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MATHEMATICAL FRONTIERS
2019 Monthly Webinar Series, 2-3pm ET

February 12: *Machine Learning for Materials Science*

March 12: *Mathematics of Privacy*

April 9: *Mathematics of Gravitational Waves*

May 14: *Algebraic Geometry*

June 11: *Mathematics of Transportation*

July 9: *Cryptography & Cybersecurity*

August 13: *Machine Learning in Medicine*

September 10: *Logic and Foundations*

October 8: *Mathematics of Quantum Physics*

November 12: *Quantum Encryption*

December 10: *Machine Learning for Text*

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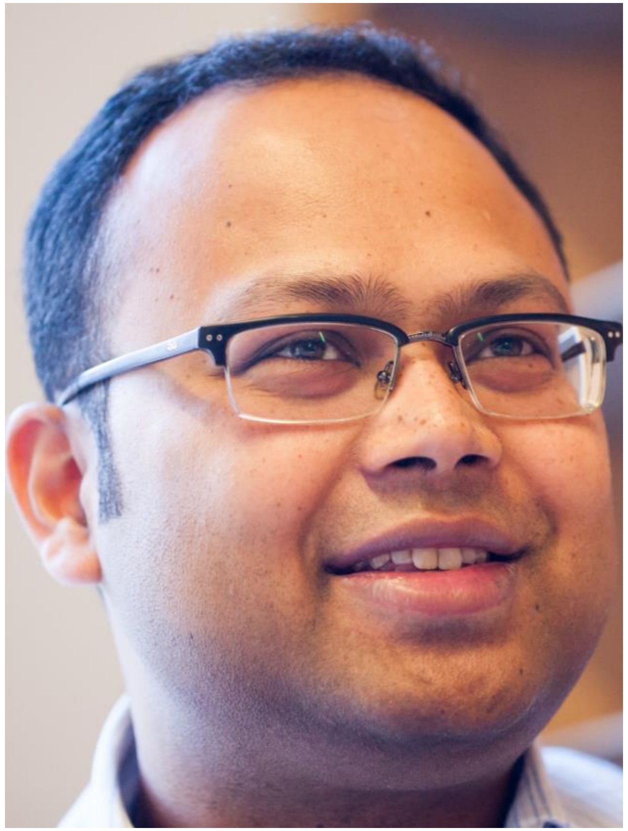
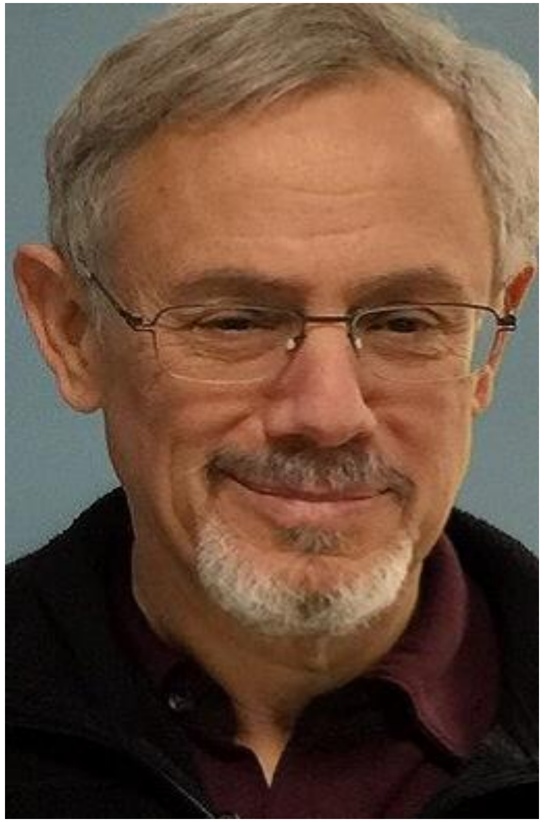
MATHEMATICAL FRONTIERS

Cryptography and Cybersecurity

Kristin Lauter,
Microsoft Research



Suman Jana,
Columbia University



Mark Green,
UCLA (moderator)

Kristin Lauter,
Microsoft Research

Principle Researcher and Research Manager for Cryptography

Private AI: Machine Learning on Encrypted Data

Private AI:
Machine Learning on Encrypted Data

Kristin Lauter

Partner Research Manager,
Principal Researcher
Cryptography Research

Microsoft Research
SEAL Team: sealcrypto.org

July 9, 2019

Microsoft

Privacy problem with AI?

- Artificial Intelligence: uses (ML) machine learning algorithms to make useful predictions
  - Input: your data
  - Output: some recommendation, decision, or classification
- **Privacy problem**: you have to input **your data** in order to get the valuable prediction!
- Typical AI services are hosted in the cloud, run by a “smart agent” (e.g. Cortana, Siri, Alexa)

New mathematical tool

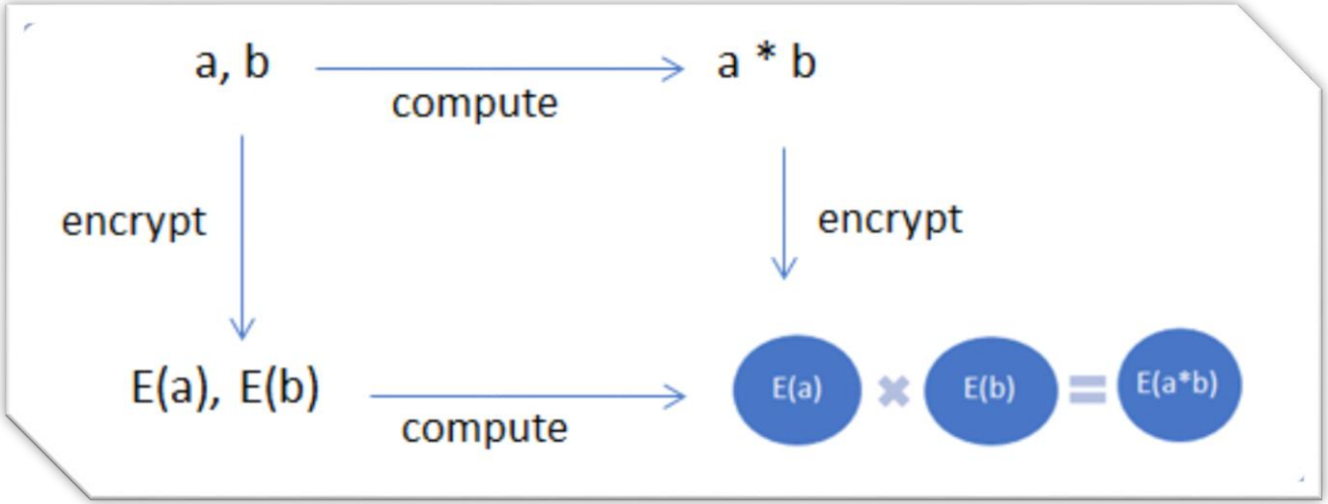
- Protect privacy and security of your data through encryption

- Need encryption which you can compute on:

    - **Homomorphic Encryption!**

Homomorphic Encryption (HE)

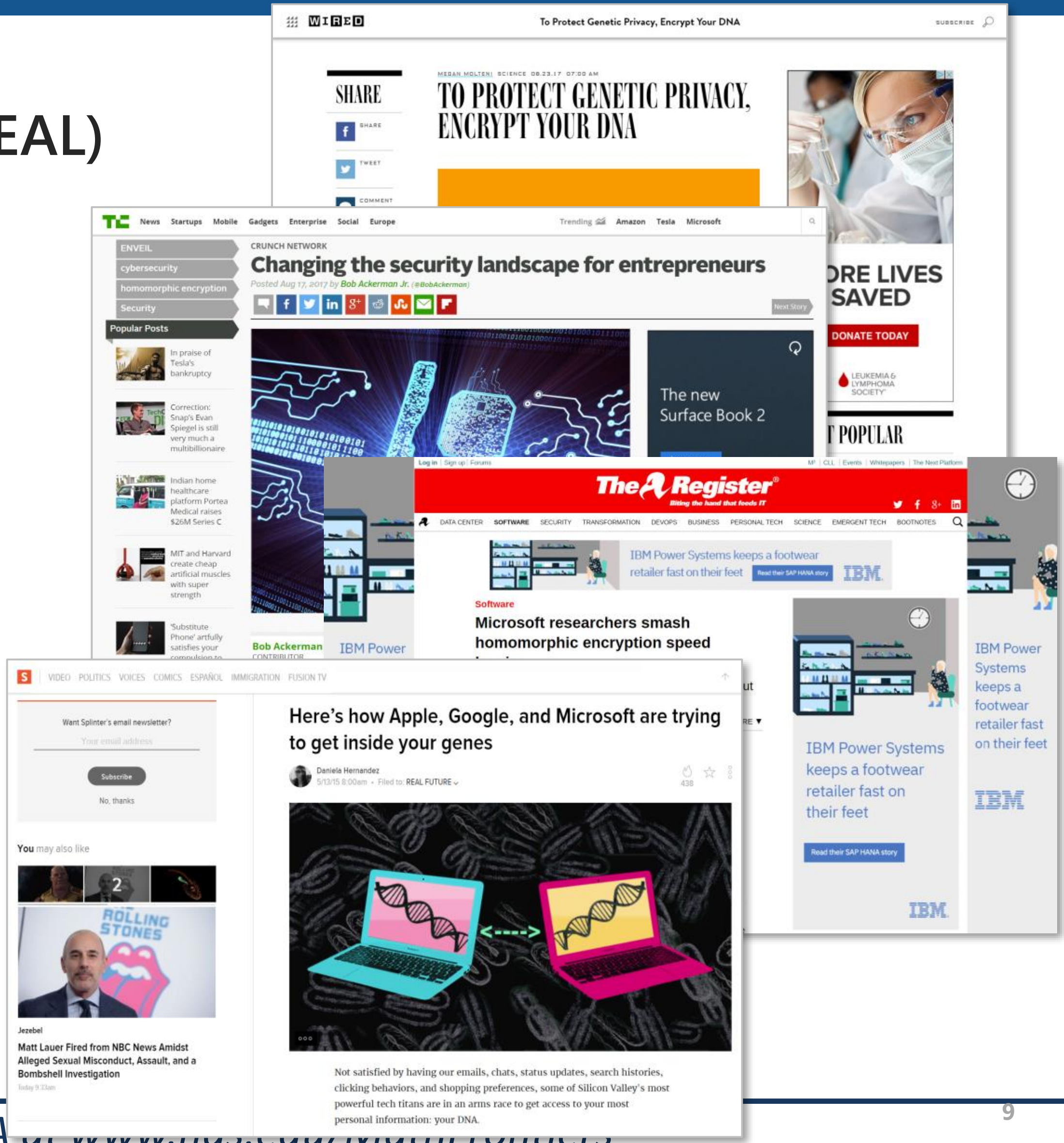
- Computation on encrypted data without decrypting it!
- 2009: First solution, considered impractical
- 2011: Surprise breakthrough at Microsoft Research
- Practical encoding: 4 orders of magnitude speed-up
- 2016: **CryptoNets** evaluates neural net predictions on encrypted data



