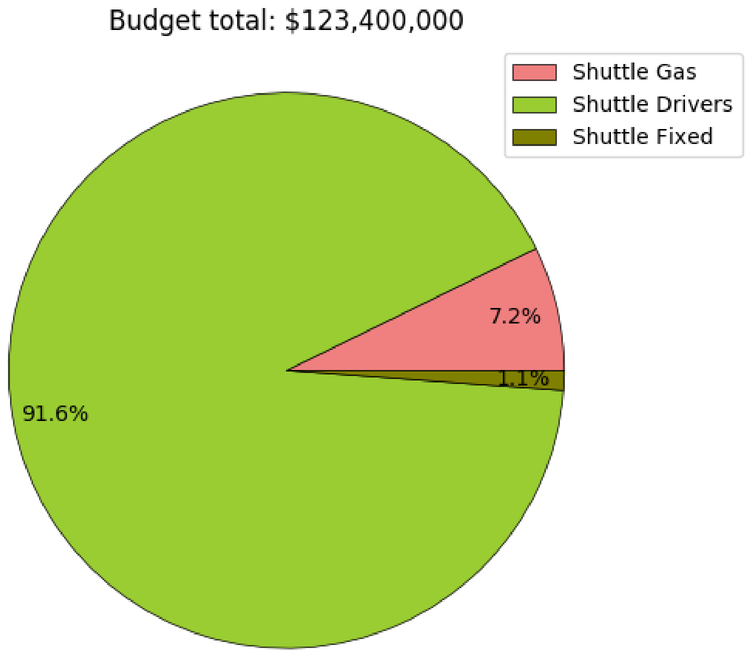
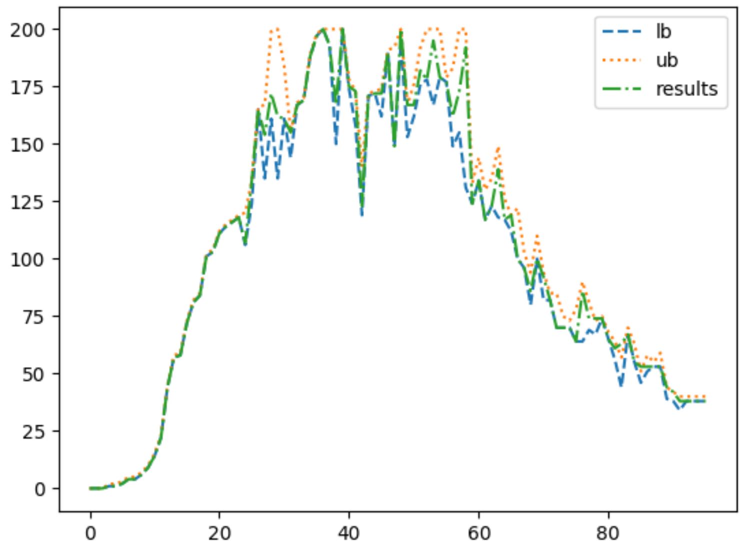


