Thermal transport and quasi-particle hydrodynamics

Kamran Behnia
ESPCI
Paris

Outline

Introduction

I. Hydrodynamics of phonons

II. Hydrodynamics of electrons

III. A boundary to thermal diffusivity?

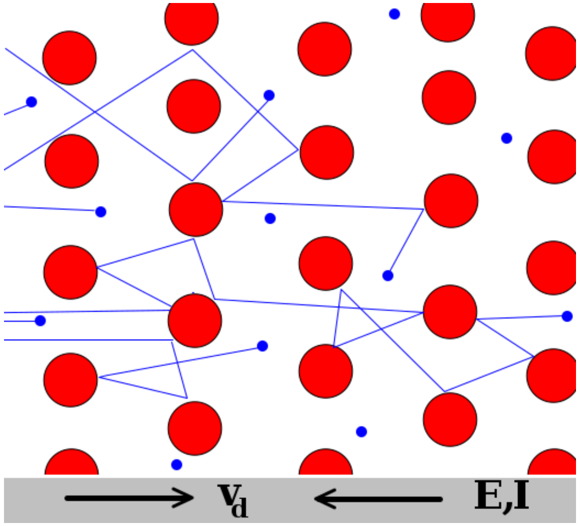
IV. Brief remarks on Berry curvature and entropy flow

Thermal and Electrical conduction

$$J_e = \sigma E$$

Ohm's law

Electric field generates a drift velocity in charge carriers!



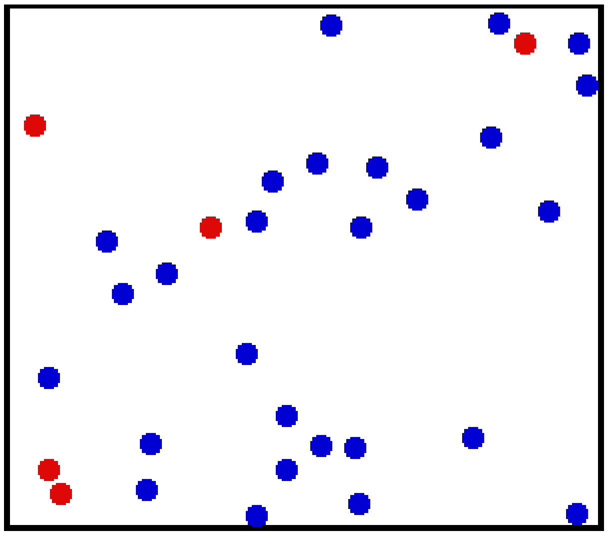
$$J_Q = -\kappa \nabla T$$

Fourrier's law

Temperature gradient generates a drift velocity in entropy carriers!

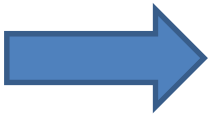
The Drude picture (circa 1900)!

Kinetic theory of gases



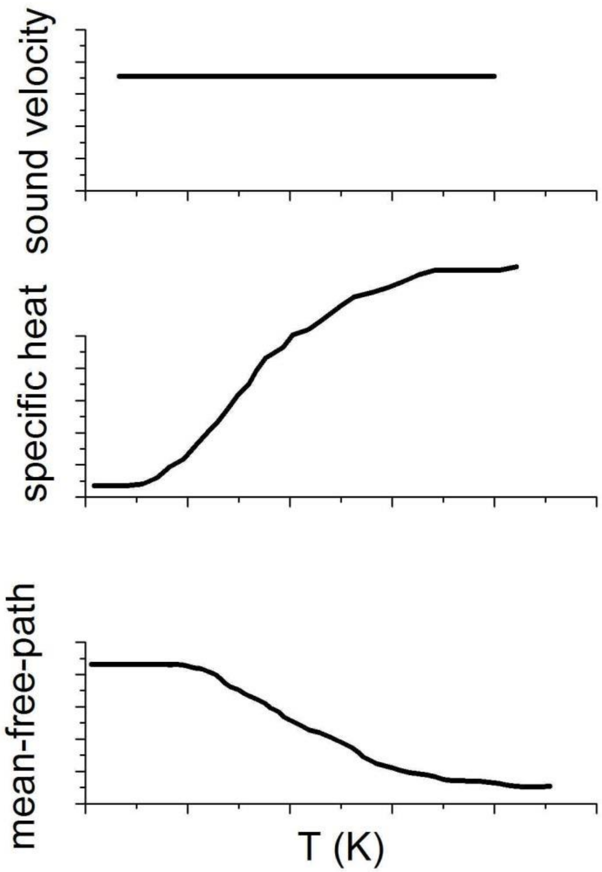
$\kappa = 1/3\ C\ v\ l$

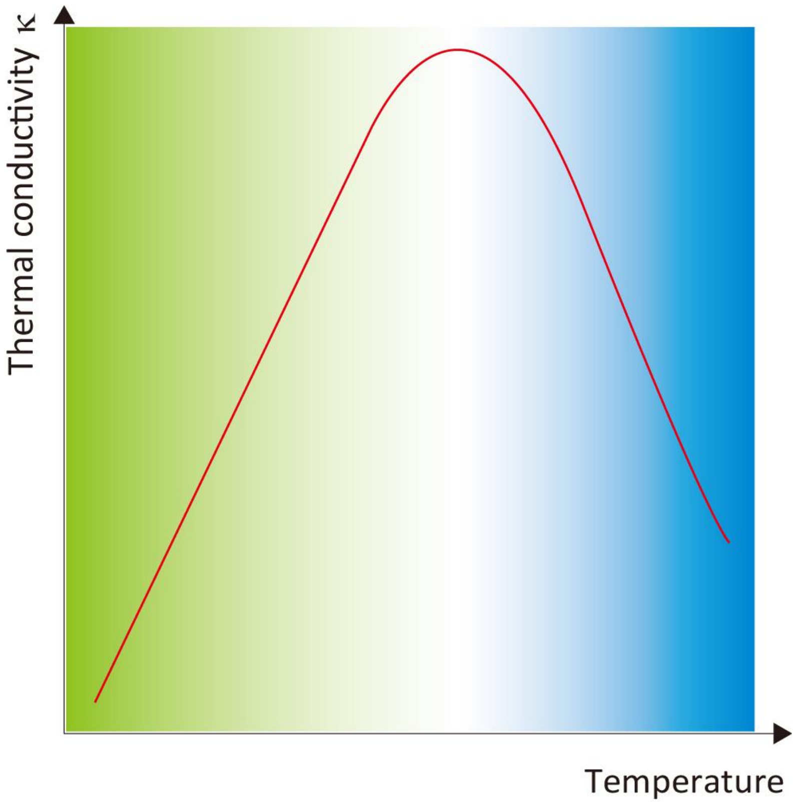
- Specific heat per volume
- Average velocity
- Mean-free-path of atomic particles

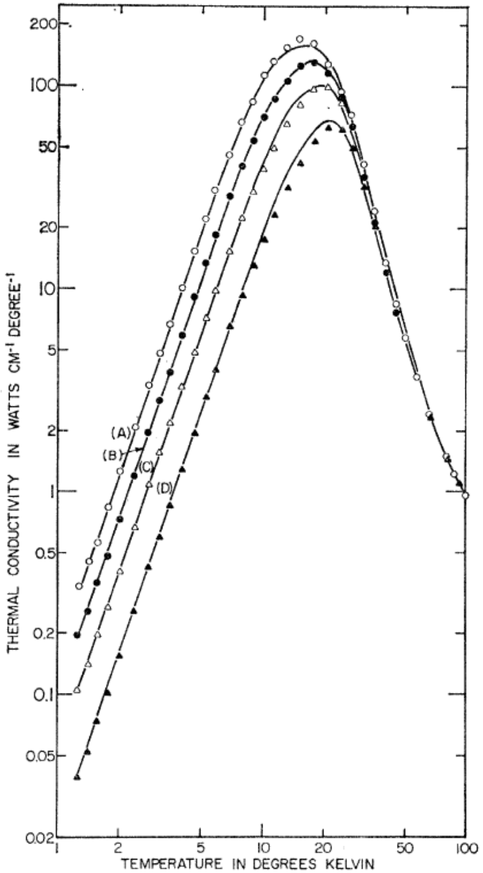


Thermal conductivity

Heat conduction in insulators







Thatcher, Phys. Rev. (1967).

Boltzmann-Peierls equation

$$\frac{\partial n_\mu(\boldsymbol{x},t)}{\partial t} + \boldsymbol{v}_\mu \cdot \nabla n_\mu(\boldsymbol{x},t) = -\frac{1}{\mathcal{V}}\sum_{\mu'} \Omega_{\mu\mu'} \Delta n_{\mu'}(\boldsymbol{x},t)$$

phonon density

mode index

velocity

Scattring matrix

Can be solved exactly!

Steady-state solution assuming a scattering time for mode μ :

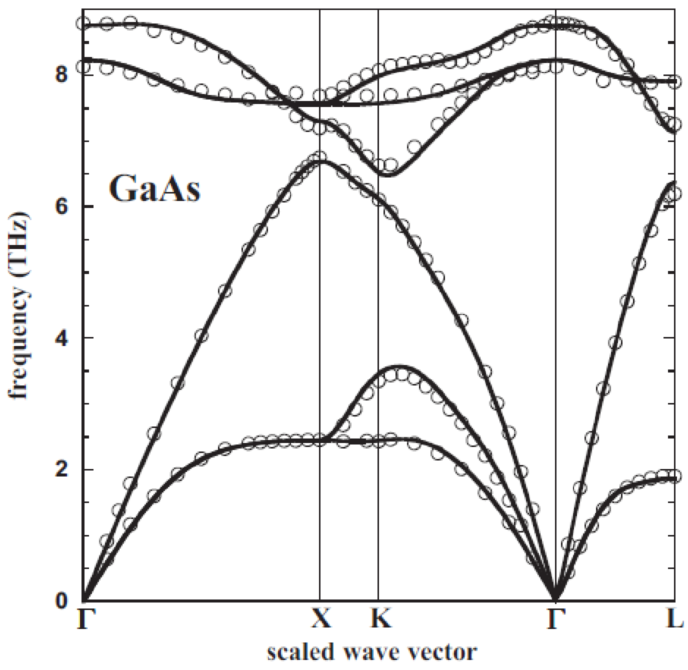
$$\kappa_i = \frac{1}{\mathcal{V}}\sum_{\mu}^{n} C_\mu v_\mu^i \ell_\mu^i$$