Microsoft SEAL

- **Simple Encrypted Arithmetic Library (Microsoft SEAL)**
    - Public release by Microsoft Research in 2015
    - Widely adopted by teams worldwide
    - OSS release of Microsoft SEAL 2018
- **Standardization of HE:**
    - November 2018

http://sealcrypto.org



Standardization: HomomorphicEncryption.org

- MSR launched in 2017
- Three workshops: Microsoft, MIT, U Toronto
- HES 1.0 Standard, November 2018
- Royal Society (PET) Report 2019
- Applications in regulated industries require standardization
- Standardization creates trust
- Open standards highly preferred in cryptography

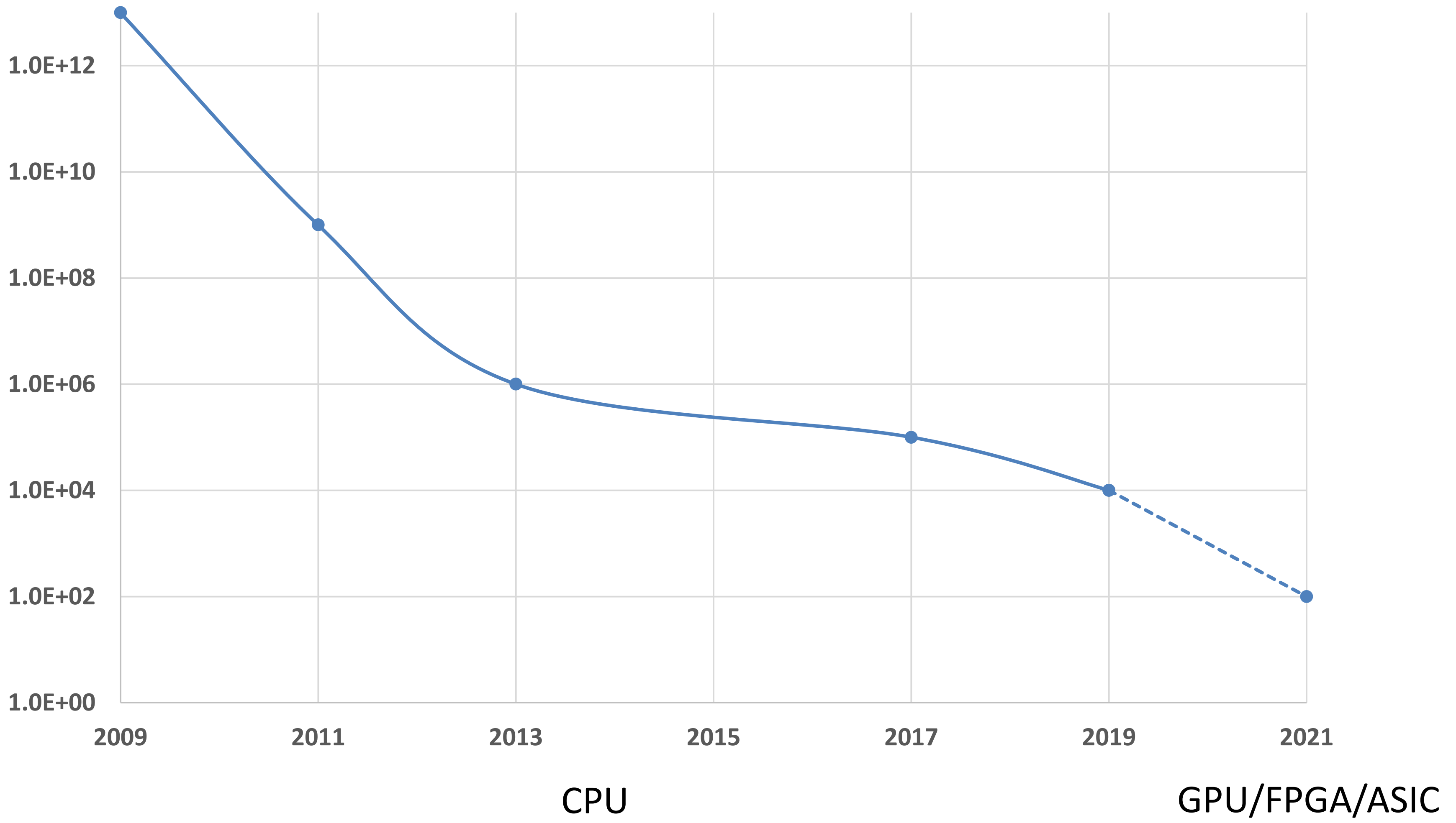
Mathematics of Homomorphic Encryption

- **New hard problems** proposed (2004-2013)
  - ❑ Small Principal Ideal Problem, Approximate GCD,
  - ❑ Learning With Errors (LWE), Ring-Learning With Errors (RLWE)
  - ❑ Related to well-known hard lattice problems:
- **Lattice-based Cryptography:**
  - ❑ Proposed by Hoffstein, Pipher, and Silverman in 1996 (NTRU), Aijtai-Dwork
  - ❑ Compare to other public key systems: RSA (1975), ECC (1985), Pairings (2000)
- **Hard Lattice Problems:**
  - ❑ Approximate Shortest Vector Problem (SVP), Bounded Distance Decoding
  - ❑ 30 year history of Lattice-basis reduction (LLL, BKZ, BKZ 2.0, FpLLL, sieving, challenges)
- **Security:**
  - ❑ Best attacks take exponential time
  - ❑ Secure against quantum attacks (so far…)

High-level Idea

- **Encryption:**
  - Encryption adds noise to a “secret” inner product
  - Decryption subtracts the secret inner product and the noise becomes easy to cancel
- **Hard Problem:**
  - Hard problem is to “decode” noisy vectors
  - If you have a short basis, it is easy to decompose vectors
- **Homomorphic property:**
  - Lattice vectors → coefficients of polynomials
  - Polynomials can be added and multiplied

Performance overhead improvement over Time (log scale!)



Microsoft SEAL 3.2 Performance Numbers

Machine : Intel Core i7-8700k @ 3.70 GHz; Single Thread

PolyModulus Degree	Encrypt	Encrypt Amort.	Decrypt	Decrypt Amort.	Add	Multiply	Multiply Amort.
4096	2683 us	1.31 us	1776 us	17 us	0.008 us	517 us	**0.252 us**
8192	7114 us	1.7 us	6091 us	72 us	0.018 us	2746 us	**0.67 us**
16384	21000 us	2.5 us	25000 us	361 us	0.044 us	17000 us	**2.066 us**
32768	69000 us	4.24 us	118000 us	1877 us	0.114 us	105000 us	**6.410 us**

Private AI Demos History

- 2014: Heart attack risk, personal health data ~ 1 second
- 2015: CryptoNets demo showing neural net prediction: MNIST data set ~80 seconds
- 2016: Genomics predicting flowering time from 200K SNPs ~ 1 second
- 2016: Pneumonia mortality risk: intelligible models ~ 8 seconds for 4,000 predictions

- 2018: Twitter sentiment analysis (150K text features) ~ less than a second
- 2018: cat/dog image classification ~ less than a second

- 2019: Asure Run (Private Fitness App)
- 2019: Chest Xray diagnostics
- 2019: Secure Weather prediction

Demos

Demo 1 : AsureRun (Private Fitness App)

01

Private Storage and Analytics

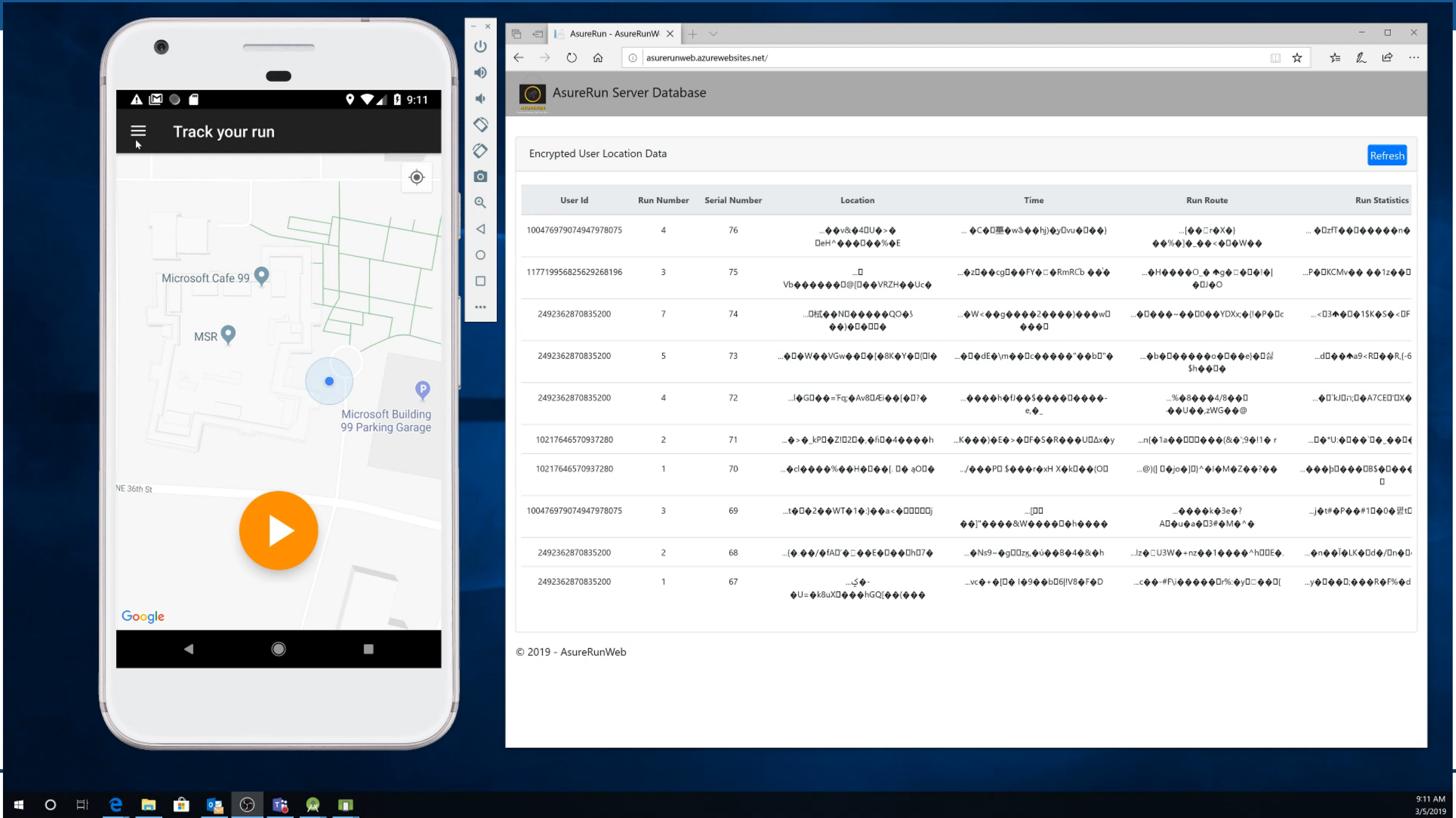
02

Private AI Prediction Services

03

Hosted Private Training

Scenario 1: AsureRun



Demo 2 : Chest X-Ray (disease prediction)