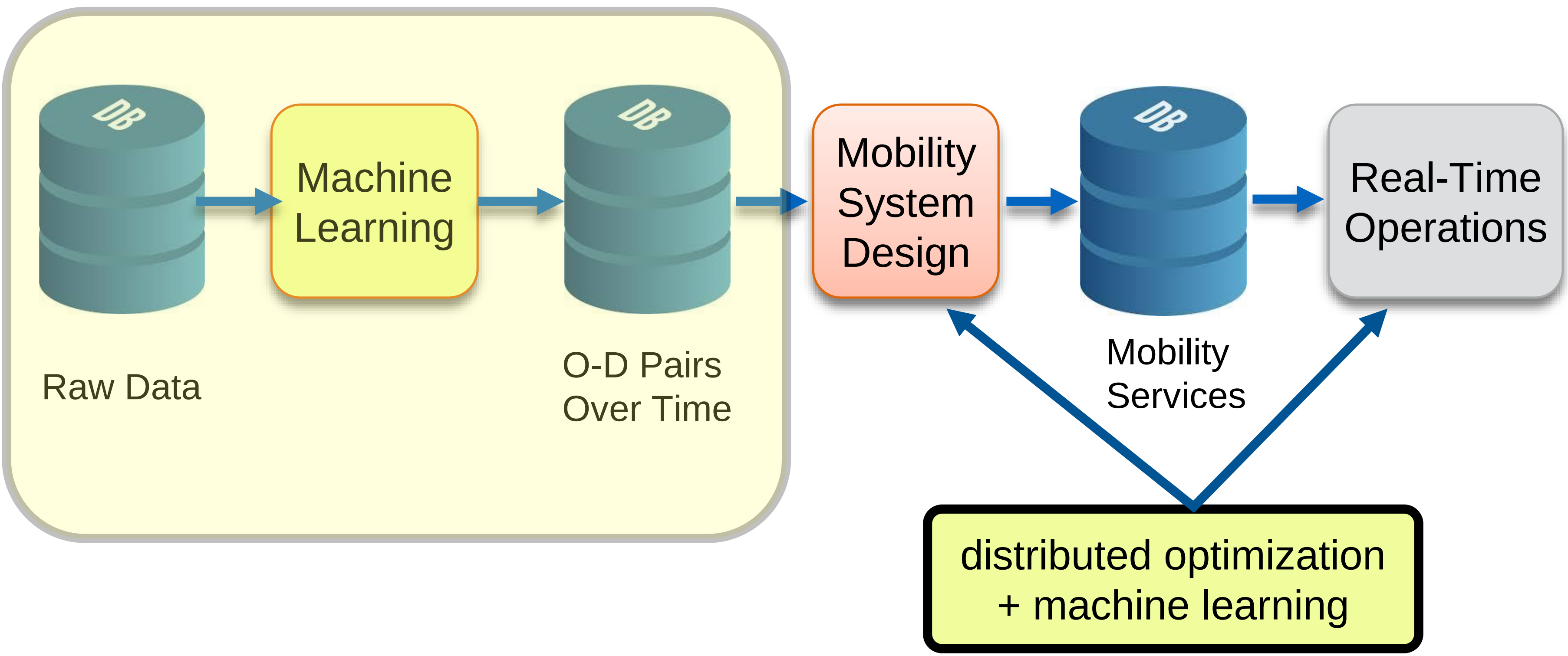



# Shuttles Only Services



# Methodology



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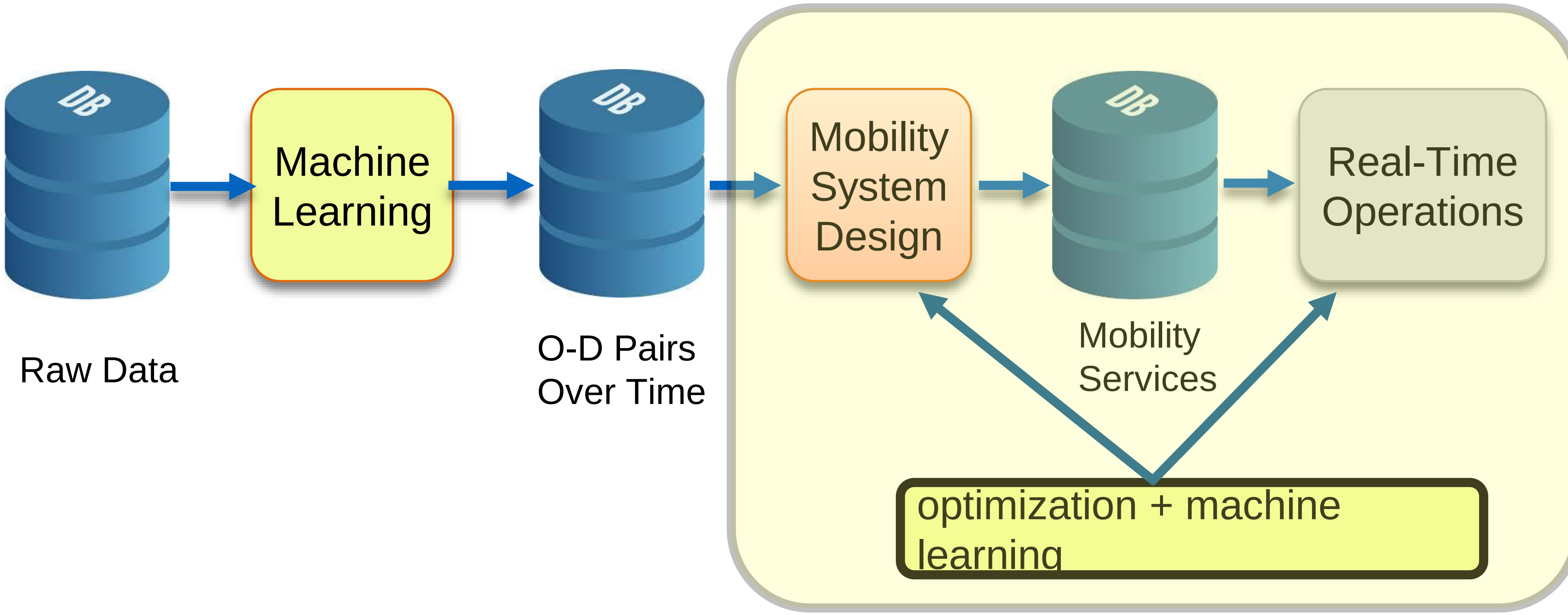
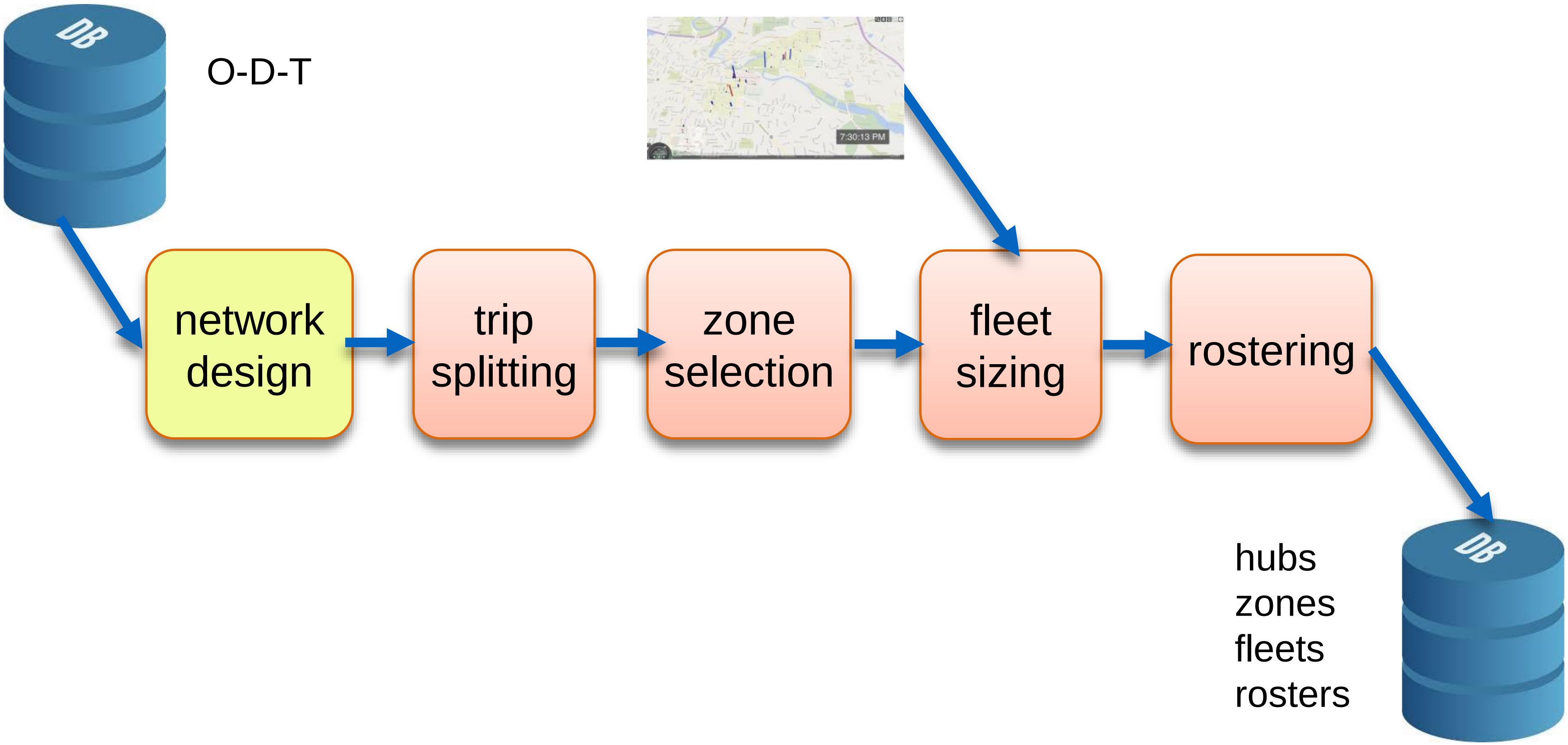