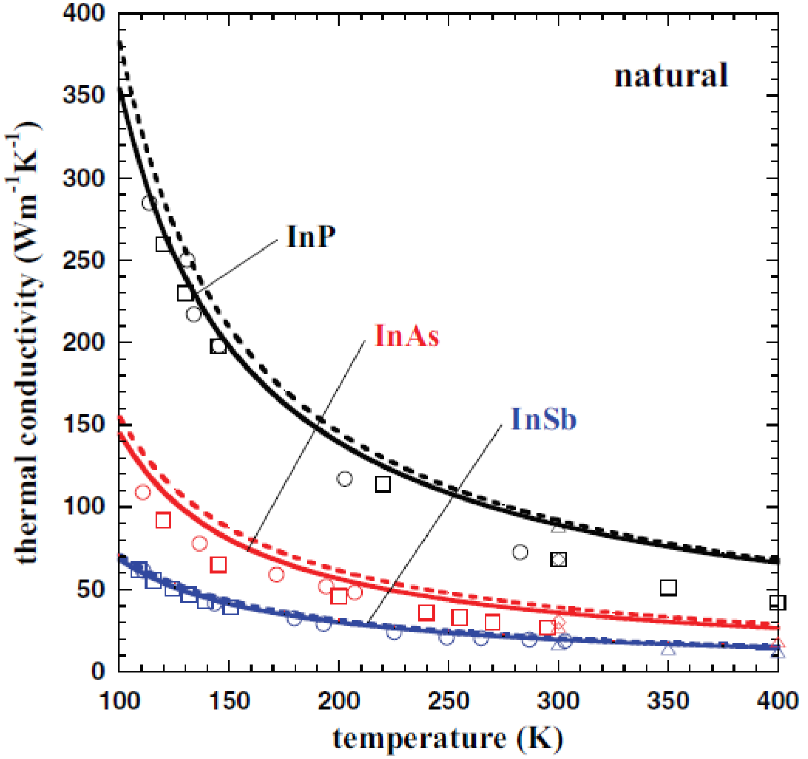
- Callaway, Phys. Rev. (1959)
- Cepellotti & Marzari, Phys. Rev. X (2016)

PHYSICAL REVIEW B **87**, 165201 (2013)

Ab initio thermal transport in compound semiconductors

L. Lindsay,[1] D. A. Broido,[2] and T. L. Reinecke[1]
[1]*Naval Research Laboratory, Washington, DC 20375, USA*
[2]*Department of Physics, Boston College, Chestnut Hill, Massachusetts 02467, USA*
(Received 30 November 2012; revised manuscript received 19 February 2013; published 2 April 2013)





Remakably successful above the peak (the intrinsic regime)!

Phononn gas? Fermi liquid?

Quasi-particles in solids

- A lattice (and its defects)
- Collisions limit the flow by giving away momentum to host solid.
- Dissipation arises even in absence of viscosity.

Hydrodynamics

- No lattice
- Collisions conserve momentum and energy and keep thermodynamic quantities well-defined.
- Viscosity is the source of dissipation.

Questions:

- What does the hydrodynamic regime correspond to?
- Where does it emerge?
- Why is it interesting if you care about collective quantum phenomena?

SOVIET PHYSICS USPEKHI VOLUME 11, NUMBER 2 SEPTEMBER–OCTOBER, 1968

530.145 + 536.48

HYDRODYNAMIC EFFECTS IN SOLIDS AT LOW TEMPERATURE

R. N. GURZHI

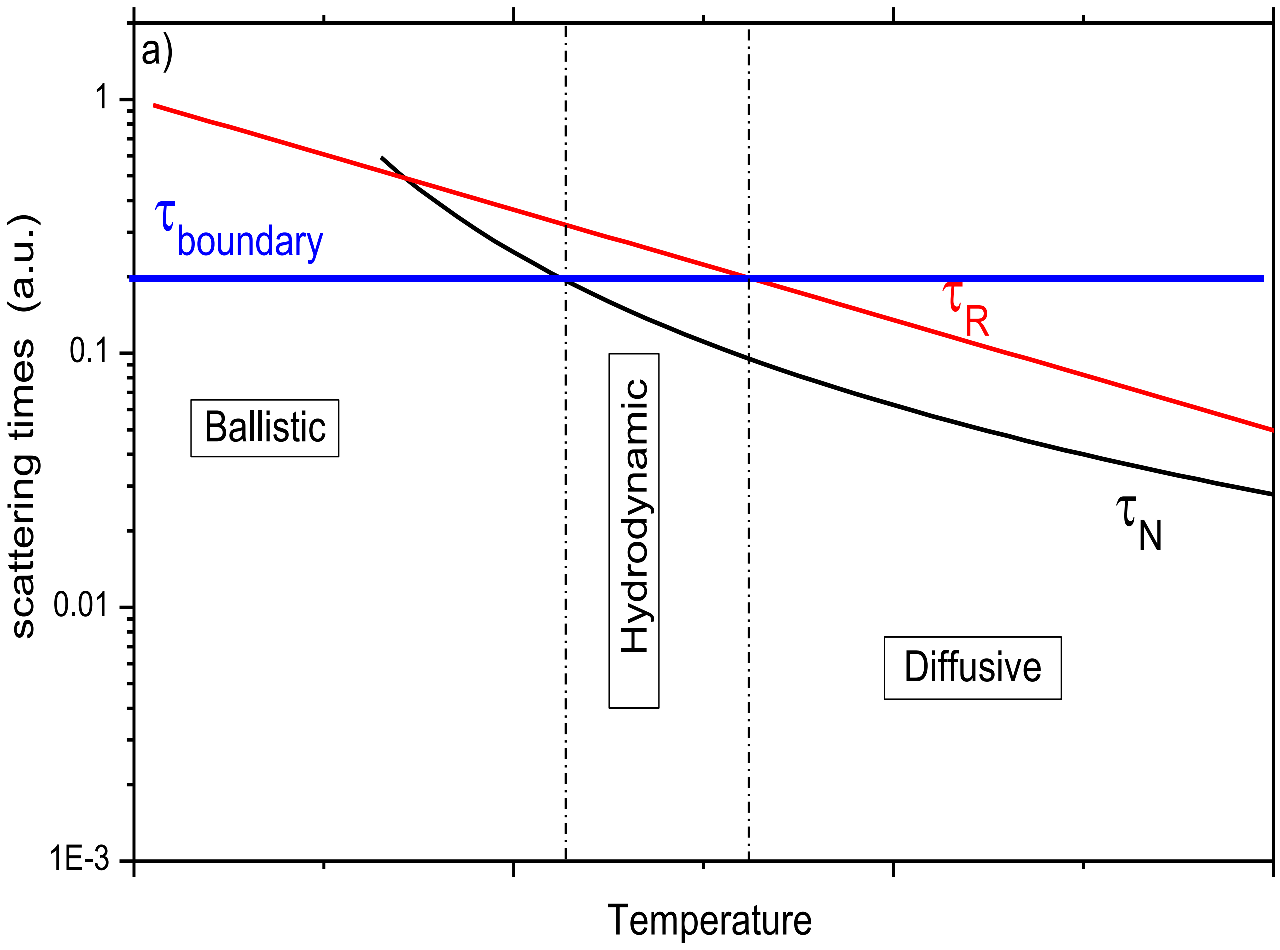
Physico-technical Institute, Academy of Sciences, Ukrainian SSR, Khar'kov

Usp. Fiz. Nauk 94, 689–718 (April, 1968)

"The phenomena of thermal conductivity of insulators and the electrical conductivity of metals have specific properties.
In both cases the total quasi-particle current turns out to be non-vanishing.
It follows that when only normal collisions occur in the system, there could exist an undamped current in the absence of an external field which could sustain it."

Without umklapp collisions, finite viscosity sets the flow rate!

The hydrodynamic window requires a specific hierarchy!



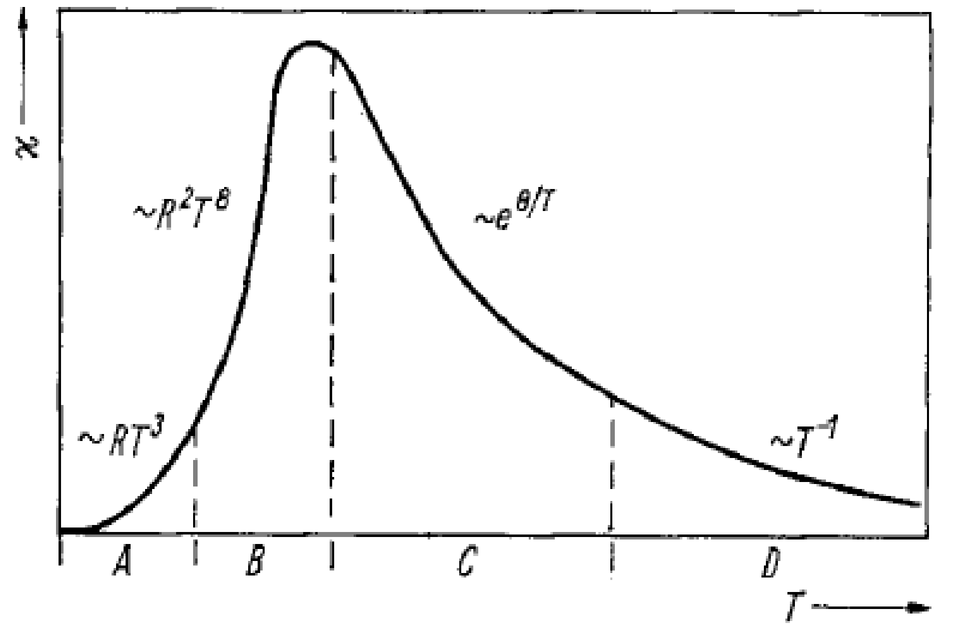
- **Abundant normal** scattering
- **Intermediate boundary** scattering
- **Small resistive** scattering

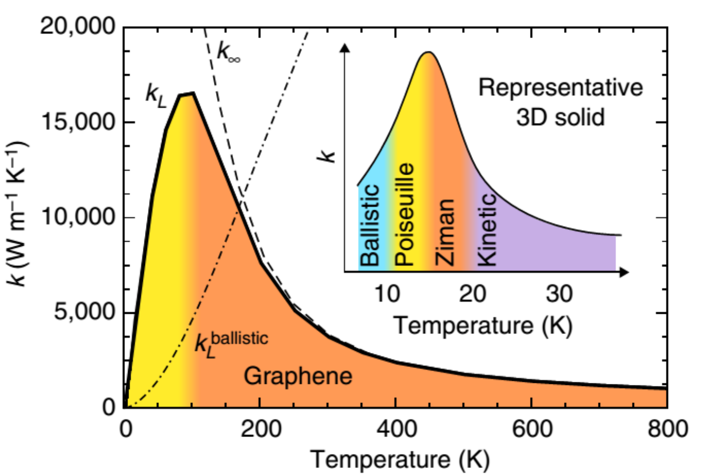
Hydrodynamic of phonons

H. Beck (a)[1], P. F. Meier (b)[2], and A. Thellung (c)

phys. stat. sol. (a) 24, 11 (1974)