01

Private Storage and Analytics

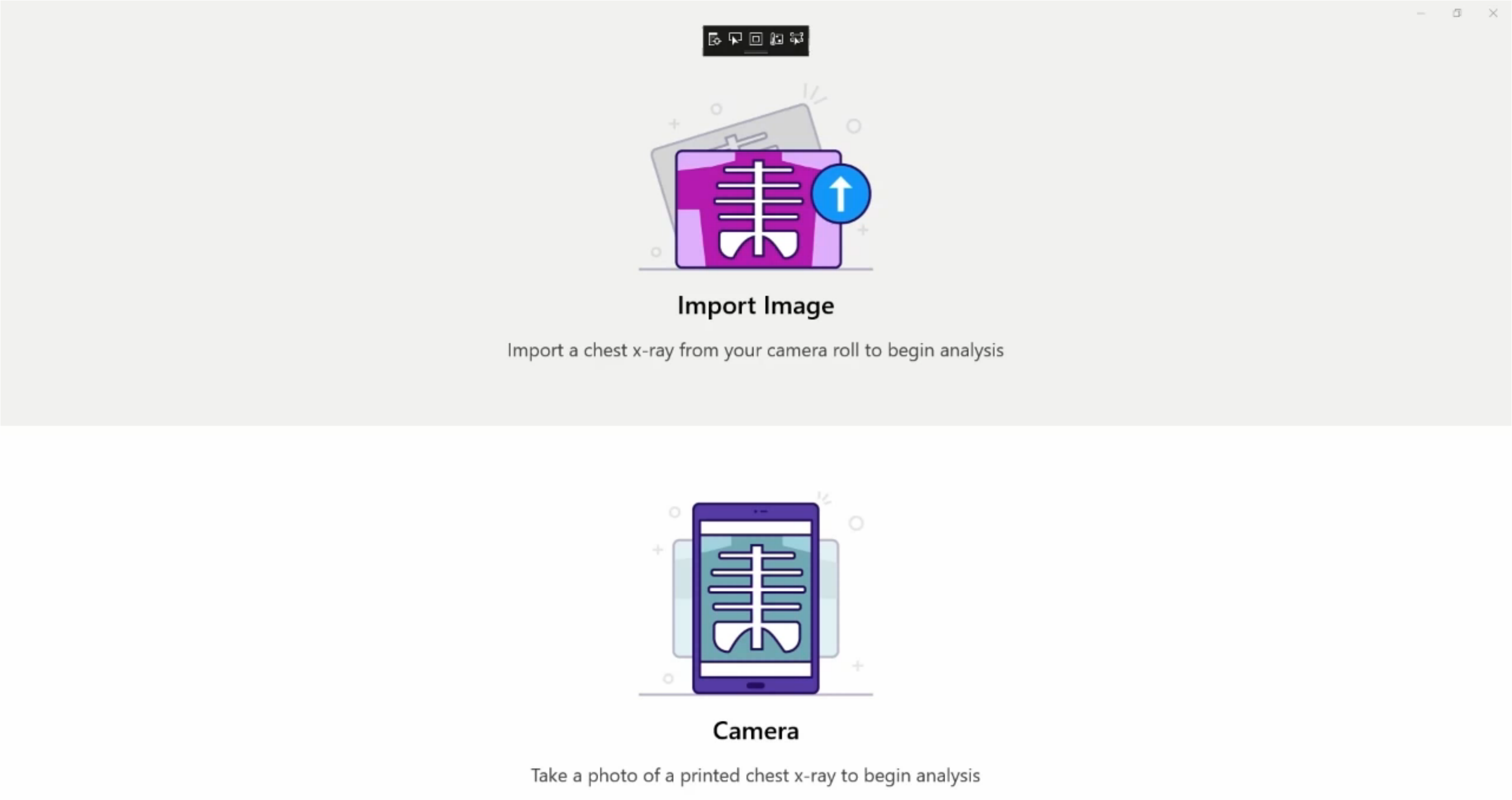
02

Private AI Prediction Services

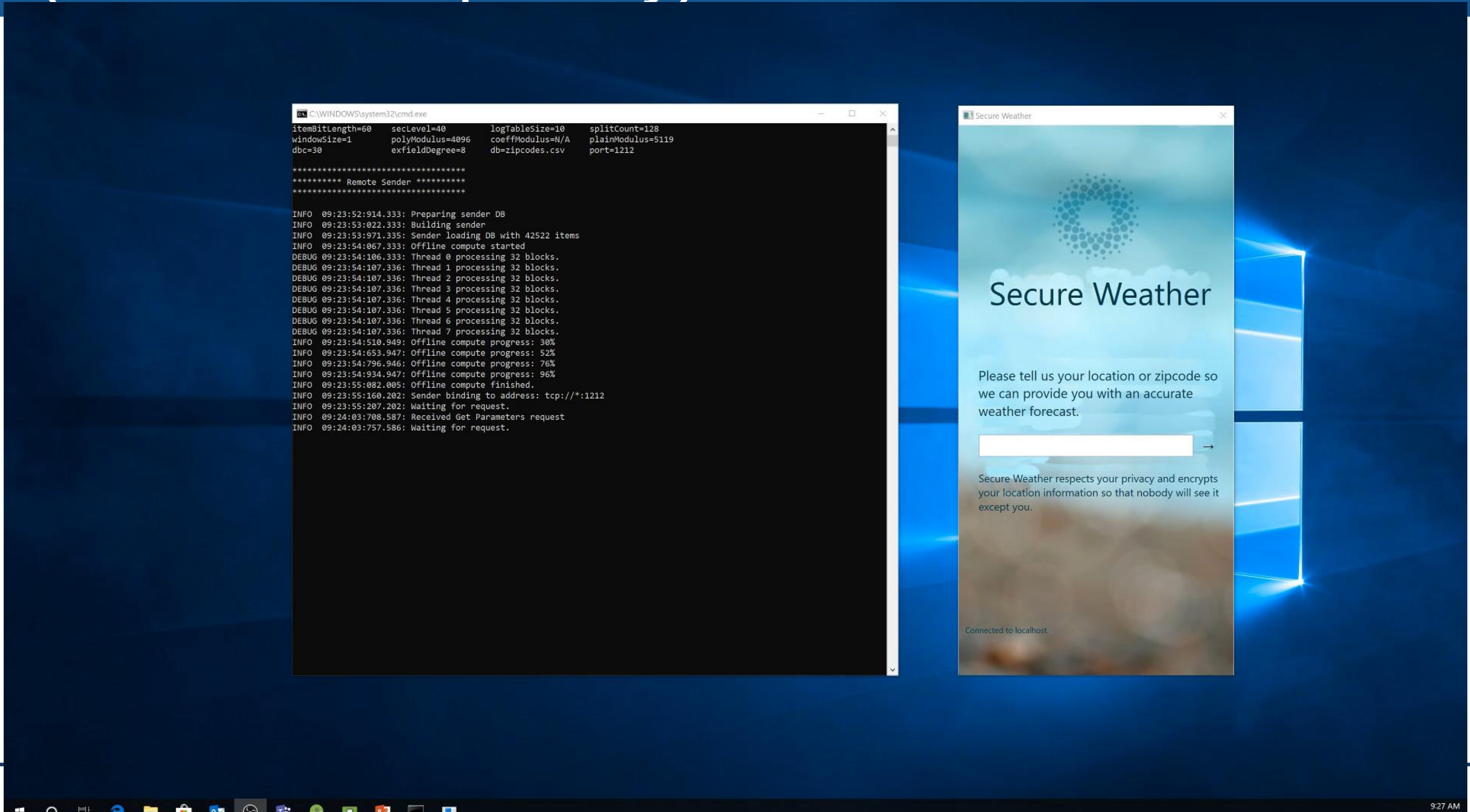
03

Hosted Private Training

Chest X-ray


Import Image
Import a chest x-ray from your camera roll to begin analysis
Camera
Take a photo of a printed chest x-ray to begin analysis

Scenario 3: Weather Prediction (with location privacy)



Resources

Microsoft Research project:

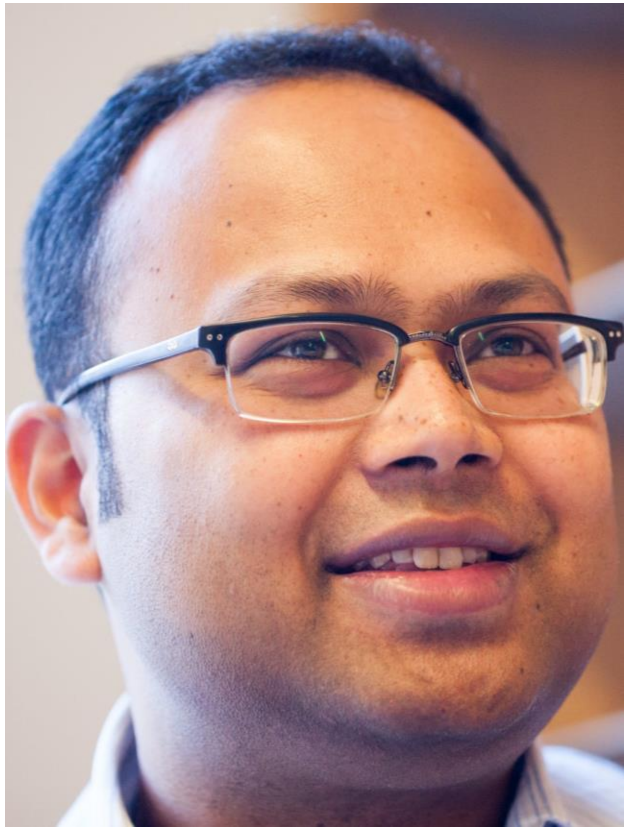
https://www.microsoft.com/en-us/research/project/homomorphic-encryption/

Standardization community:

http://homomorphicencryption.org/

SEAL code to download:

https://www.microsoft.com/en-us/research/project/microsoft-seal/



Suman Jana,
Columbia University

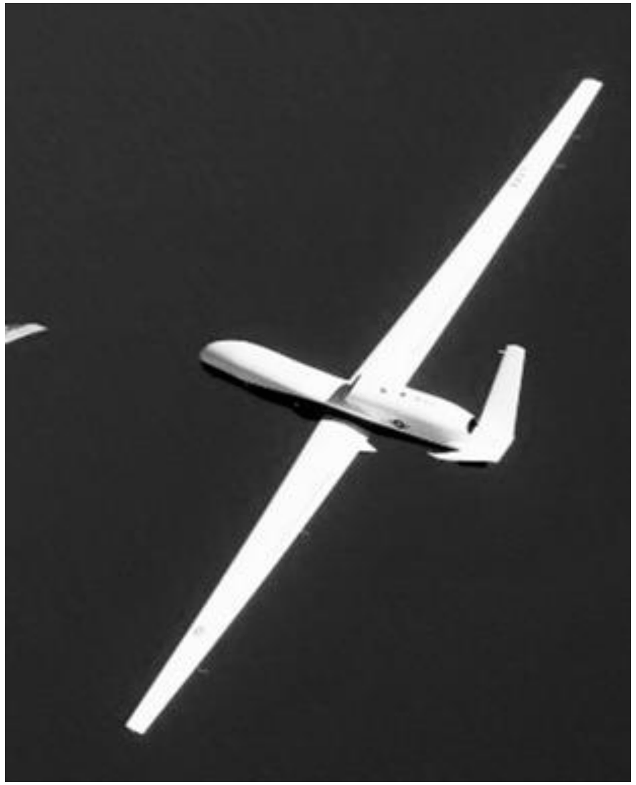
Assistant Professor of Computer Science

Formal verification of Neural Networks

Machine Learning Systems in Security-critical Domains

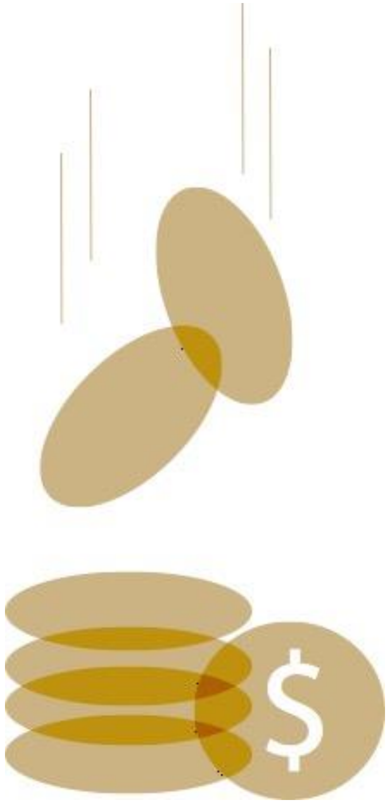
Autonomous Vehicles

(Tesla, Google, ...)