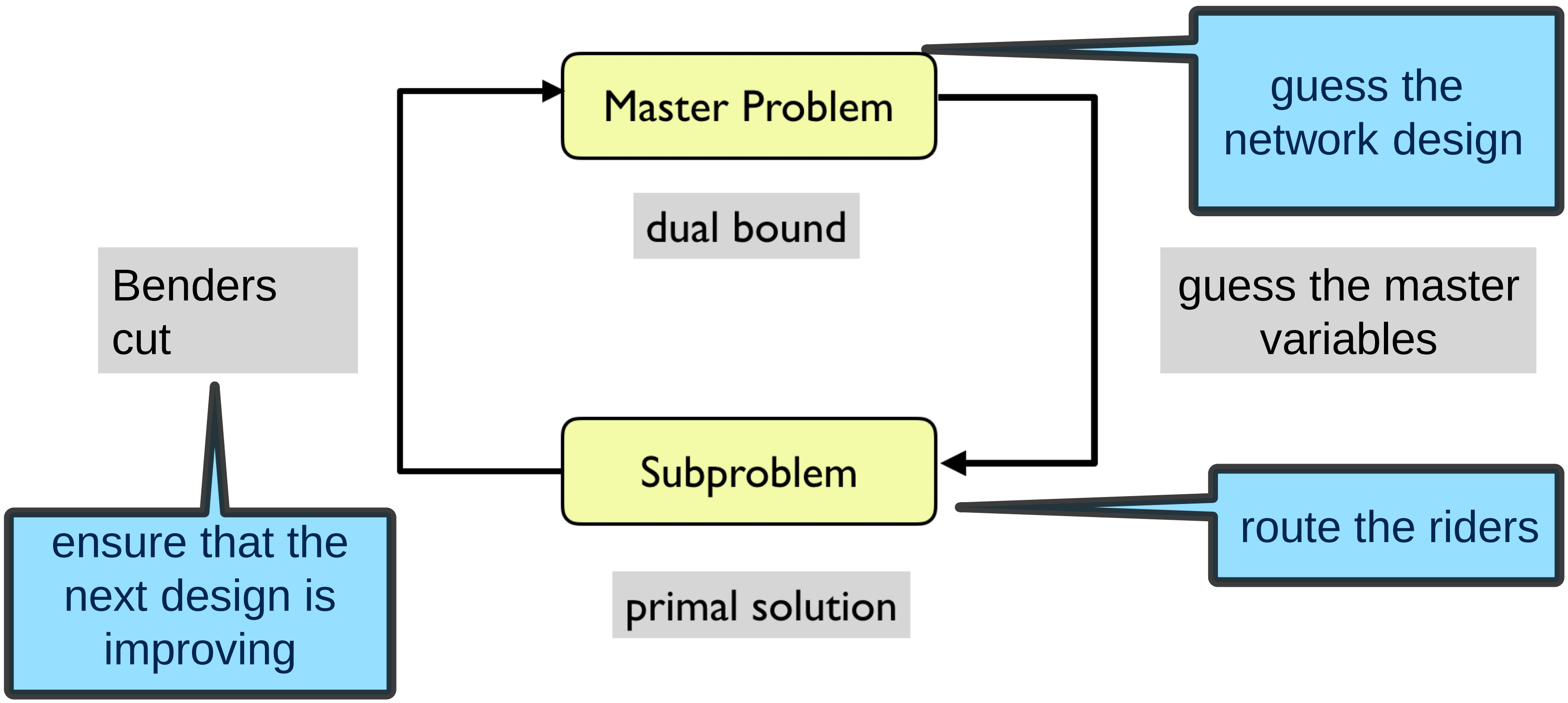
# Network Planning Pipeline



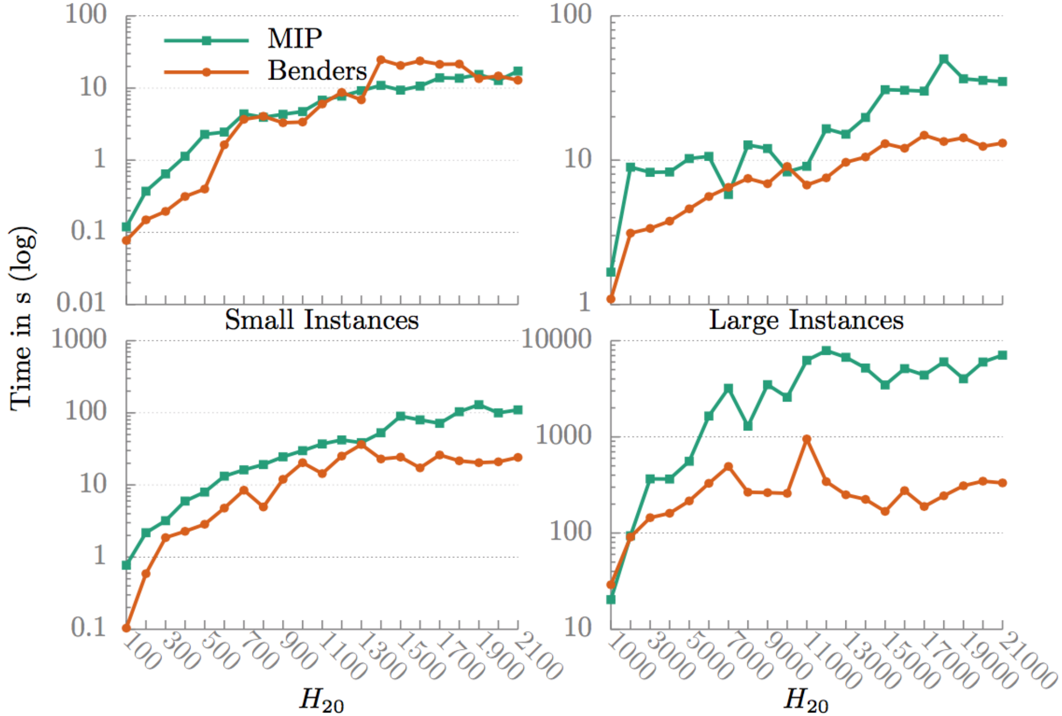
# Network Design

- Key question
  - where to open high-frequency corridors for buses and rails
- Large-scale optimization problem
  - input: all O-D pairs
  - output: where to open the rail and bus lines and with which frequencies
- Objective function
  - combination of convenience and cost
    - also accessibility

# Benders Decomposition



# MIP Versus Benders




**Comparing the MIP Model and the Final Benders Decomposition.**

# Conclusions

- On-Demand Scalable Mobility
  – inventing innovative mobility services
- Fundamental societal impact
  – mobility is central to almost all human activities
  – strong need for change in accessibility for various population segments
- Many science and engineering challenges
  – optimization, machine learning, simulation, statistics, …
- Interaction with social sciences
  – Mobility systems affect every individual
  – fairness and privacy

# MATHEMATICAL FRONTIERS
## Mathematics of Transportation

**Alain Kornhauser, Princeton University**

*Professor of Operations Research & Financial Engineering*

*Director, Transportation Program*

*Faculty Advisor, PAVE (Princeton Autonomous Vehicle Engineering)*

# Mathematical Optimization in Transportation

# Fundamental Math of Transportation

- Transportation is an Indirect Good…
  - It has little value in and of itself, but…
  - It **adds** **value** to **goods** and **people**
    - By taking them from $P_{\underline{x},\underline{y}}(t) \rightarrow P_{x',y'}(t+\Delta t)$ (P: Place, t: time)
    - Where $V_i(P_{x',y'}(t+\Delta t)) > V_i(P_{\underline{x},\underline{y}}(t)) + \$^k(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$

($V_i$: Value of $i^{th}$ entity, $\$^k$: cost of transport from $P_{\underline{x},\underline{y}}(t)$ to $P_{x',y'}(t+\Delta t)$ ) via mode k)

# Fundamental Math of Transportation

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($V_i$ : Value of $i^{th}$ entity, $\$^k$: cost of transport from $P_{\underline{x},\underline{y}}(t)$ to $P_{x',y'}(t+\Delta t)$ ) via mode k)