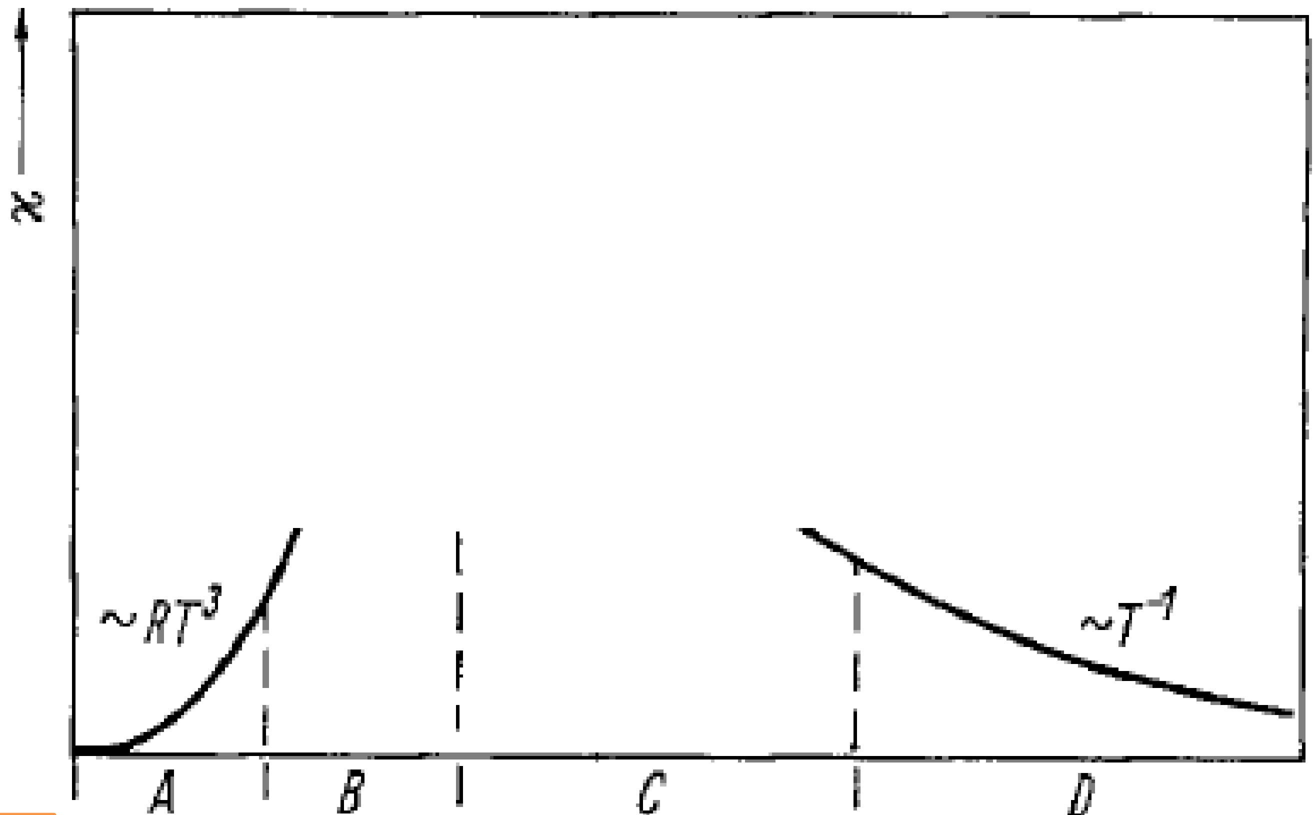
Fig. 1. Different regions of thermal conductivity:
A: Casimir region; $\tau_B \ll \tau_N, \tau_B \ll \tau_R$
B: Poiseuille flow region; $\tau_N \ll \tau_B \ll \tau_R$
C: Ziman region; $\tau_N \ll \tau_R \ll \tau_B$
D: kinetic region; $\tau_R \ll \tau_N \ll \tau_B$
Here τ_N, τ_R, τ_B denote the relaxation times for normal processes, resistive processes, and boundary scattering, respectively





A. Cepellotti et al., Nature Comm. 2014

Regimes of heat transport



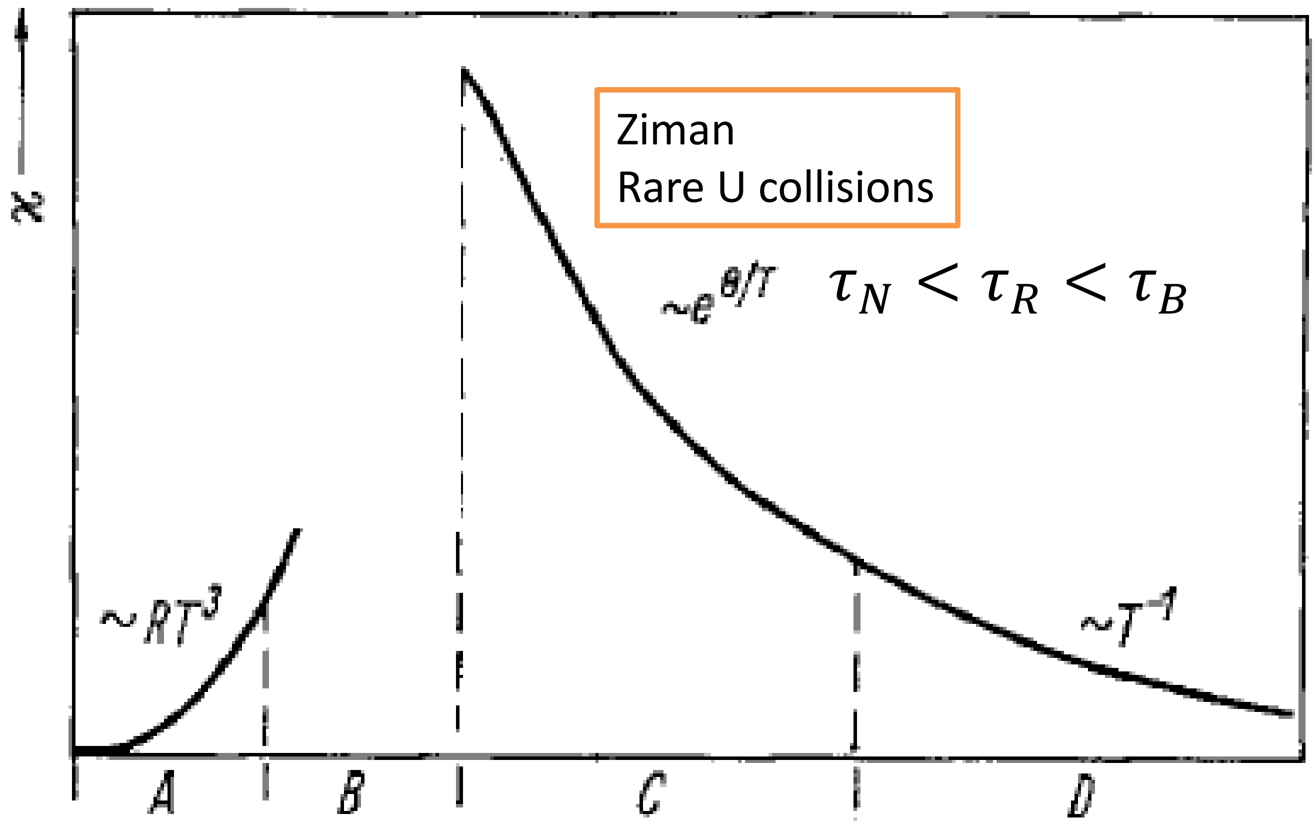
Ballistic
mfp constant

$\tau_B < \tau_R, \tau_N$

Kinetic
Abundant U collisions

$\tau_R < \tau_N < \tau_B$

Regimes of heat transport



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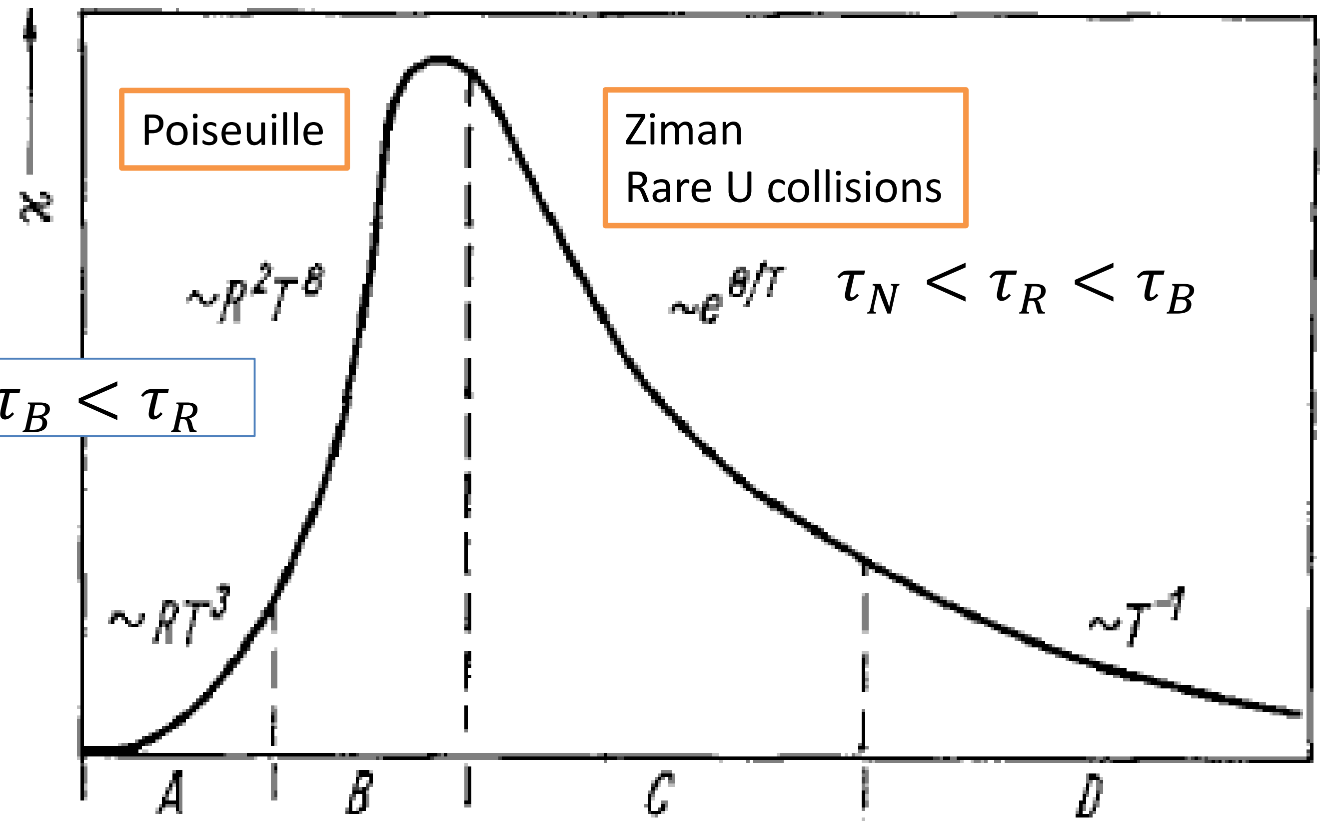
Regimes of heat transport