Unmanned Aircraft Systems

(Navy MQ-4C Triton)



Huge potential benefits

Machine Learning: Safe and Secure?

Tesla Autopilot fatal crash, June 2017

Tesla Autopilot fatal crash, March 2018

Uber fatal crash, June 2018

Not robust or reliable

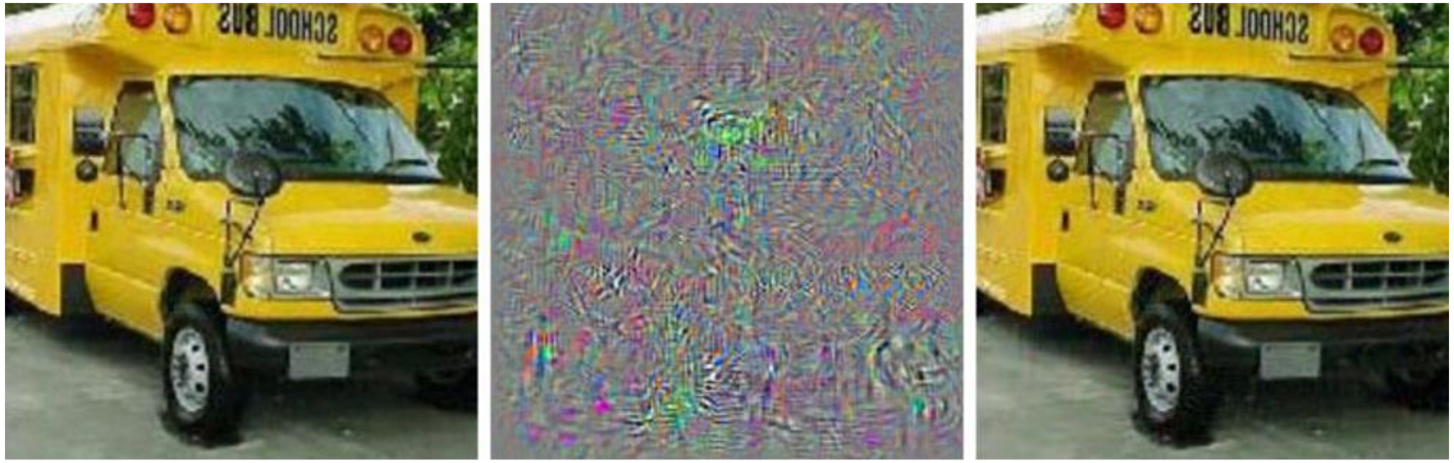
Danger

Adversarial Examples

Adversarial examples:
- Minor perturbations will lead to misclassifications with high confidence

An inherent weakness of neural networks

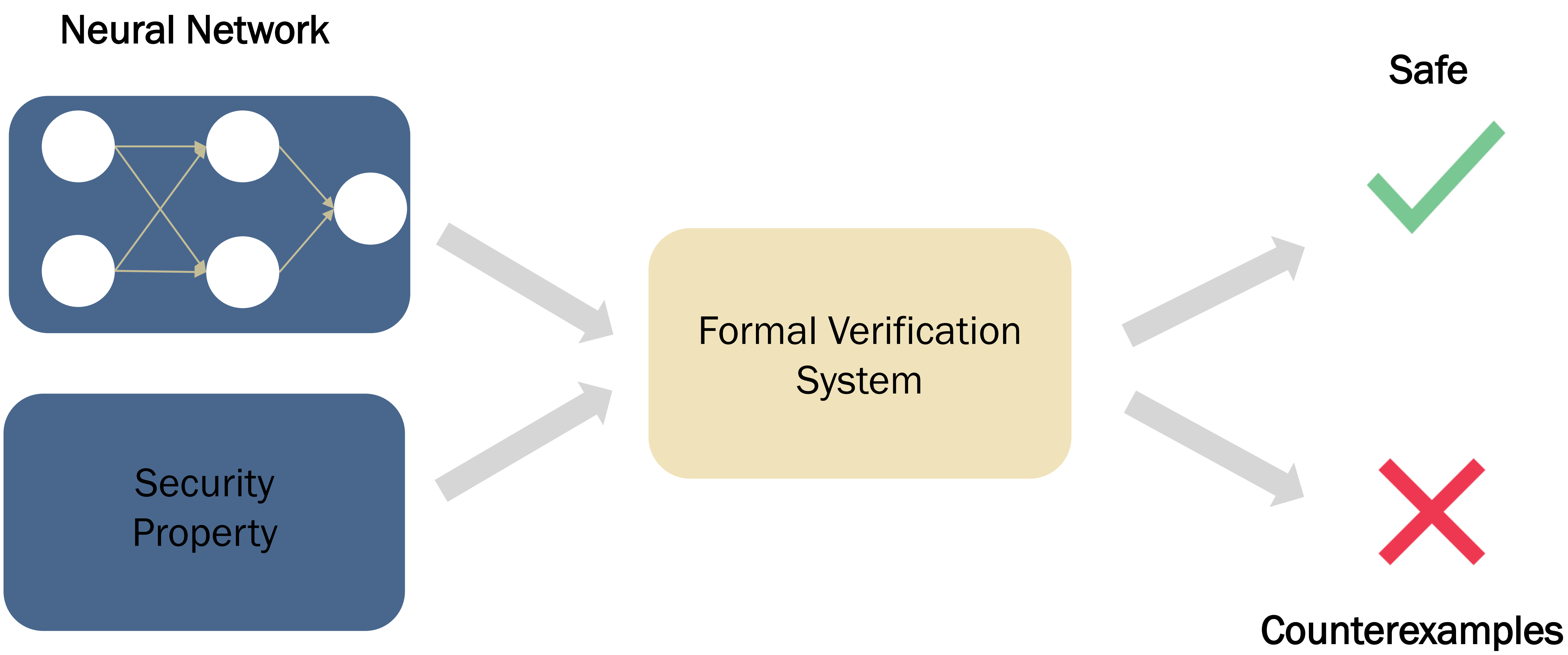
No scalable technique to prove non-existence of adversarial examples



Bus | Adversarial Noise | Ostrich

Formal analysis is urgently needed

Formal Verification of Neural Networks



Security Properties

Given a neural network

Given input ranges

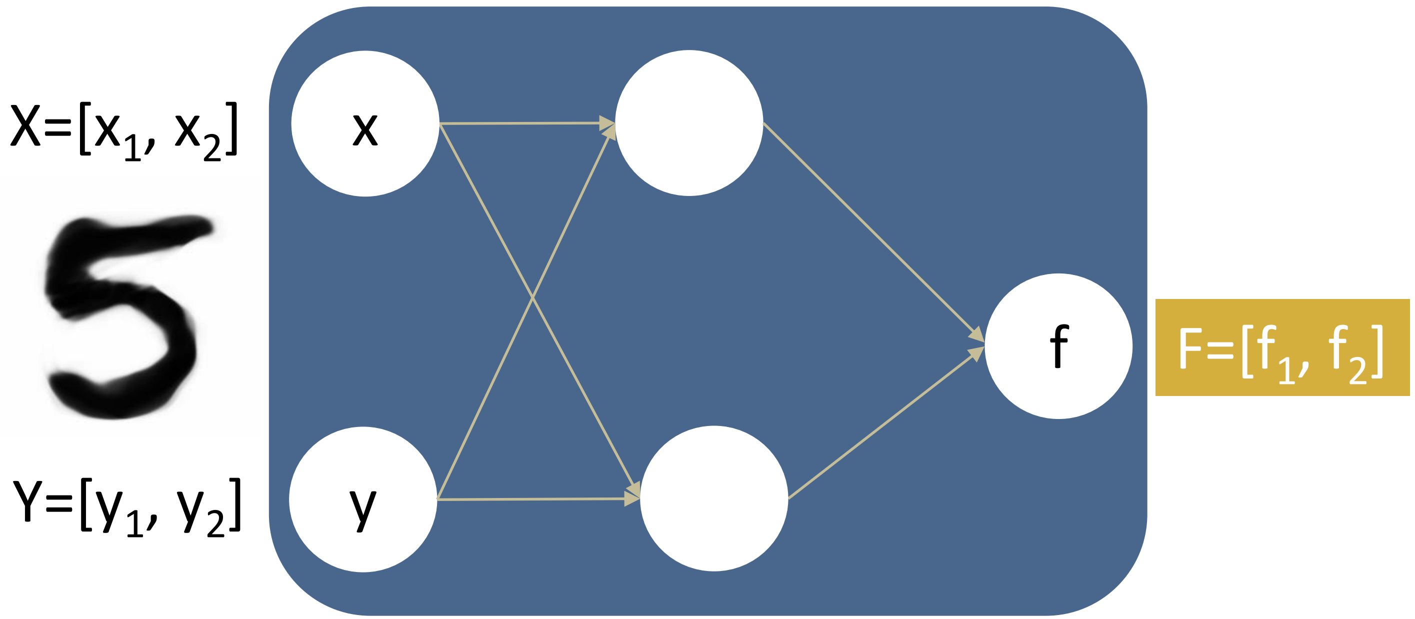
- p-norm for images
- Customized ranges

Check output range F

✓ Safe: F satisfies property P

✗ Unsafe: counterexample found

Input layer Hidden layers Output layer



Hard to analyze

Hardness of Formally Bounding Output Ranges

NN without activation functions

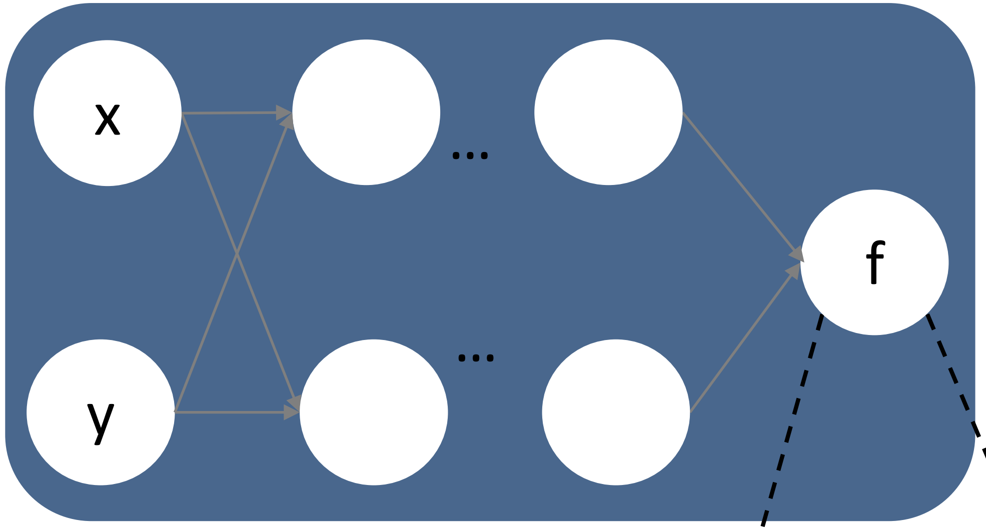
- Linear, easy to analyze
- Not very useful in practice