# Let's focus on Transportation of **People**

- Transportation is the Summation of Individual Human Decisions… The human is in the Loop!
- We are each always rational:
  - We choose: $P_{x',y'}(t+\Delta t) = \text{Max}_{\text{over all } x,y} \{V_i\,(P_{x,y}\,(t+\Delta t))\}$
    - $V_i\,(P_{x',y'}(t+\Delta t)) > V_i\,(P_{\underline{x},\underline{y}}\,(t)) + \$^k\,(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$

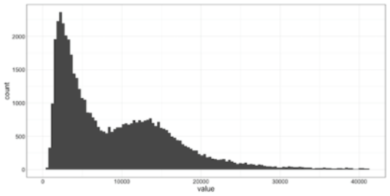

# Let's focus on Transportation of **People**

- We are each always rational:
  - We choose: $P_{x',y'}(t+\Delta t) = \text{Max}_{\text{over all } x,y} \{V_i(P_{x,y}(t+\Delta t))\}$
- Where we choose to go $\mathbf{P_{x',y'}(t+\Delta t)}$ is all about Maximizing **Happiness**

- But, it's going to cost us something to get there
  - $V_i(P_{x',y'}(t+\Delta t)) > V_i(P_{\underline{x},\underline{y}}(t)) + \$^k(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$

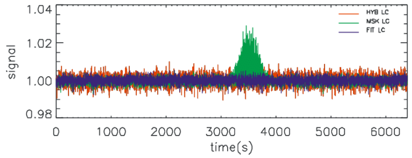
# Let's focus on Transportation of People

- Seems so simple, but… How do we begin to represent Mathematically **Happiness**$_i$ $V_i(P_{x,y}(t))$
- We know..
  - Everyone is different
    - So we are dealing with distributions not scalars



  - It has many dimensions,
    - Some dominant, some tie breakers

  - It is non-stationary
    - "time" is important




- But, in the end we each pick what is best: Mathematics of Optimization

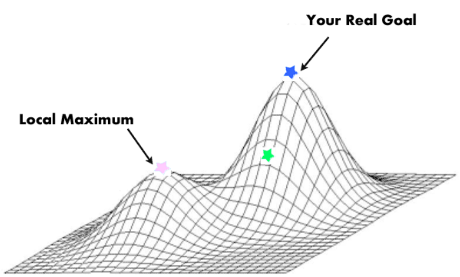
# Mathematics of Optimization

*maximize* $_x$ $f(x)$ *(objective function)*

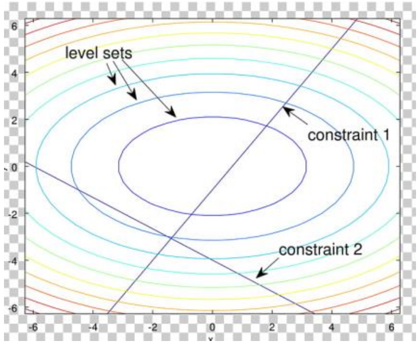
*( x is n-variable vector)*

*Subject to:* $g_i\,(x) < 0\ ,\ i = 1, \ldots .\ m$ *(inequality)*

$h_j\,(x) = 0\ ,\ j = 1, \ldots .\ P$ *(equality)*







# Back to Transportation of People

- Maximizing **Happiness**

$$V_i(P_{x',y'}(t+\Delta t)) > V_i(P_{\underline{x},\underline{y}}(t)) + \$^k(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$$