Poiseuille

Ziman
Rare U collisions

$\tau_N < \tau_R < \tau_B$

$\tau_N < \tau_B < \tau_R$

Ballistic
mfp constant

$\tau_B < \tau_R, \tau_N$

Kinetic
Abundant U collisions

$\tau_R < \tau_N < \tau_B$

Theoretical Poiseuille flow of phonons

- Predicted by Gurzhi (1959-1965)
- Expected to follow T^8!

$$\kappa = 1/3\ C\ v\ l_{eff}$$

$$\ell_{eff} = \frac{d^2}{\ell_N}$$ ← Distance between two normal collisions!

$$\ell_N \propto T^{-5}$$

Experimental Poiseuille flow

- Diagnosed in a handful of solids!
- Whenever thermal conductivity evolves faster than specific heat!

$$\kappa \propto T^{\gamma}$$

$$\gamma > \gamma'$$

$$C \propto T^{\gamma'}$$

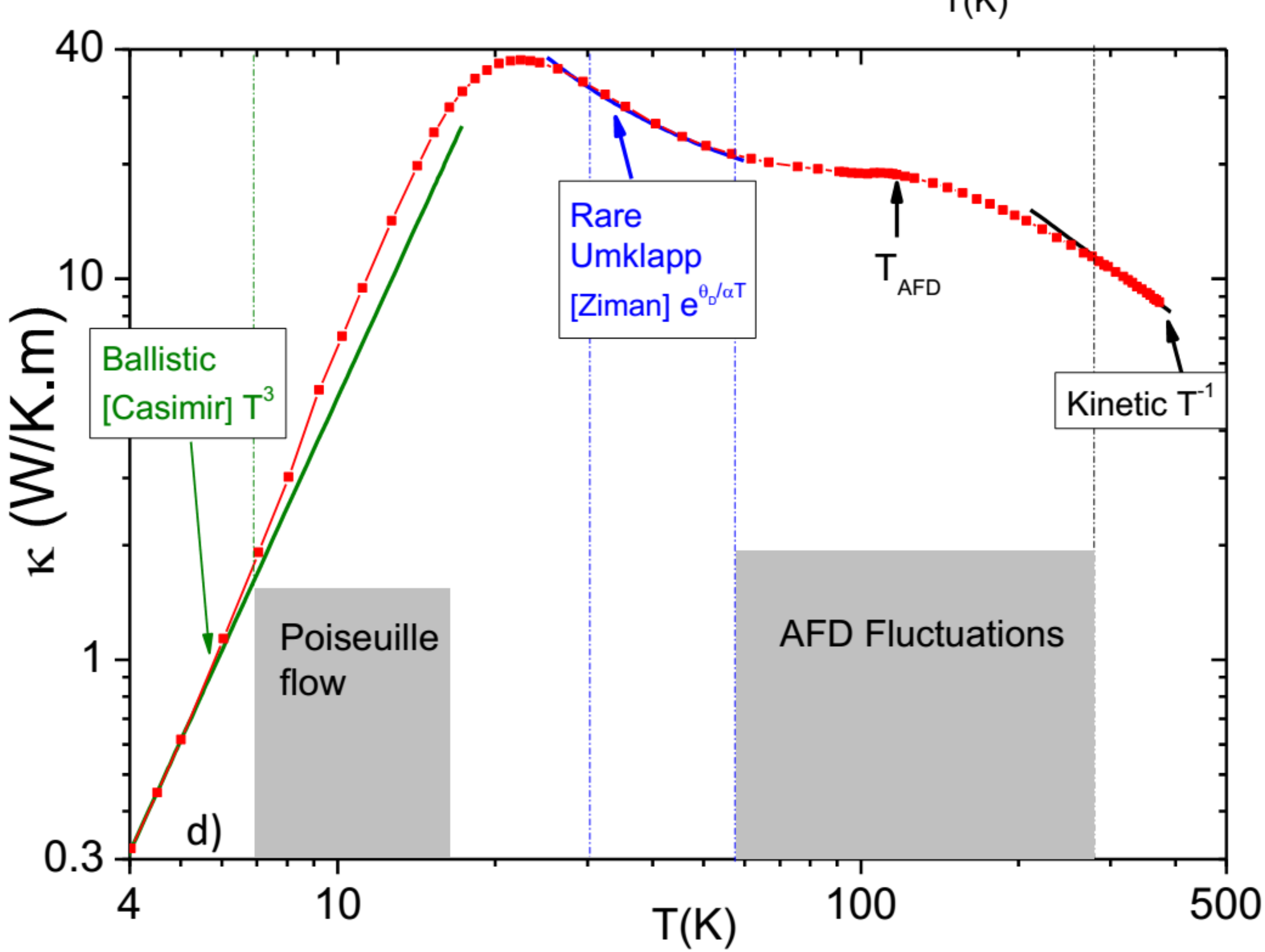
γ and γ' both close to 3!

- He^4 solid (Mezhov-Deglin 1965)
- He^3 solid (Thomlinson 1969)
- Bi (Kopylov 1971)
- H (Zholonko 2006)
- Black P (Machida 2018)
- $SrTiO_3$ (Martelli 2018)
- Sb (2019 unpublished)
- Graphite (2019 unpublished)