ReLU

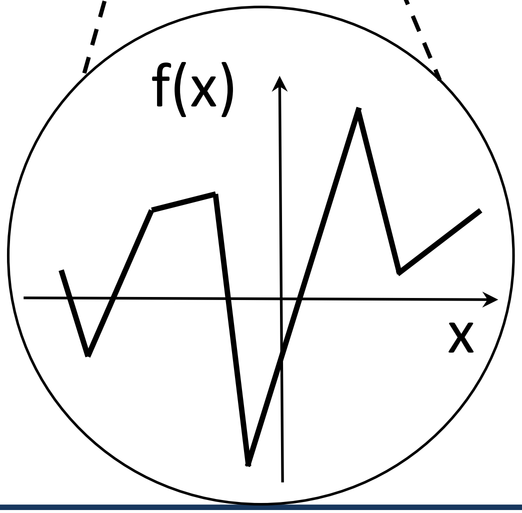
- ReLU(x)=max(x,0)
- Two linear pieces

ReLU-based NN

- Outputs are determined by combination of ReLU pieces
- Linear pieces exponentially increase as NN gets larger



d^n linear pieces



Very expensive to track all linear pieces

Difference with formal verification of traditional software

Complete specification is not feasible
- Mostly local security properties with respect to an existing dataset (e.g., adversarial perturbations to images in a dataset should not change classifier output)

ML verification can provide verified lower bounds on robustness of a ML model against different attackers
- All perturbations bounded by L_inf norm
- All rotation angles within some bounded range

Generalizability
- Will verified robustness on a test dataset hold for new datasets?
- Machine learning classifiers are already making this assumption for regular accuracy
- Verification is simply leveraging generalizability to provide strong robustness guarantees

Existing Approaches: Customized Solvers

Extend SMT/linear/MILP solvers

- ReluPlex by Katz et al. [CAV'17]
- Sherlock by Dutta et al. [NFMS'18]

Limitations

- High overhead even for simple properties (>hours or days)
- Hard to scale for large networks (<=1000s of ReLU nodes)

Existing Approaches: Relaxation

Over-approximate NN output ranges

- Over-approximate with abstract domain
- Relax to convex optimization problems

Several existing works

- Kolter et al. [ICML'18]
- Gehr et al. [Oakland'18]
- Dvijotham et al. [arXiv:1803:06567]

Drawbacks:

- High false positive rates
- Inefficient at finding concrete violations

We want the best of both worlds!

Efficiently compute sound (i.e., over-approximated) bounds on NN outputs

Iteratively refine the bounds by spending increasingly more computation power

- Ensure that bound soundness is maintained at each step
- Perform cheap, targeted search for counterexamples in each iteration

Support fine-grained trade-off between over-approximation errors and computational effort

Interval Analysis

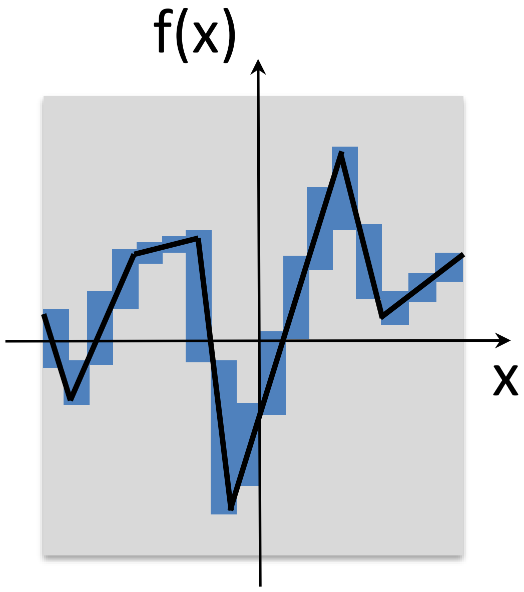
Sound (always over-approximate output)

Fast bound propagation

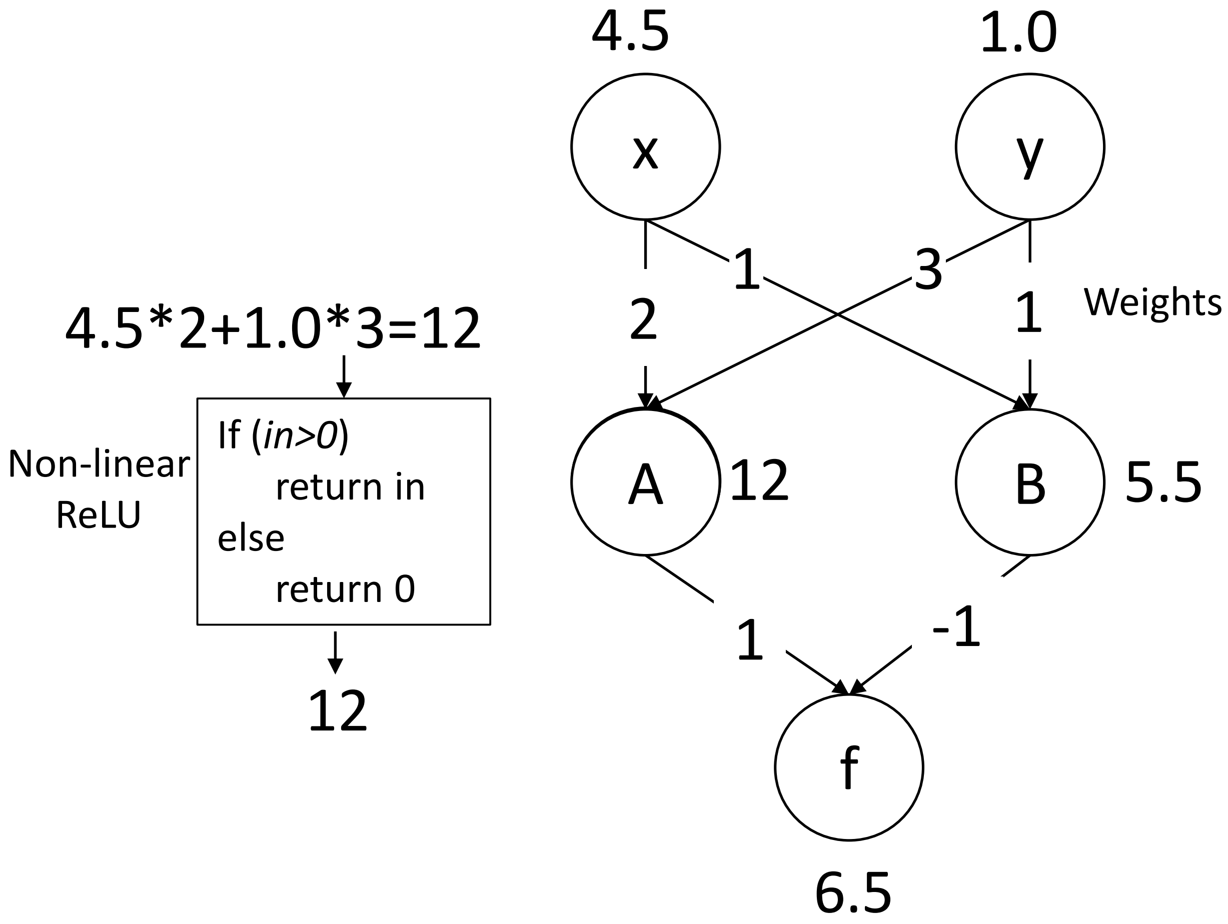
- Interval versions of common NN operations (addition and multiplication) are efficient

Easy and efficient refinement through bisection

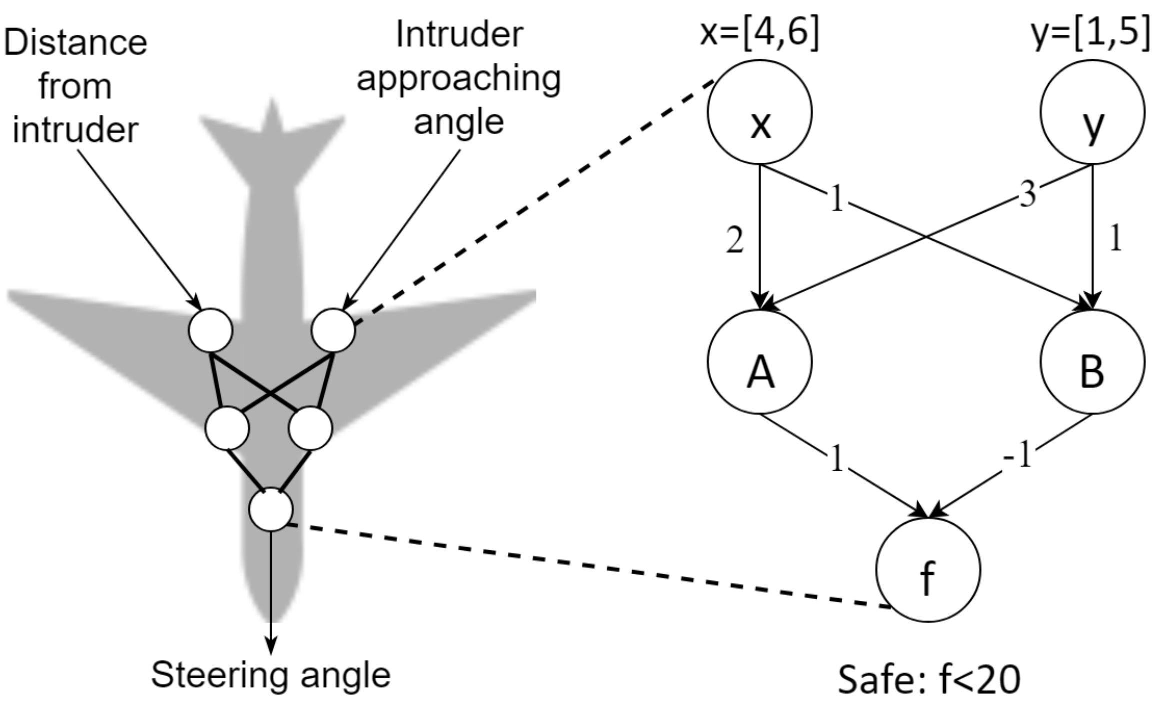
- Can approximate NN outputs up to any desired precision
- Amenable to highly parallelizable Branch and bound techniques