The decision to go someplace is all about its **happiness/attractiveness** relative to staying put**,** **discounted** by the **unhappiness/cost** imparted by Transportation ($\$^k(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$ )

Transportation is all about

minimizing $\$^k(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$

# Attributes Important to Transportation of **People**

**Transportation is governed by:** $\$^k(\underline{x}, \underline{y}, t, x', y', t+\Delta t)$

Time

ride, wait, time-of-day, frequency, …

Monetary costs

cash, credit, terms, …

Comfort

safety, noise, stand/seated, A/C, …

Convenience

accessibility, …

Societal

energy, pollution, global warming, inclusiveness

As perceived by individuals

# Major Tech Advances in Transportation of **People**

**Baseline is Walking**

Animals

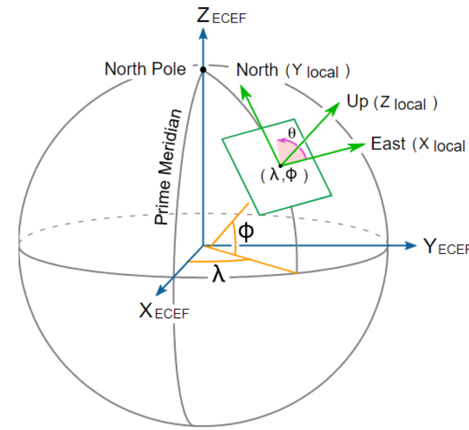
Homo erectus??

Hull

"Archamedes"

Wheel

Mesopotamia??

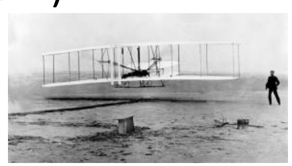



Rail

George Stephenson (Steam)

Airfoil

Wright Bros. / Bernoulli



Rubber tire

Benz (Jan 31, 1886), Ford…

PersonTrips USA: ~$1.1\text{x}10^9$ /day,

85% drive ourselves, 10% Walk, 4% Transit, 1% other

Automated Driving

DARPA Challenges (mid 2000)