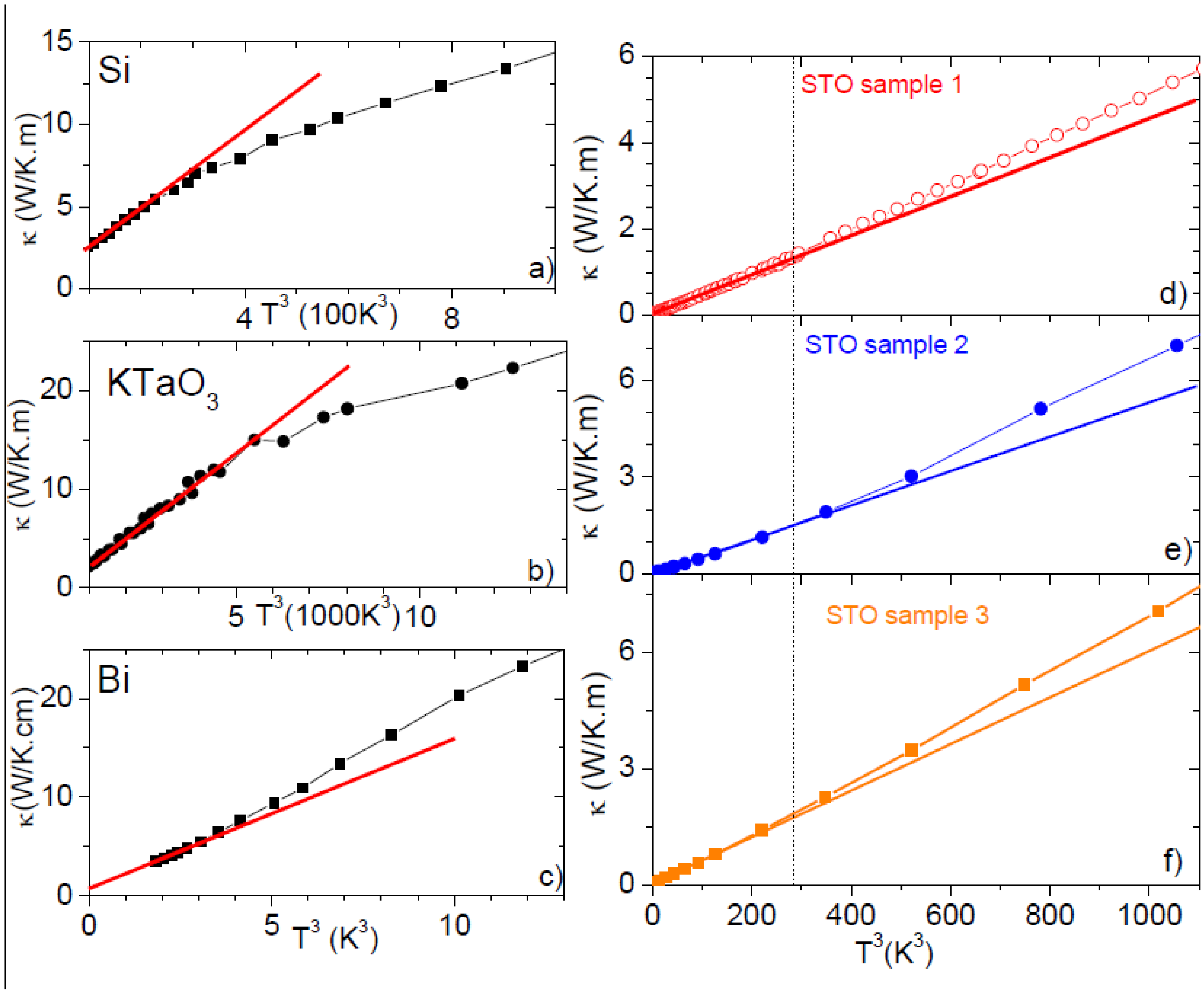
Editors' Suggestion

Thermal Transport and Phonon Hydrodynamics in Strontium Titanate

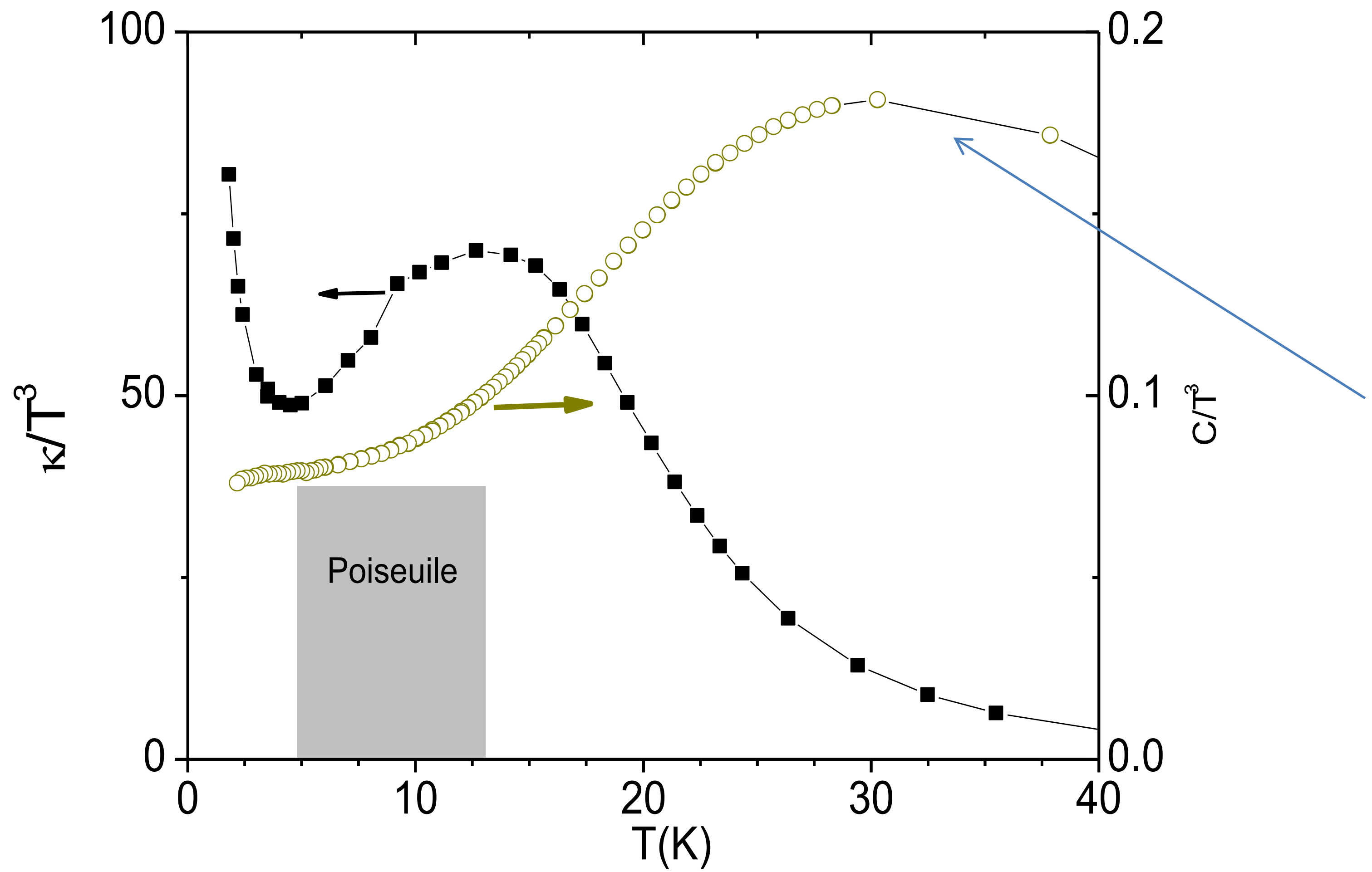
Valentina Martelli,[1] Julio Larrea Jiménez,[2] Mucio Continentino,[1] Elisa Baggio-Saitovitch,[1] and Kamran Behnia[3,4]



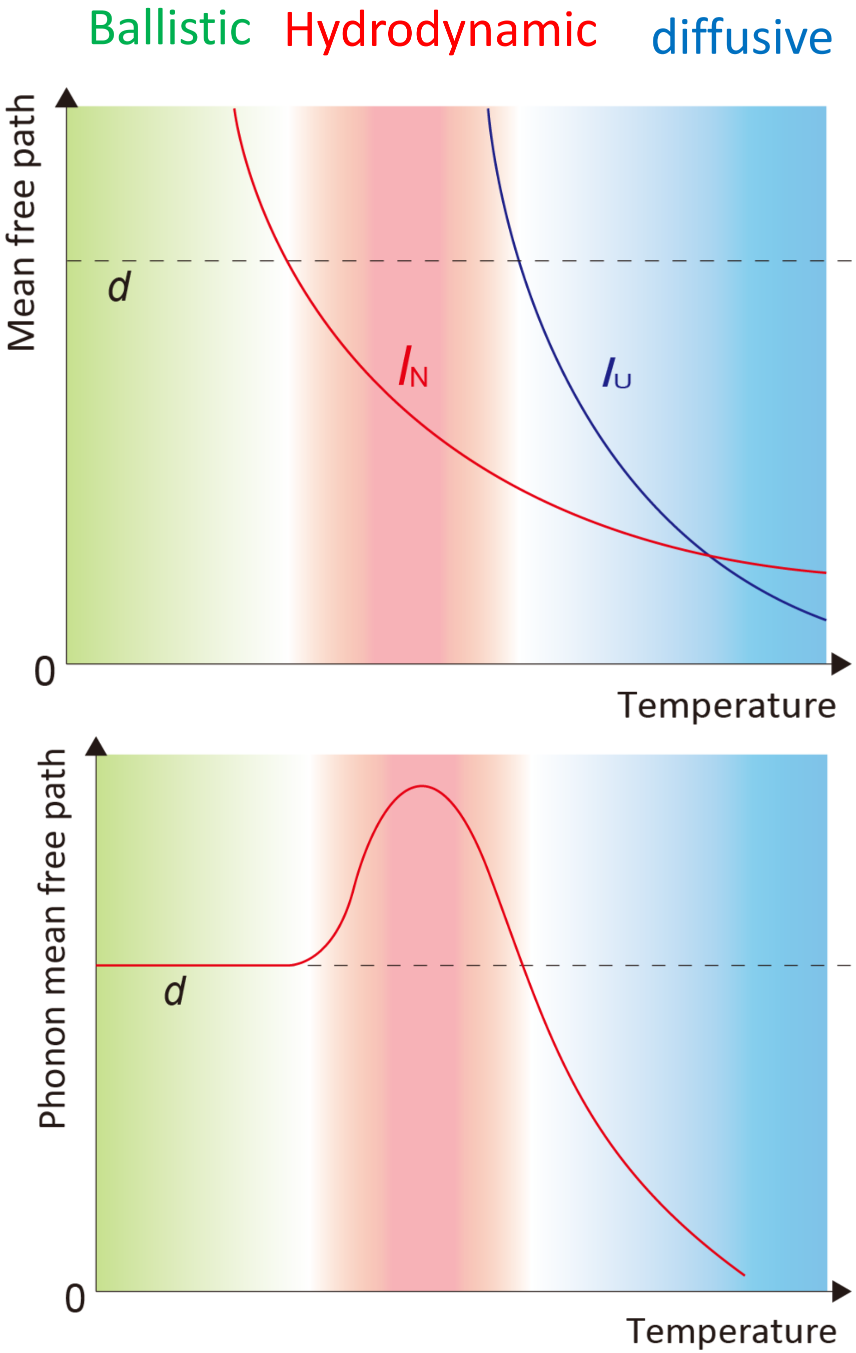
Faster than T^3 thermal conductivity



Thermal conductivity and specific heat



Two distinct deviations from the cubic behavior!

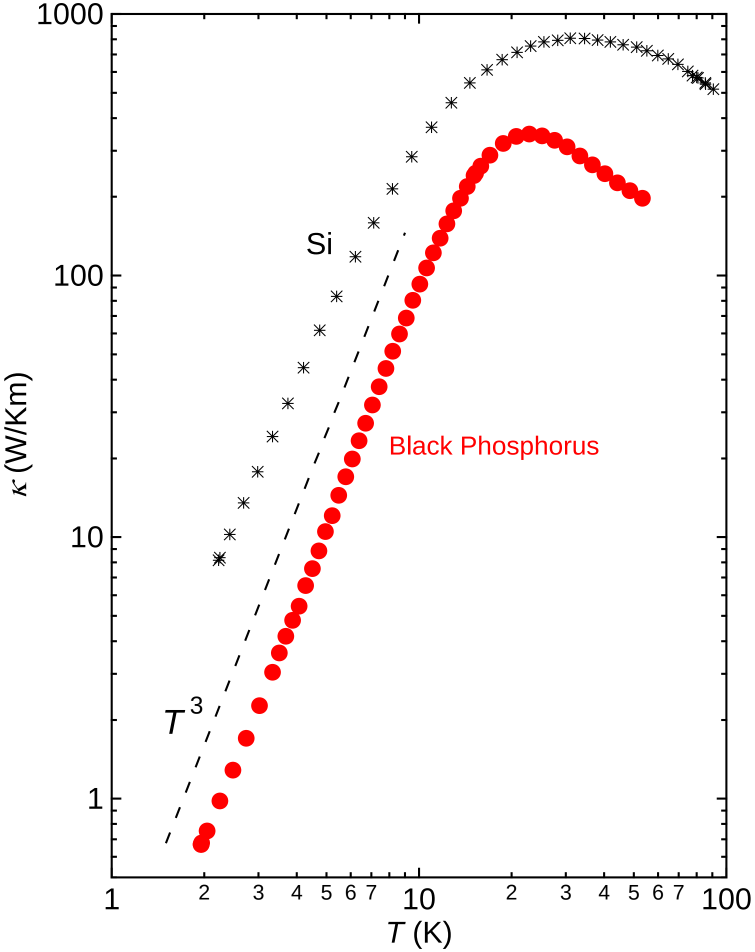


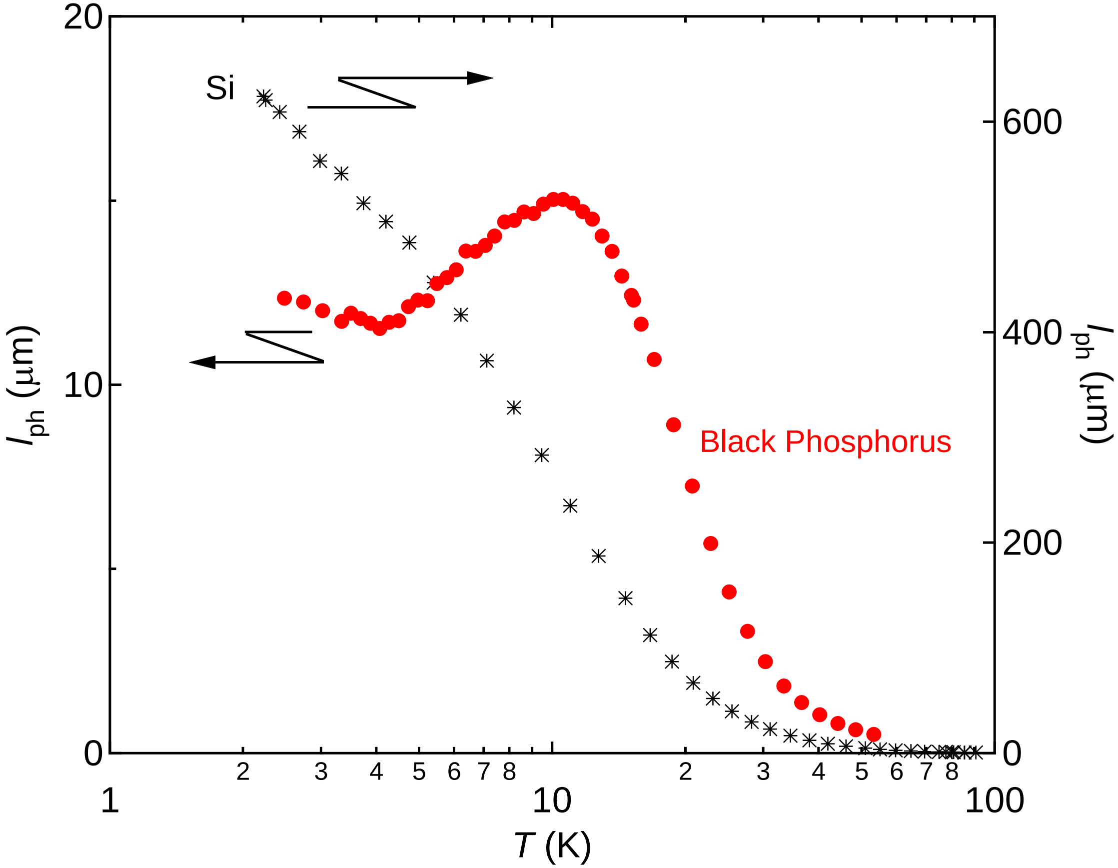
In this hydrodynamic regime :

- [Momentum-consevering] collisions enhance mfp!