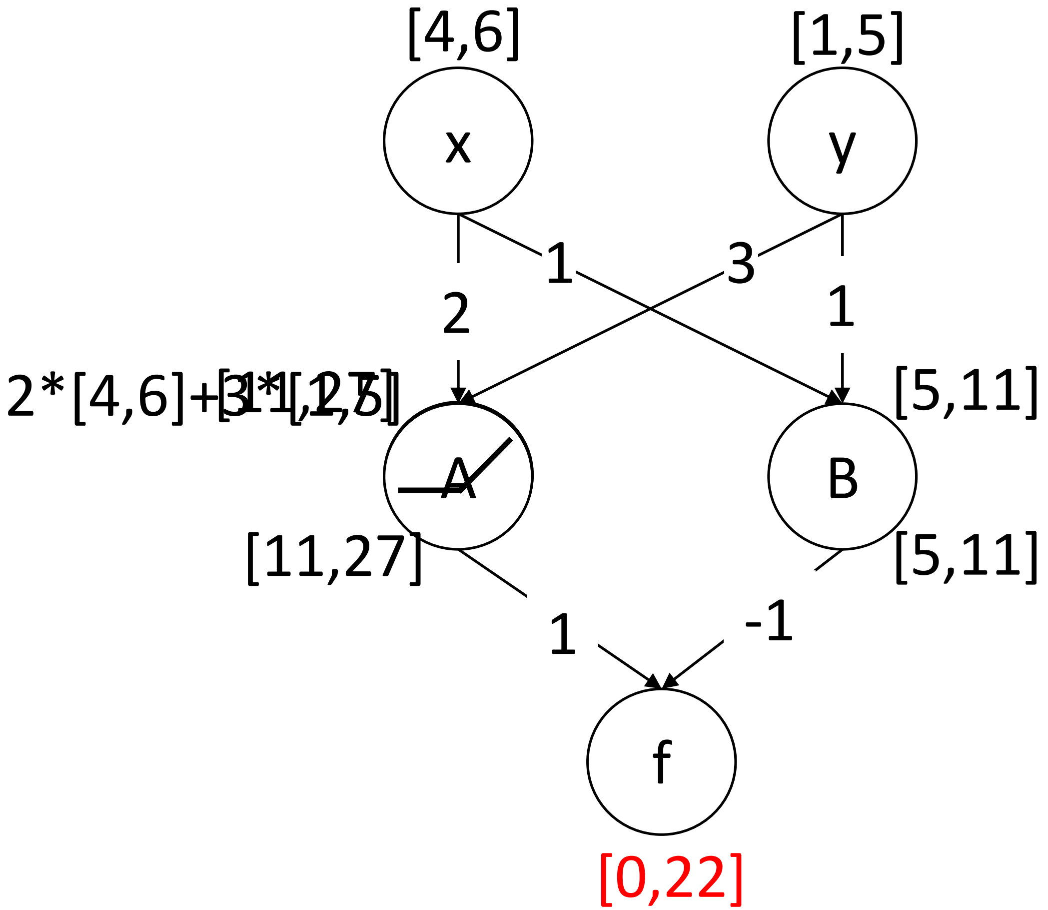


Neural network basics (inference)



Sample of Naive Interval Analysis





True security violation?

Security property:

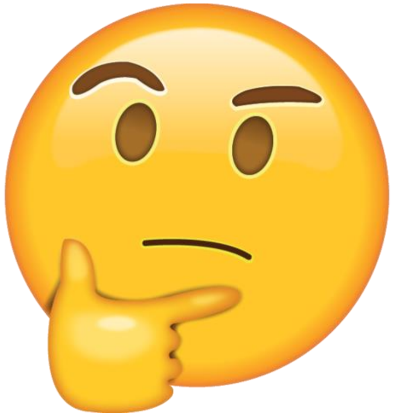
input ranges: x=[4,6], y=[1,5]

output: f<20

Ignoring Dependencies Cause Overestimation!

$x = [-1,1]$

$f = x - x = ?$



Naive interval:

$f = [-1,1] - [-1,1] = [-2,2]$

True bound:

$f = x - x = [0,0]$

Dependency matters

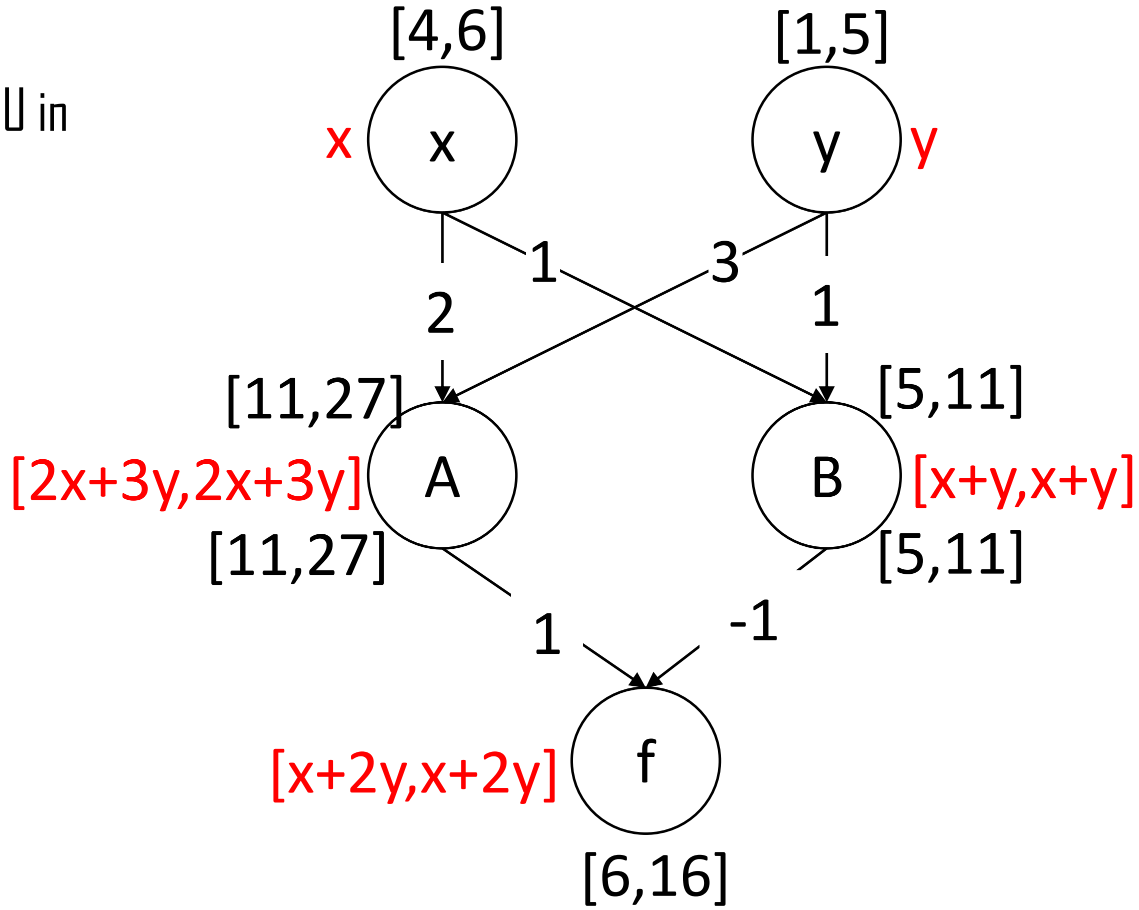
Symbolic Interval Analysis

Symbolic interval [Eq_{low}, Eq_{up}]

- Symbolic linear upper and lower bound equations for each ReLU in terms of inputs

Tighter output approximation

- Track input dependencies



f is always less than 20

Safe!

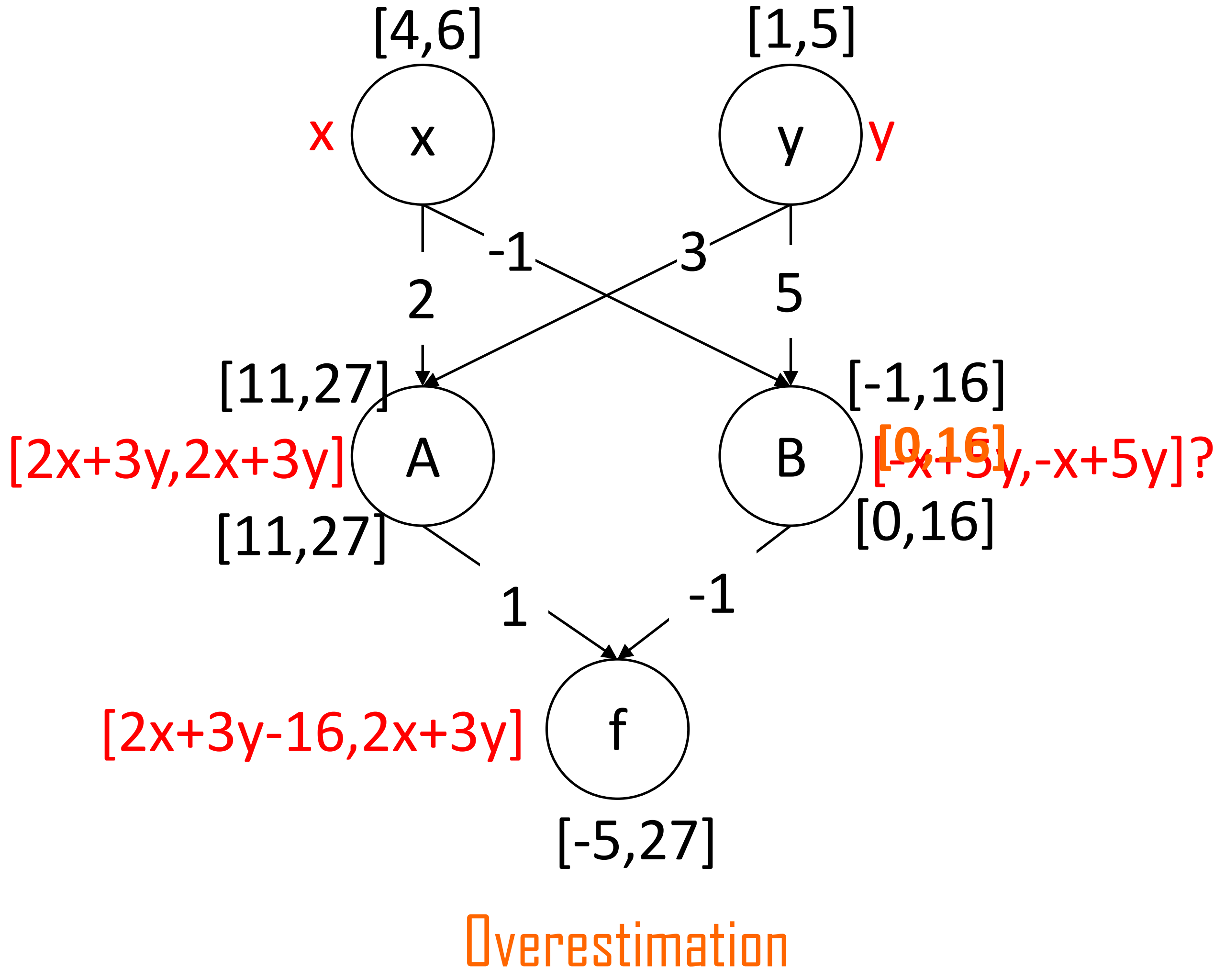
Security property:

input ranges: x=[4,6], y=[1,5]

output: f<20

Symbolic Interval Analysis is Not Perfect

Concretization
- Only when nonlinearity occurs
- Overestimation
- May have many false positives for large NNs