# Why Automated Driving

**Transportation is governed by:** $\$^k (\underline{x}, \underline{y}, t, x', y', t+\Delta t)$

Time

ride, wait, time-of-day, frequency, …

**Monetary costs**

cash, credit, terms, …

Comfort

**Safety**, noise, stand/seated, A/C, …

~ 90% crashes involve human error

Convenience

accessibility, …

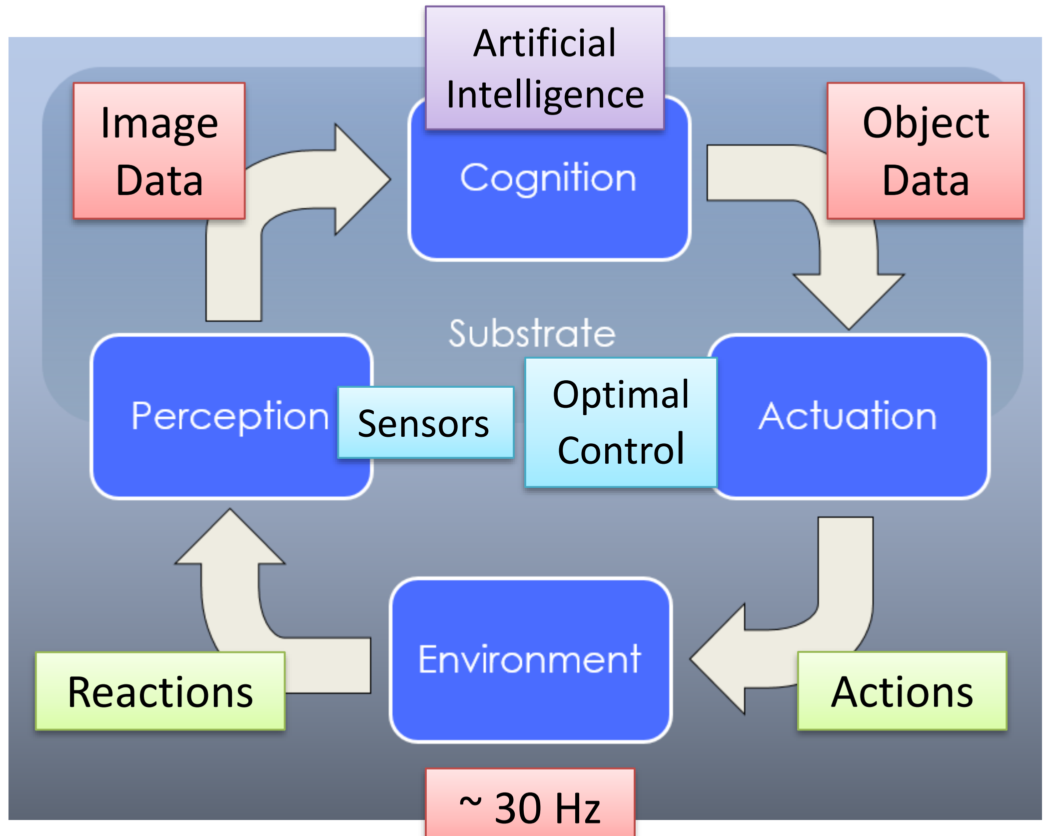

Societal

**energy, pollution, global warming, inclusiveness**

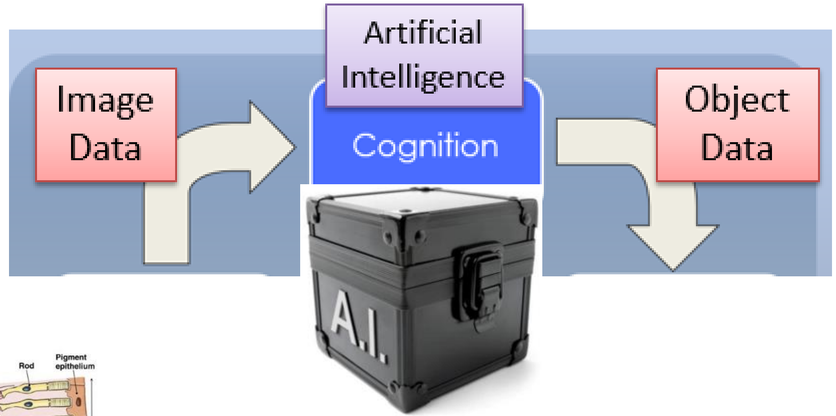
# Major Elements of Automated Driving

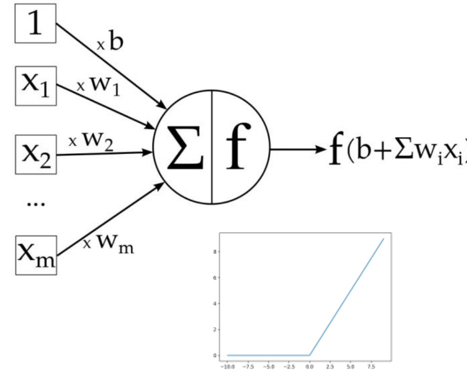
Key is 

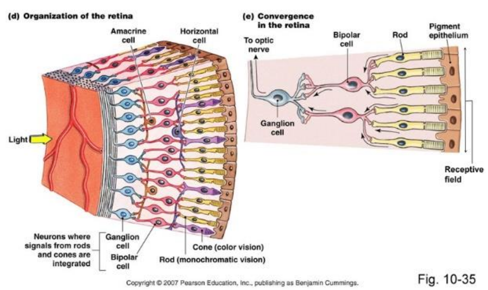
... Place all of the intelligence in the vehicle



# Major Elements of Automated Driving




## Deep Convolutional Neural Network (CNN)

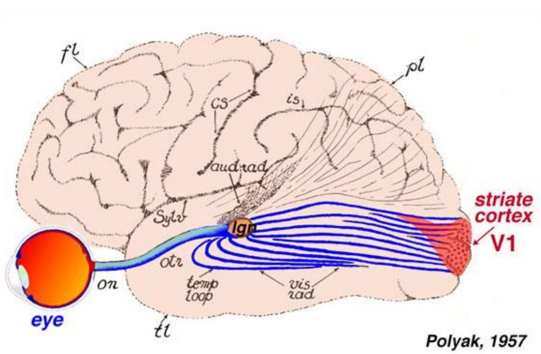



Figure 8. Visual input to the brain goes from eye to LGN and then to primary visual cortex, or area V1, which is located in the posterior of the occipital lobe.
Adapted from Polyak (1957).

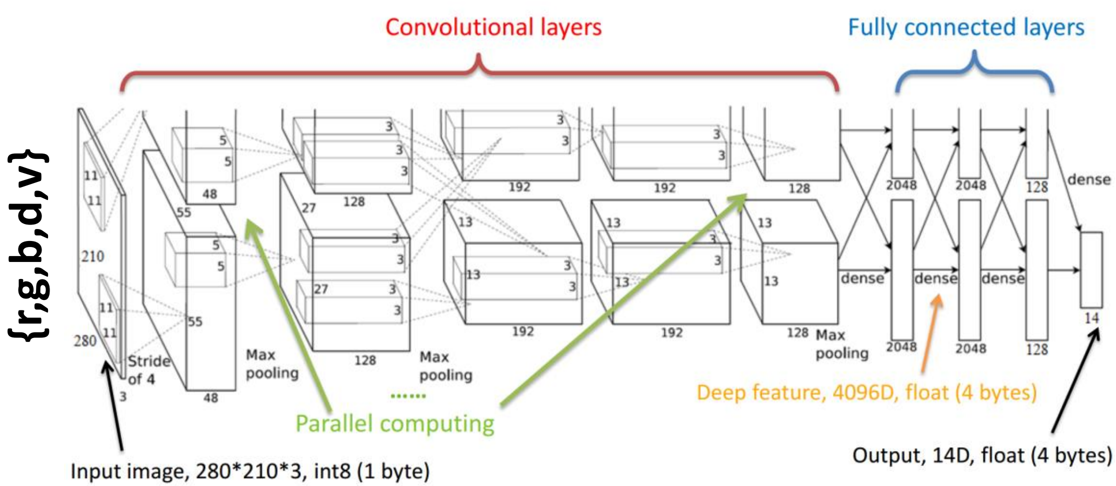



Figure courtesy and modified of Alex Krizhevsky, Ilya Sutskever, Geoffrey E. Hinton

# Major Elements of Automated Driving

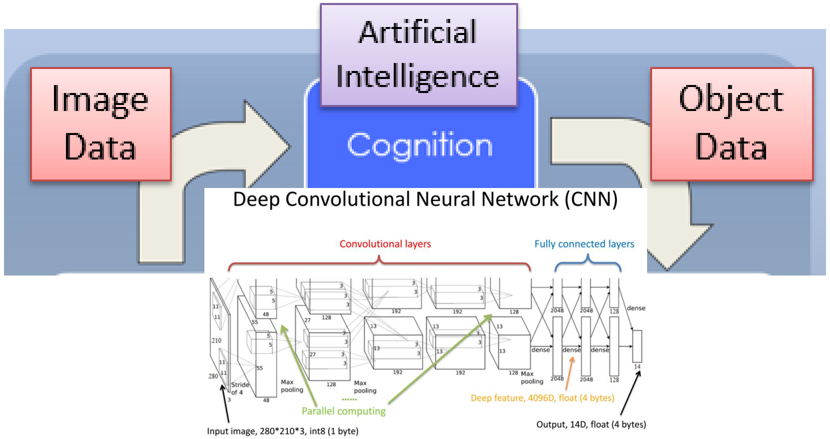
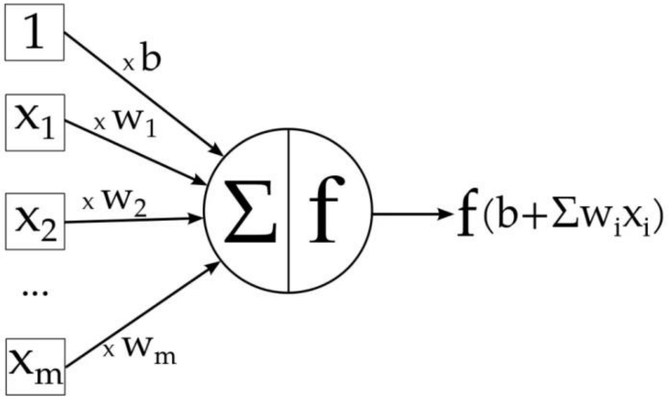