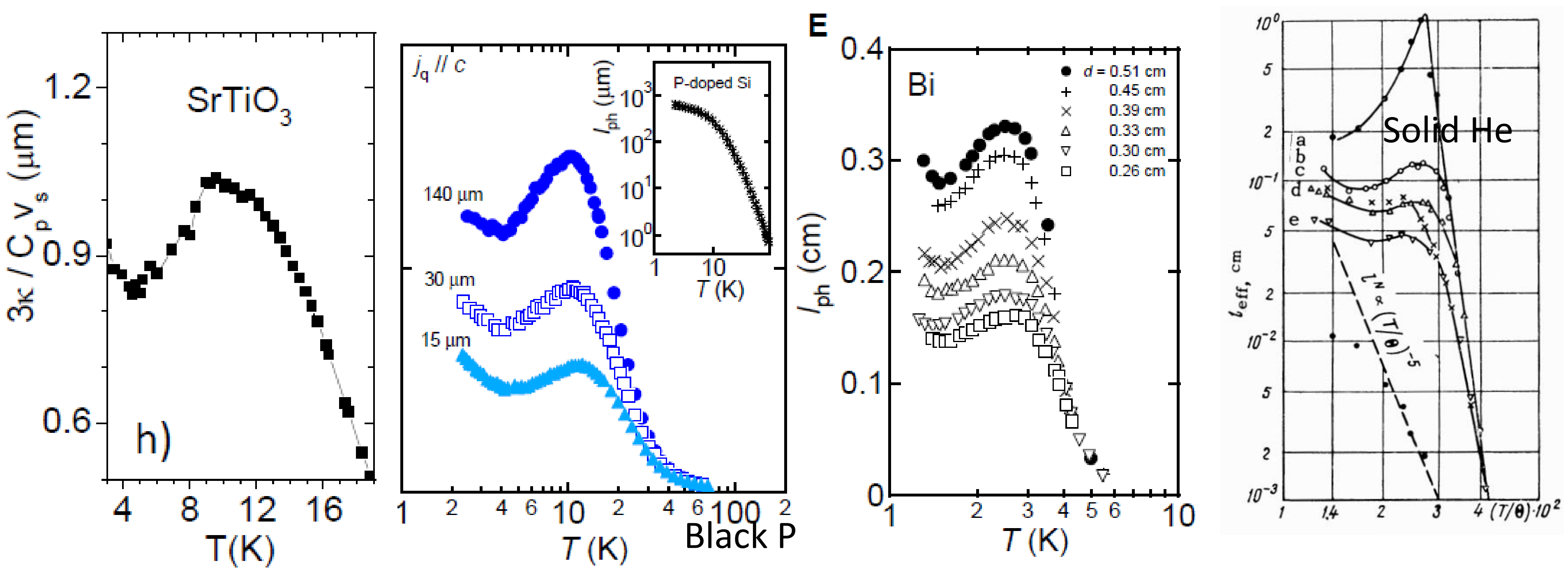
Silicon and black P

Machida Sci. Adv. (2018)





A Knudsen minimum and a Poiseuille peak



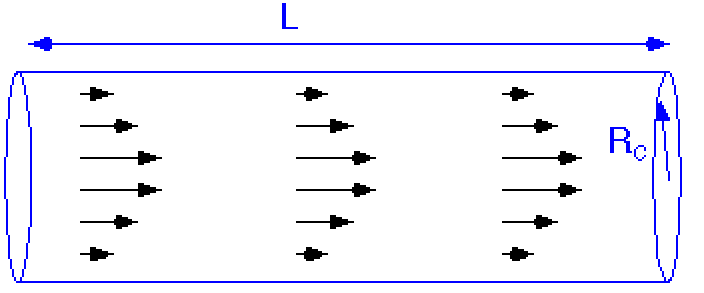
In this hydrodynamic regime : Collisions enhance mfp!

The higher the rate of momentum-conserving collisions the lower the viscosity!

Superlinear size dependence of thermal conductivy has never been seen. Why?

$$\kappa \propto d^{\alpha} T^{\beta}$$

- According to Gurzhi, α=2 and β=8
- But experiment yields $\alpha \approx 1$ and $3<\beta<4$

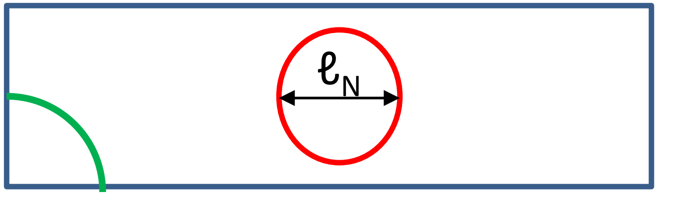


Poiseuille flow in a NEWTONIAN fluid

Phonon viscosity is NOT homogeneous!

$$\nu_{ph} = v_T \ell_N$$

Far from boundary full N scattering!



Close to boundary less N scattering!

POISEUILLE FLOW IN NON-NEWTONIAN FLUIDS IS NOT PARABOLIC

18 A. Morozov and S.E

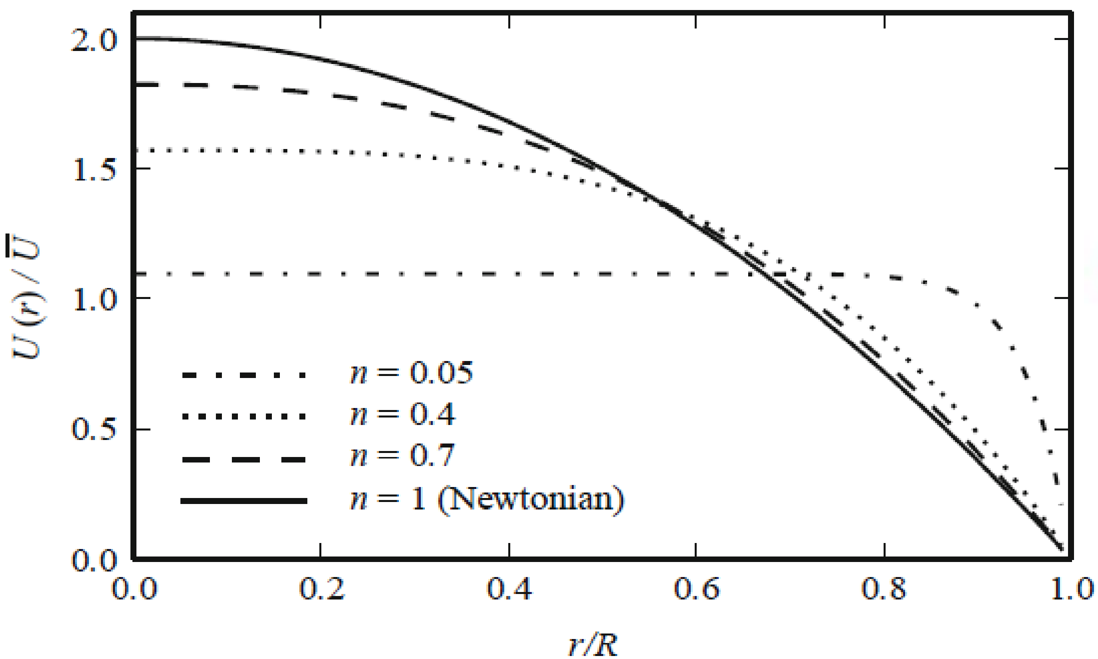


Fig. 1.5 The normalized velocity profile of a pressure-driven pipe flow, from Eq. (1.40), for various values of the power-law index n. As the fluid becomes more shear-thinning (decreasing n), the high-shear region of the flow moves progressively towards the wall and the region near the center of the pipe becomes more plug-like

The profile is not parabolic in shear-thinning fluids!

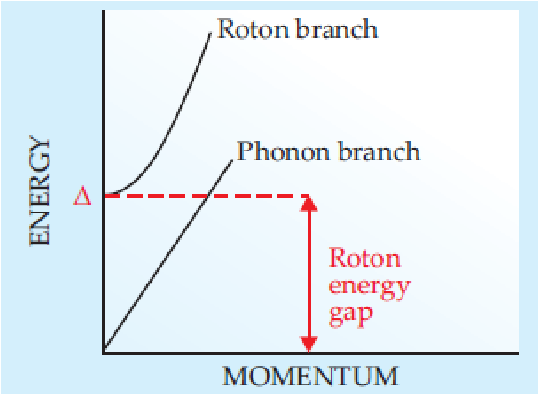
Compensation amplifies the signal

Gurevich and Shklovskii, Soviet Physics –Solid State (1967)

In presence of a large and equal concentration of electrons and holes, phonons can [NORMALLY NOT RESISTIVETLY] exchange momentum through the electron bath!

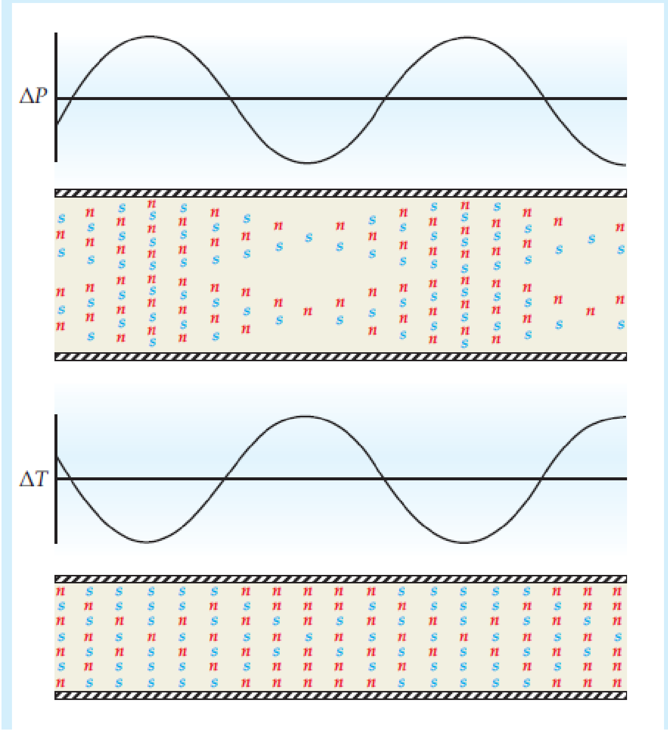
Second sound: a brief history

- **1938:** Laszlo Tisza proposes the two-fluid model of He II: the existence of two propagating waves.
- **1941:** Lev Landau dubbed “second sound” the velocity associated with the roton branch.