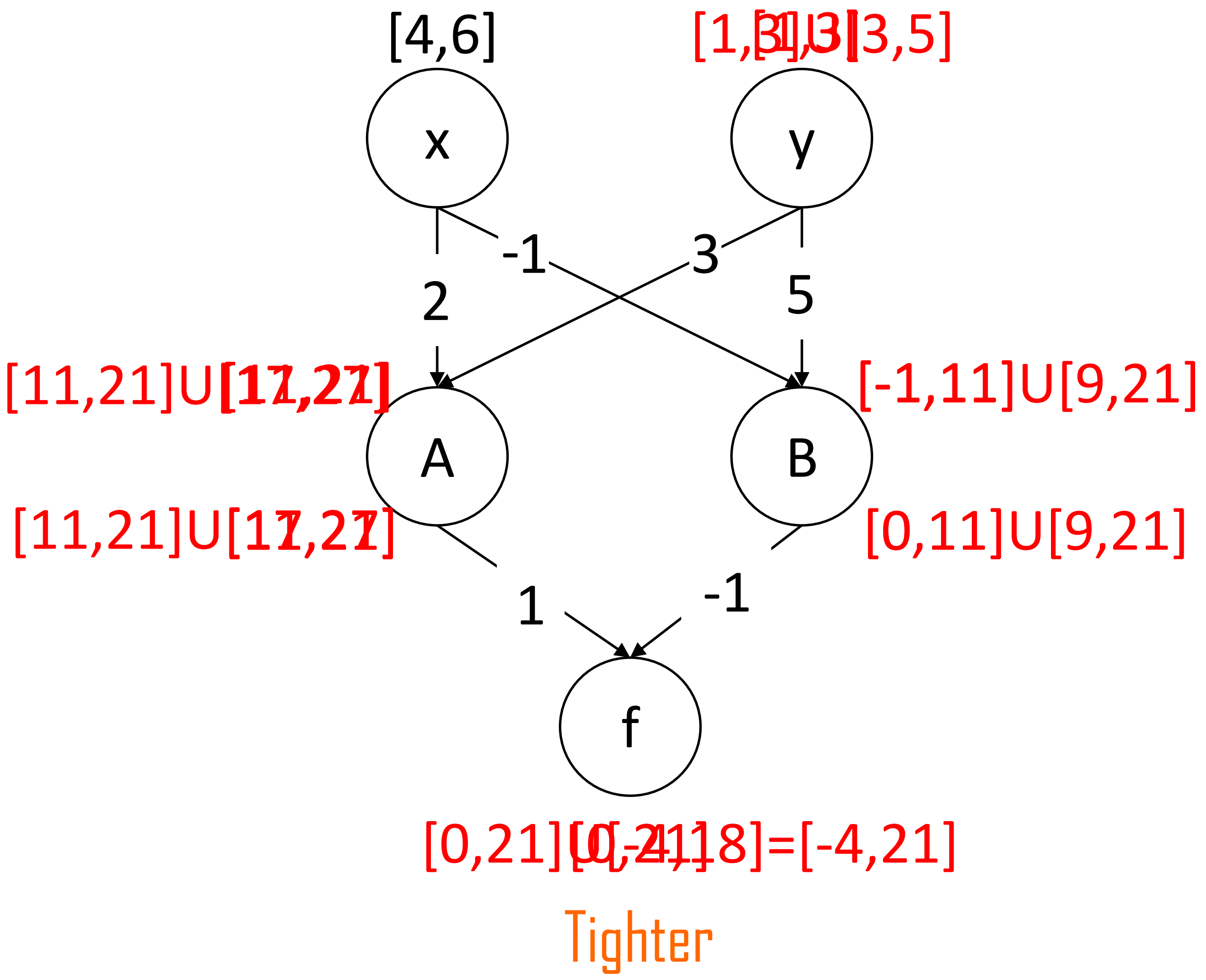
Security property:
input ranges: x=[4,6], y=[1,5]
output: f<20

Iterative Interval Refinement with One Bisection

Iterative refinement

- Bisect input ranges
- Compute union of output ranges
- Iteratively refine

Overestimation error decreases as input width becomes smaller



Security property:
input ranges: x=[4,6], y=[1,5]
output: f<20

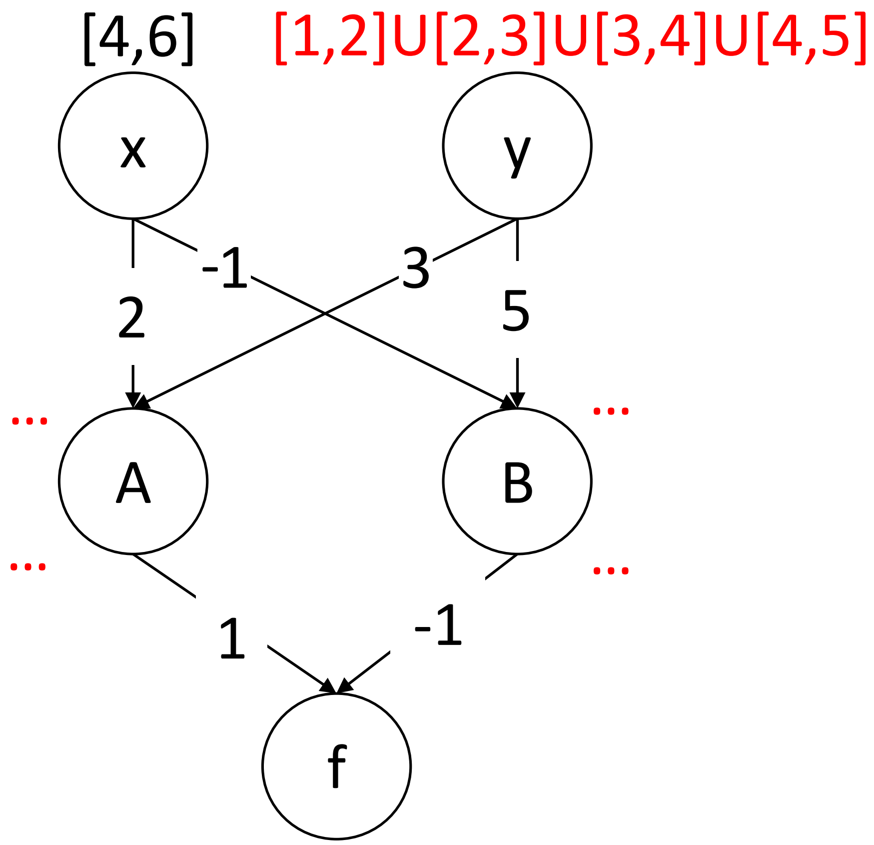
Iterative Interval Refinement with Two Bisections

Iterative refinement

- Bisect input ranges
- Compute union of output ranges
- Iteratively refine

Overestimation error decreases as input width becomes smaller

Security property:
input ranges: x=[4,6], y=[1,5]
output: f<20



[5,18]U[3,17]U[1,15]U[0,19]=[3,19]

f is always less than 20

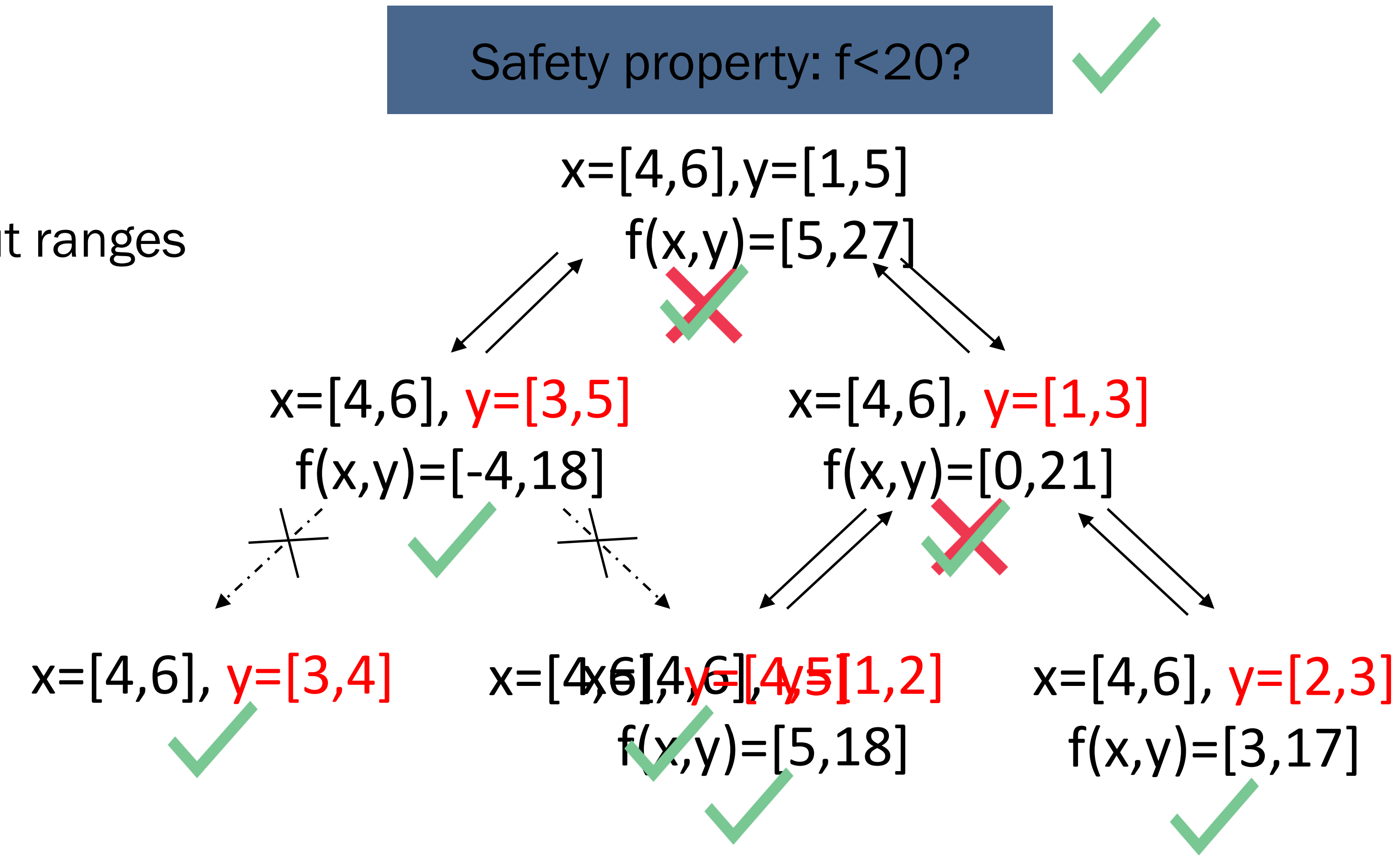
Safe!