Figure courtesy and modified of Alex Krizhevsky, Ilya Sutskever, Geoffrey E. Hinton

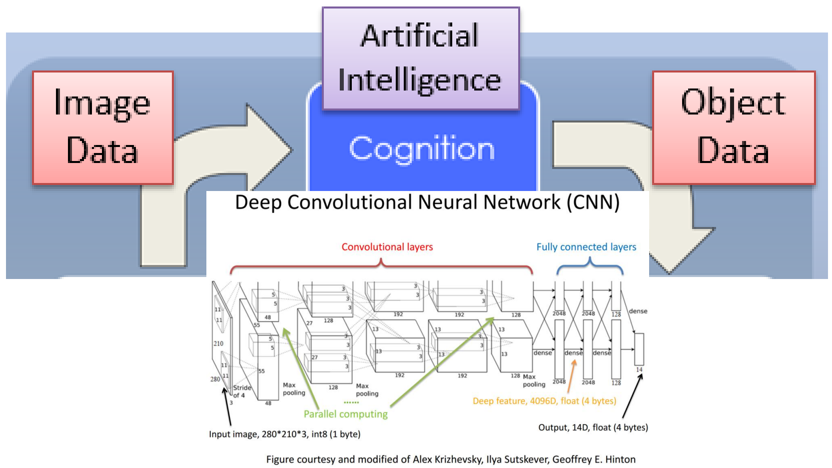
$>10^5$ bytes → $10^4$ bytes → $\sim 10^2$ bytes



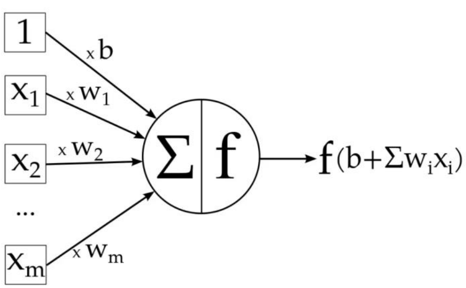

$w_i \quad i = 1, \ldots, 10^8$

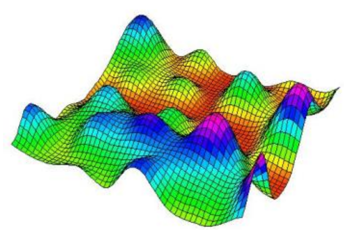
# Major Elements of Automated Driving

Training: "computing/finding" correct values of $w_i \quad i = 1, \dots, 10^8$




$$\min_{w_i,\; i = 1, \dots, 10^8} \sum \left[ \text{Computed Object data} - \text{Correct Object data} \right]^2$$



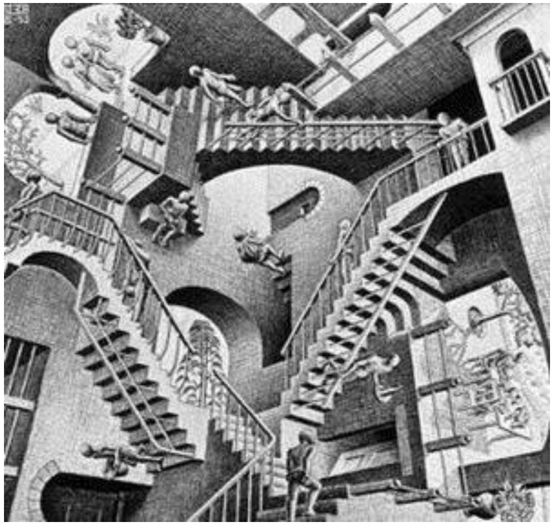



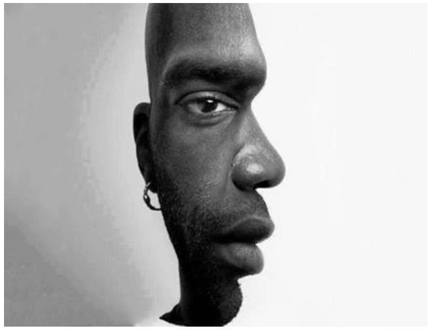
# Limitation of CNNs? Adversarial Results

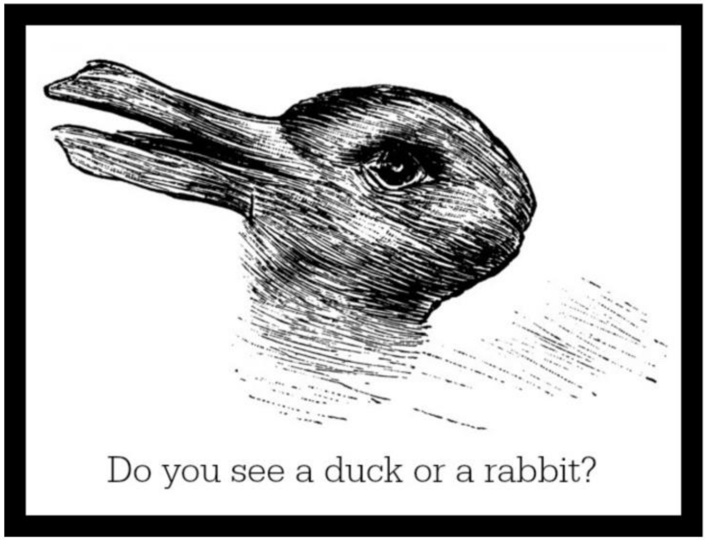
When does 

give the wrong answer?