Two distinct waves :

- Wave-like propagation of density (ordinary sound)
- Wave-like propagation of temperature (second sound)



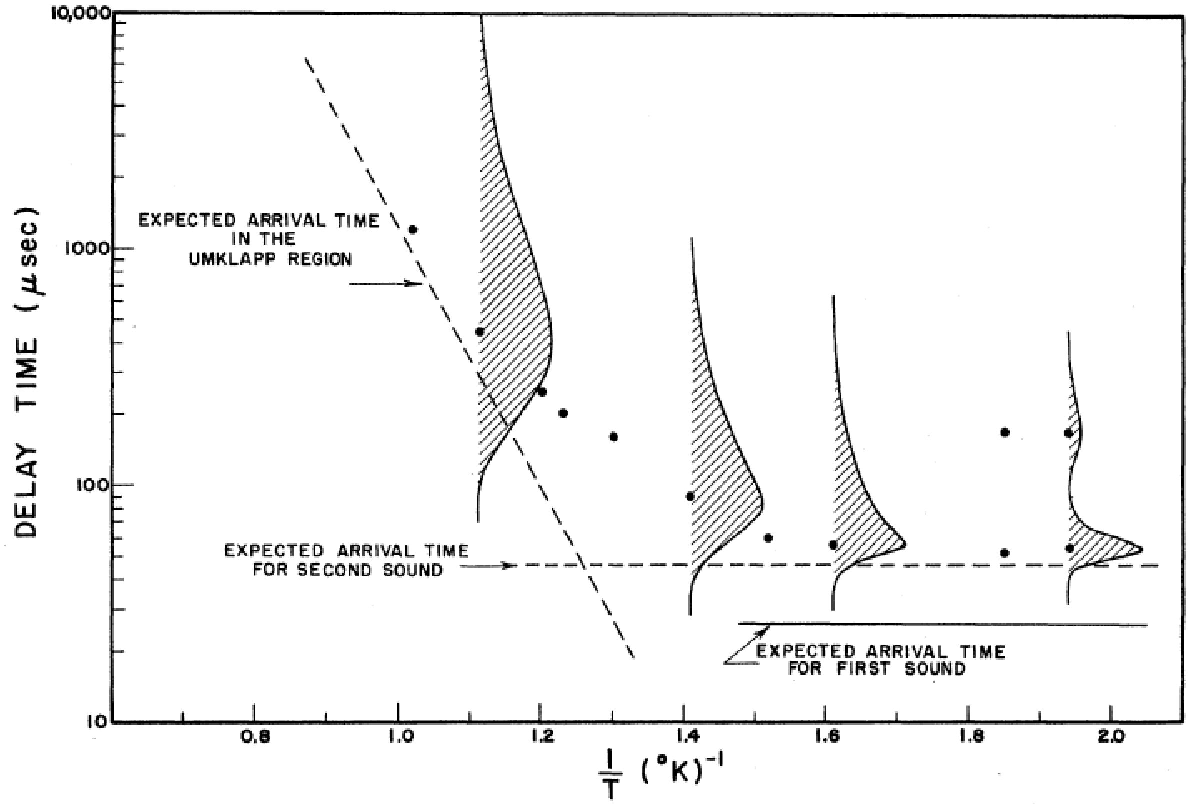
- **1944:** Observation of second sound by Peshkov in HeII

- **1952:** Dingle suggests that a density fluctuation in a phonon gas can propagate as a second sound.

- **1966:** Gruyer & Krumhansl argue that second sound and Poiseuille flow in solids require the same hierrachy of scattering rates.

Experimental observation

- **1966:** Observation of second sound in ^{4}He by Ackerman *et al.*



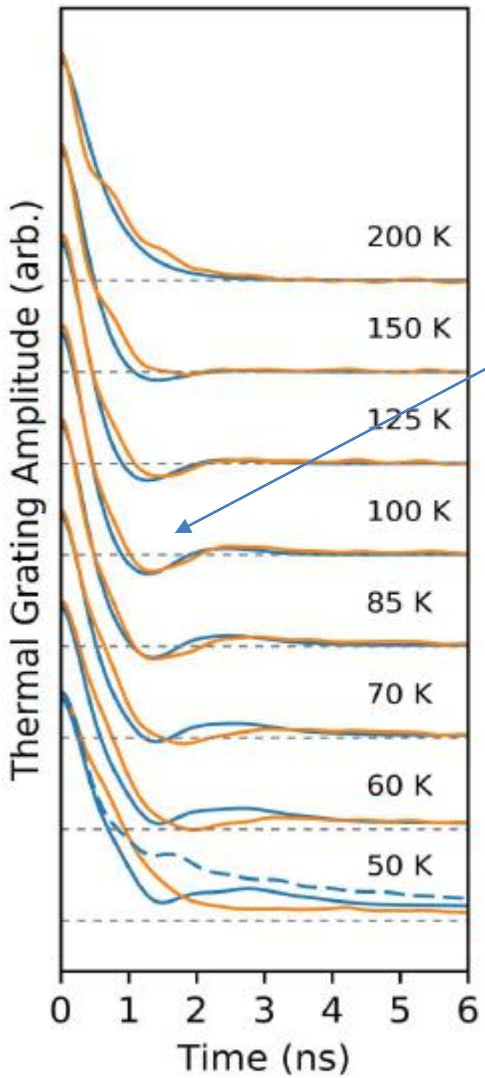
- **1969:** Observation of second sound in ^{3}He (Ackerman & Overton)
- **1970:** Observation of second sound in NaF (Jackson *et al.)*
- **1972:** Observation of second sound in bismuth (Narayanamurti and Dynes)

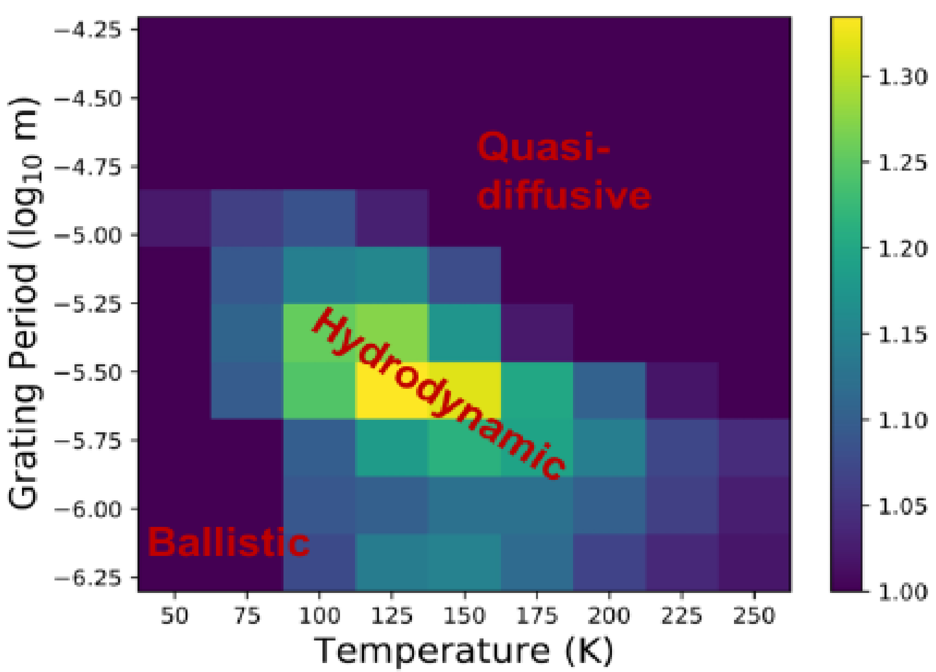
Observation of second sound in graphite at temperatures above 100 K

Authors: S. Huberman[1]†, R. A. Duncan[2]†, K. Chen[1], B. Song[1], V. Chiloyan[1], Z. Ding[1], A. A. Maznev[2], G. Chen[1]*, K. A. Nelson[2]*.

Science 14 Mar 2019:
eaav3548
DOI: 10.1126/science.aav3548

The signature





The two signatures of phonon hydrodynamics

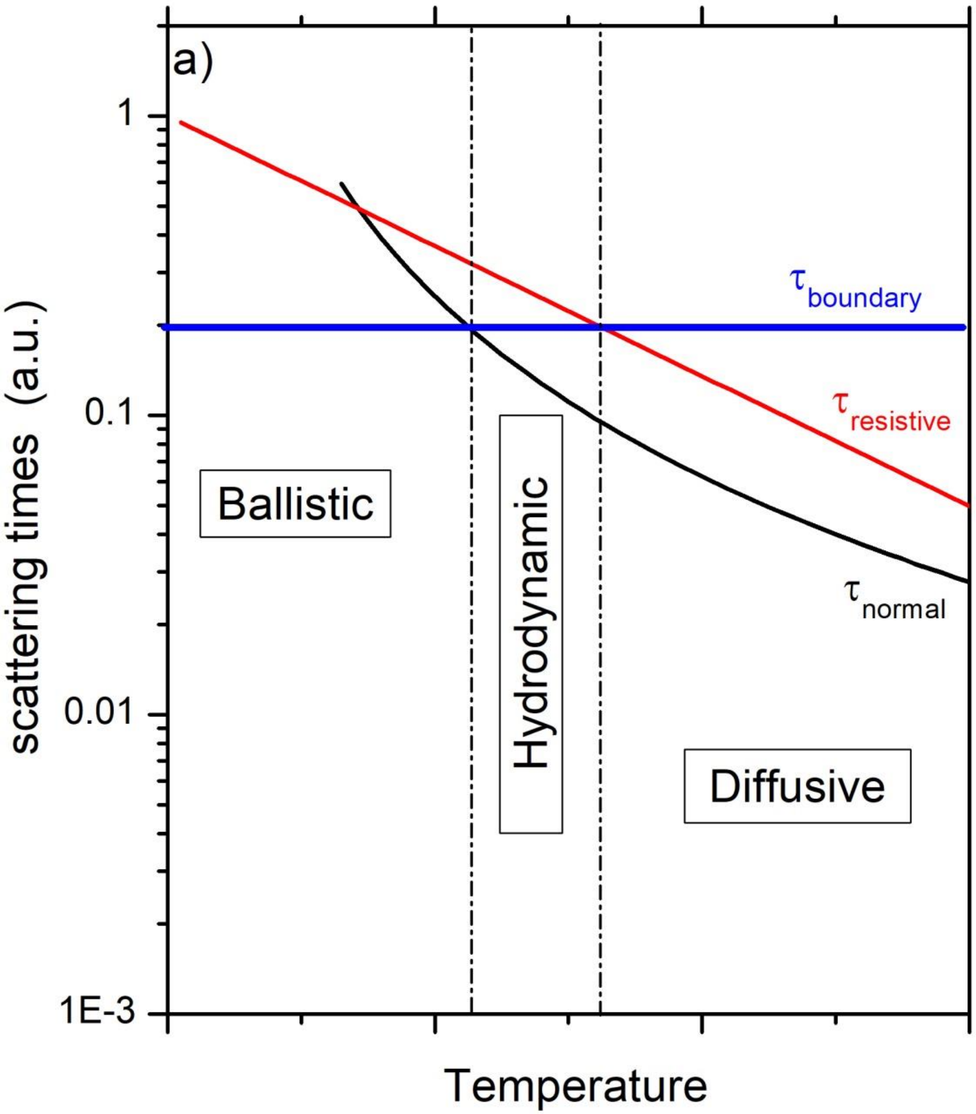
- **Poiseuille flow:** Steady drift of phonon gas COLLECTIVELY

- **Second sound:** Propagation of a heat pulse representing local phonon POPULATION

Individual phonons travelling ballistically are NOT hydrodynamic.