Benefits of Iterative Interval Refinement

Highly uneven distribution
- No need to split safe subtrees
- Flexibly refine output based on the property

Iteratively narrow down violating input ranges
- Easily locate counterexamples

Highly parallelizable
- Subtrees are independent



ACAS Xu Models

Airborne collision avoidance system

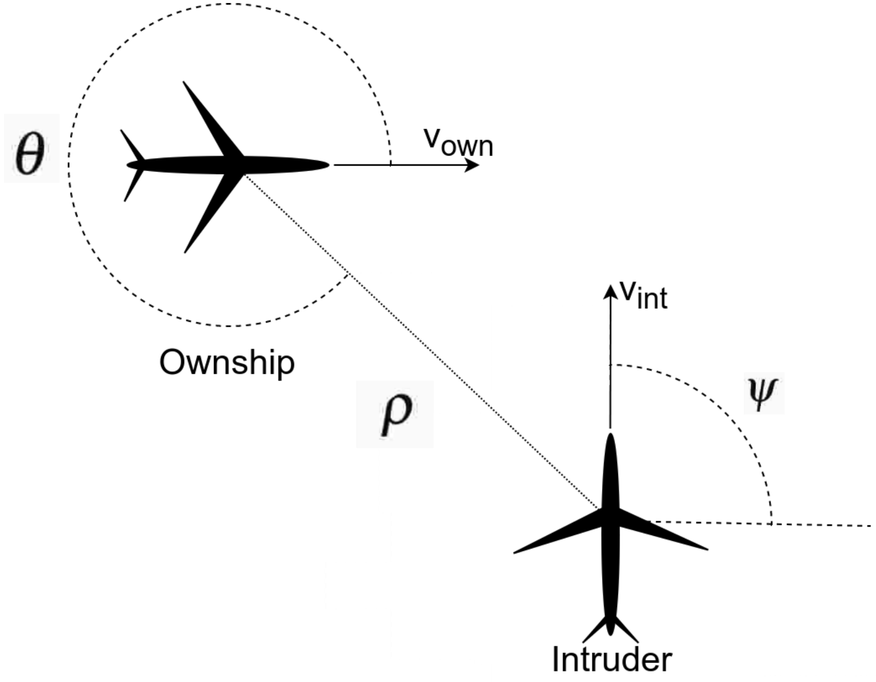
- Planned to be installed on Navy MQ-4C Triton
- Tested by NASA and FAA

Networks

- 5 inputs: $\rho, \theta, \psi, v_o, v_i$
- 5 outputs: COC, weak left, weak right, strong left, strong right
- 6 hidden layers, each with 50 ReLU nodes

Markov decision process

- 45 models=5*9
- previous decision: 5
- time until loss of vertical separation: 9



Sample Example of ACAS Xu Safety Properties

Safety property:

Given input ranges:

$\rho = [1000, 1200]$

$\theta = [\pi, \pi]$

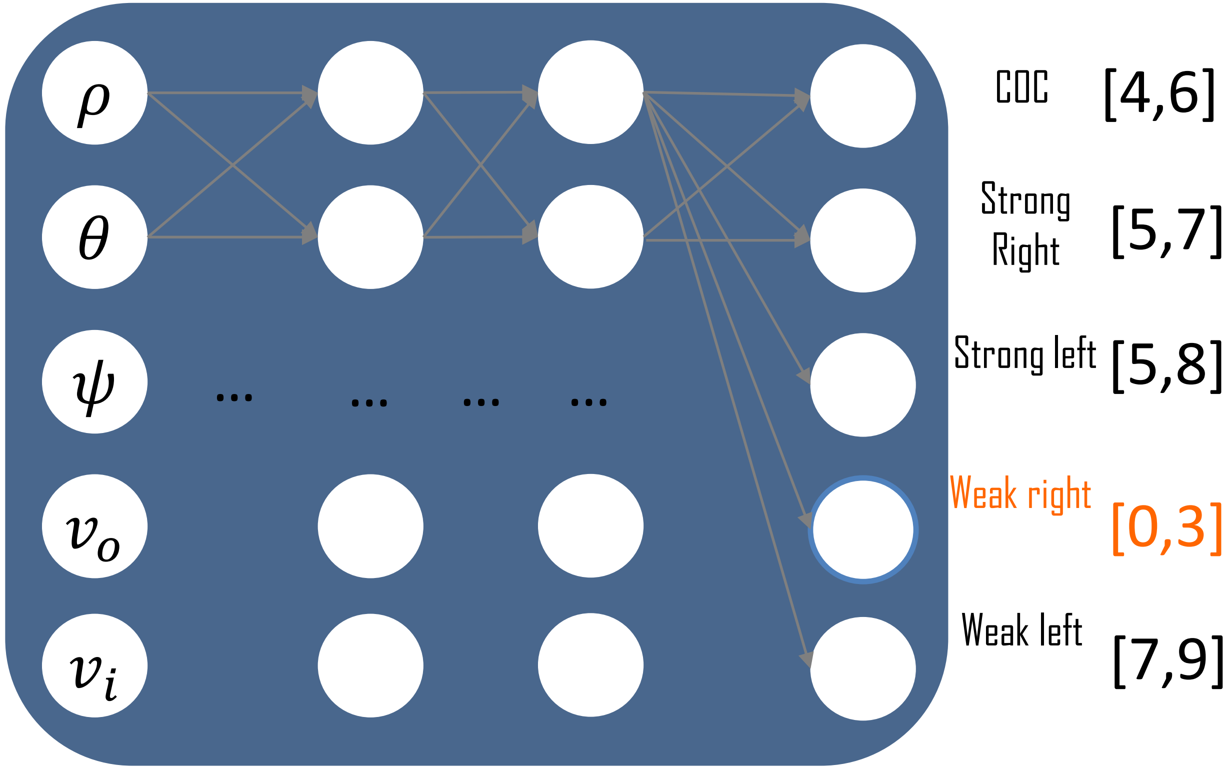
$\psi = [-\pi, -0.5\pi]$

$v_o = [60, 80]$

$v_i = [120, 140]$

Desired output:

Weak right



Verified safe!

ReluVal vs. Reluplex on ACAS Xu

Source	Networks	Reluplex Time (sec)	ReluVal Time (sec)	Speedup
10 Security Properties Proposed by Guy Katz et al.	45	>443,560.73	14,603.27	>30x
	34	123,420.40	117,243.26	1x
	42	35,040.28	19,018.90	2x
	42	13,919.51	441.97	32x
	1	23,212.52	216.88	107x
	1	220,330.82	46.59	4729x
	1	>86400.0	9,240.29	>9x
	1	43,200.01	40.41	1069x
	1	116,441.97	15,639.52	7x
	1	23,683.07	10.94	2165x
5 Additional Security Properties	1	4,394.91	27.89	158x
	1	2,556.28	0.104	24580x
	1	>172,800.0	148.21	>1166x
	2	>172,800.0	288.98	>598x
	2	31,328.26	876.86	36x

Over 200 times faster than Reluplex on average

Neurify [NeuRIPS'18]

Use Linear solver together with symbolic interval analysis

Instead of splitting input features, split inputs to intermediate nodes
- Only relaxed neurons can be overestimated
- Fewer nodes are overestimated
- We can split them instead of inputs

Iteratively split overestimated neurons into two linear cases
- Use linear solver to solve these cases separately

Verification of DAVE Models

DAVE-2 self-driving dataset
- Nvidia end-to-end deep learning for self-driving cars
- Map visual data to steering angles

Convolutional Networks
- 30,000 inputs: raw pixels
- 1 output: steering angle
- Over 10,000 ReLUs

Safety property
- Difference between the predicted steering angle and the ground-truth angle is always less than some threshold

Conclusions and Future Work

Symbolic Interval Analysis is a great tool for verifying NNs
- Efficient and sound over-approximations
- Iterative refinements with bisection
- Can be easily augmented with linear solvers

Exciting new directions
- Use formal verification tools for NNs as part of robust training
- Support verification of complex natural transformations like fog or rain
- Support other activation functions besides ReLUs and other types of networks like RNNs

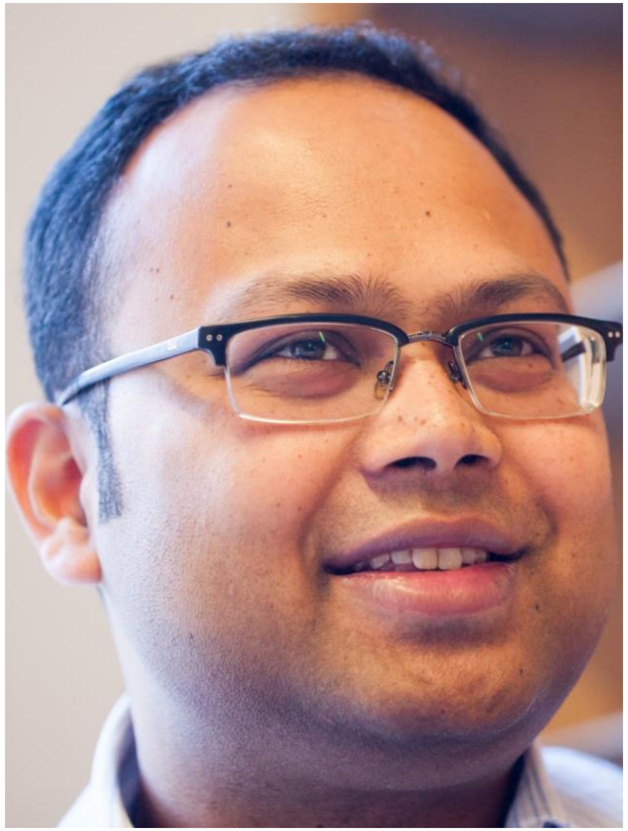
ReluVal: https://github.com/tcwangshiqi-columbia/ReluVal
Neurify: https://github.com/tcwangshiqi-columbia/Neurify

? Questions

MATHEMATICAL FRONTIERS

Cryptography and Cybersecurity

Kristin Lauter,
Microsoft Research



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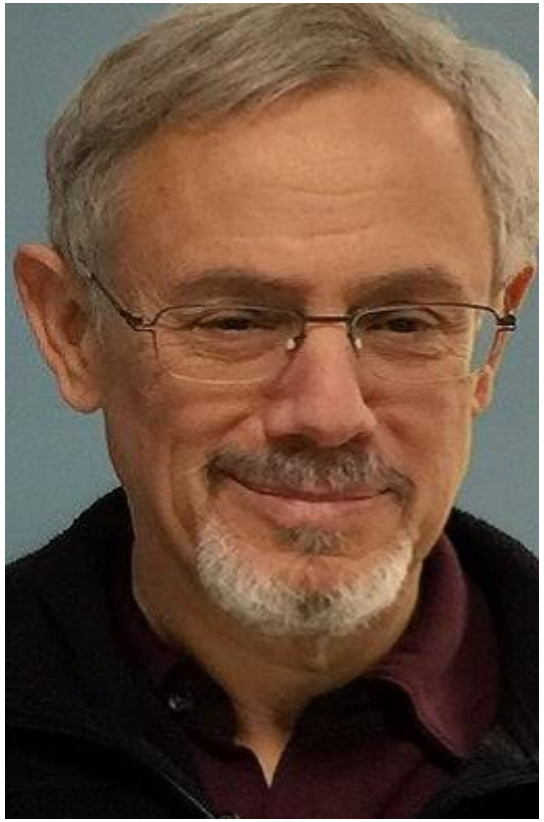

Mark Green,
UCLA (moderator)

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