We know we get fooled…





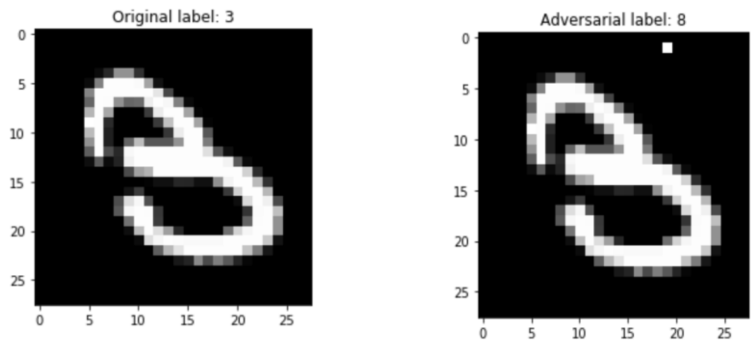


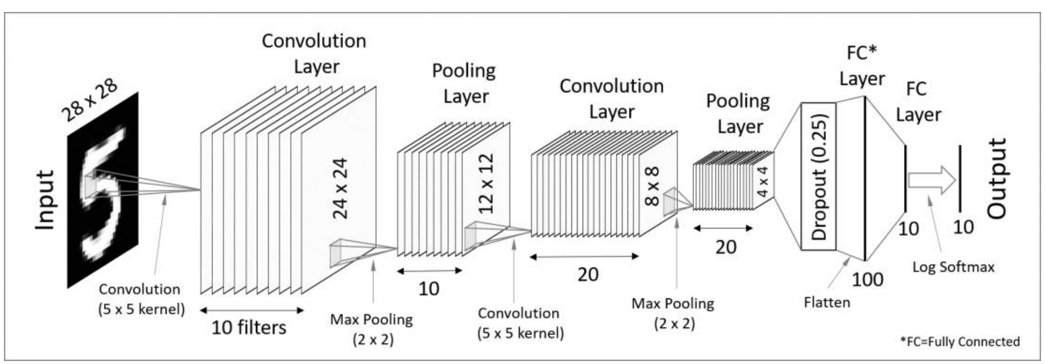

Escher

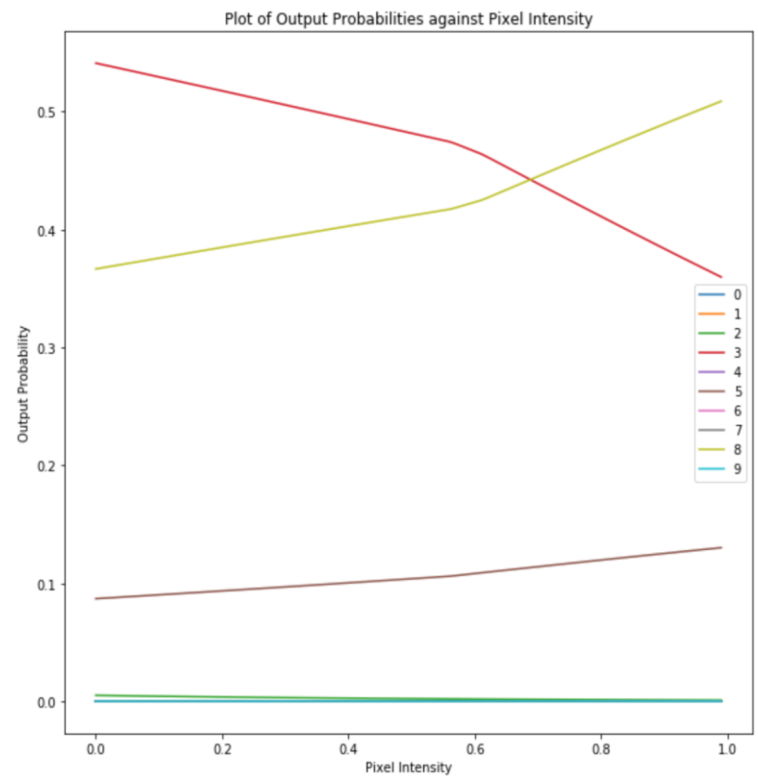
# Limitation of CNNs? Adversarial Results

When does  give the wrong answer?

## Digit Recognition Problem



# Much more Math to be done

***Thank You***

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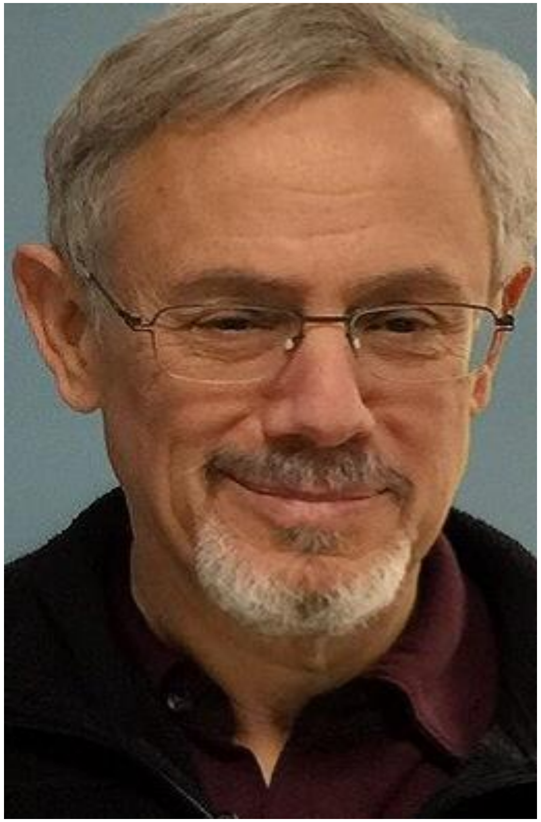
## Mathematics of Transportation

**Pascal Van Hentenryck,**
**Georgia Institute of Technology**

**Alain Kornhauser,**
**Princeton University**



**Mark Green,**
**UCLA (moderator)**

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**July 9:** *Cryptography & Cybersecurity*

**August 13:** *Machine Learning in Medicine*

**September 10:** *Logic and Foundations*

**October 8:** *Mathematics of Quantum Physics*

**November 12:** *Quantum Encryption*

**December 10:** *Machine Learning for Text*

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