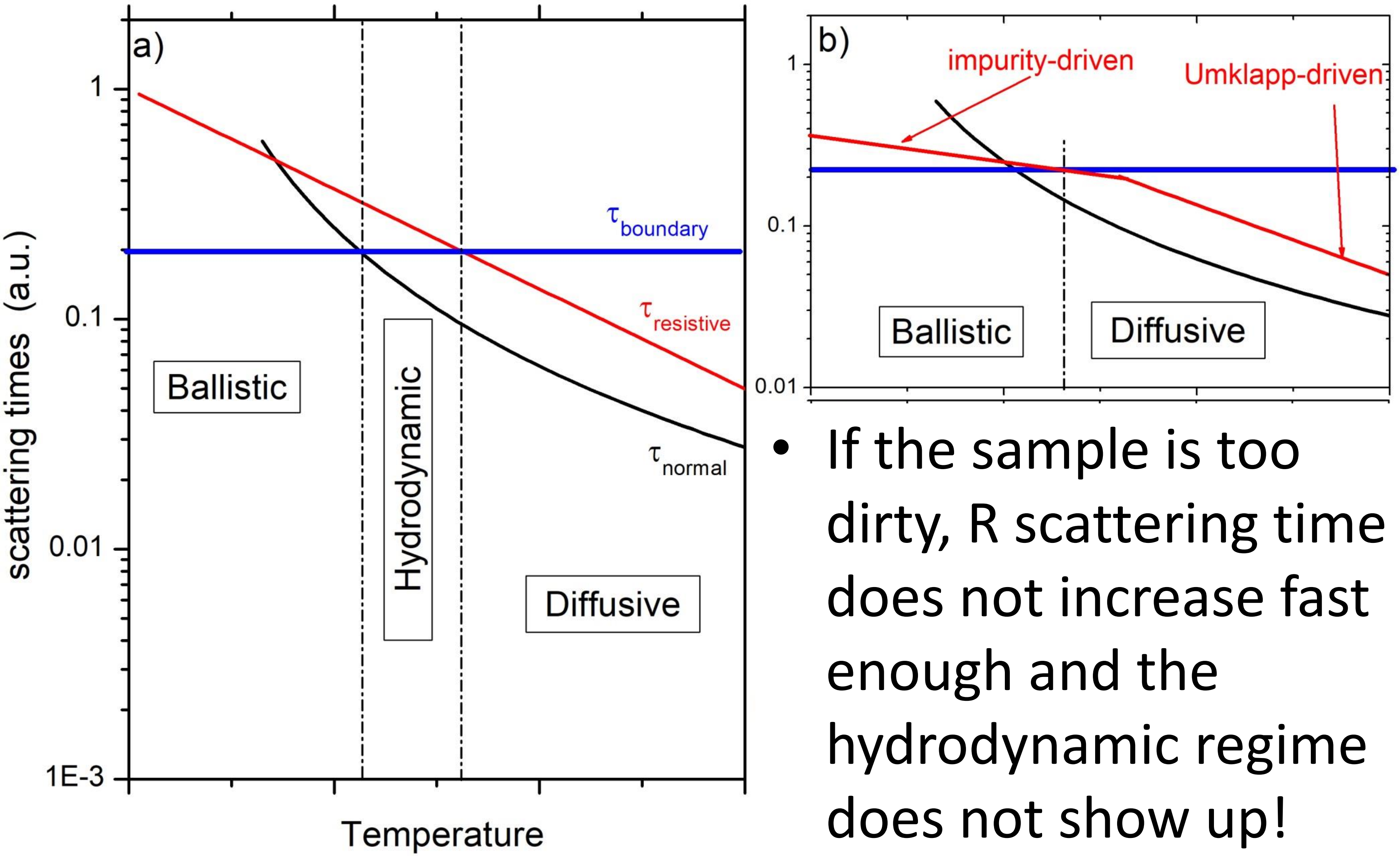
Both are corrections to diffusive flow in a limited temperature window!

The hydrodynamic regime is fragile!



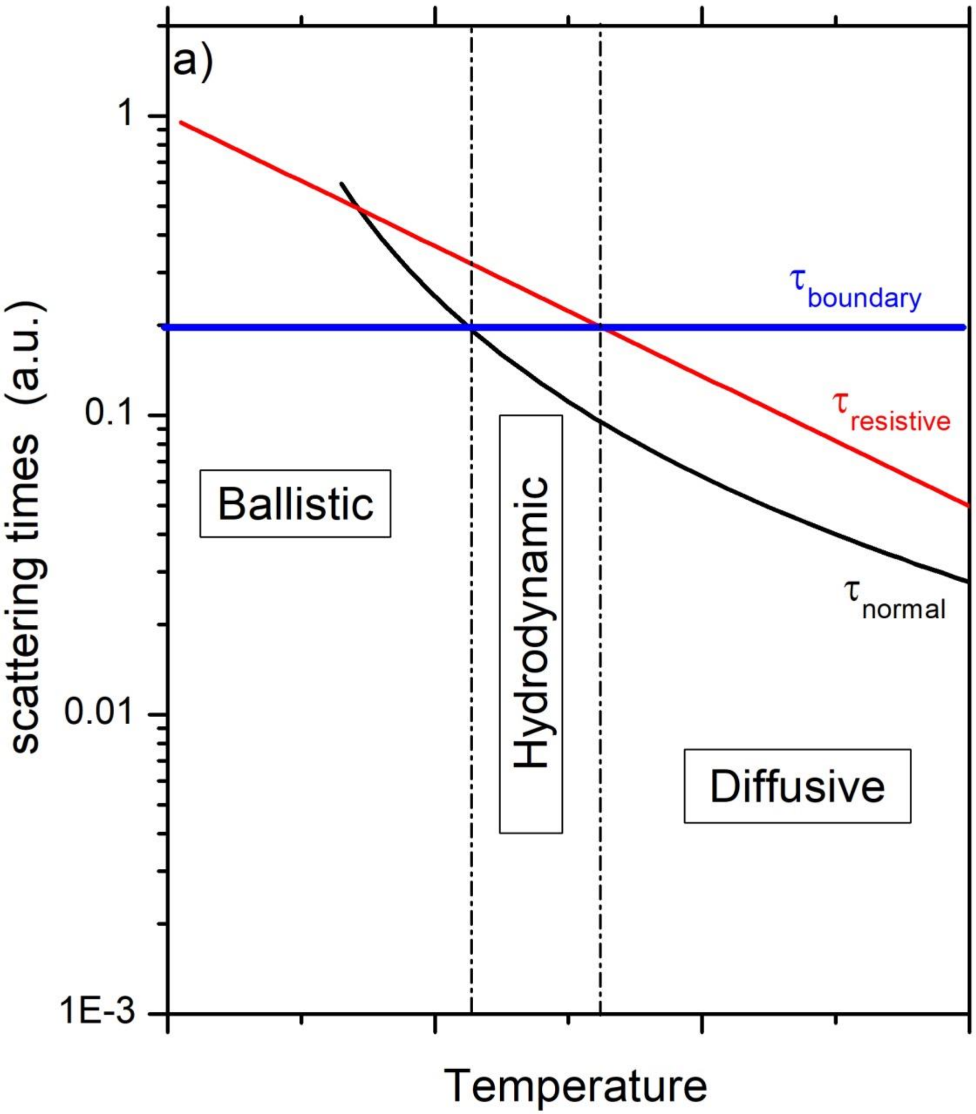
- This is NOT a zero-temperature phenomenon
- Umklapp scattering time grows faster than Normal scattering time with cooling!
- In a narrow window , the boundary scattering time lies between the two!

The hydrodynamic regime is fragile!

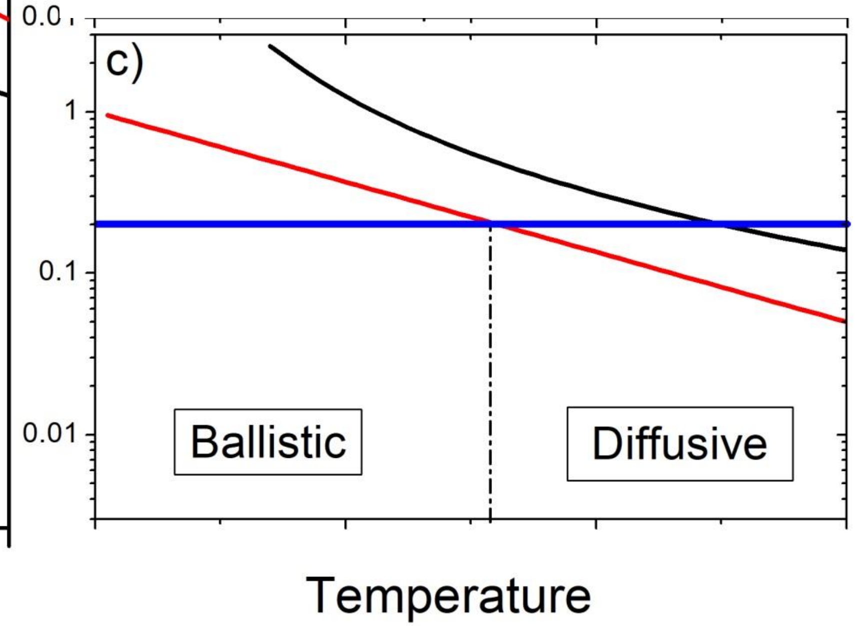


- If the sample is too dirty, R scattering time does not increase fast enough and the hydrodynamic regime does not show up!

The hydrodynamic regime is fragile!



- If not enough N scattering then it will always remain below R scattering and the hydrodynamic regime will not show up either!



Why is it interesting?

- Normal scattering between phonons is poorly understood.

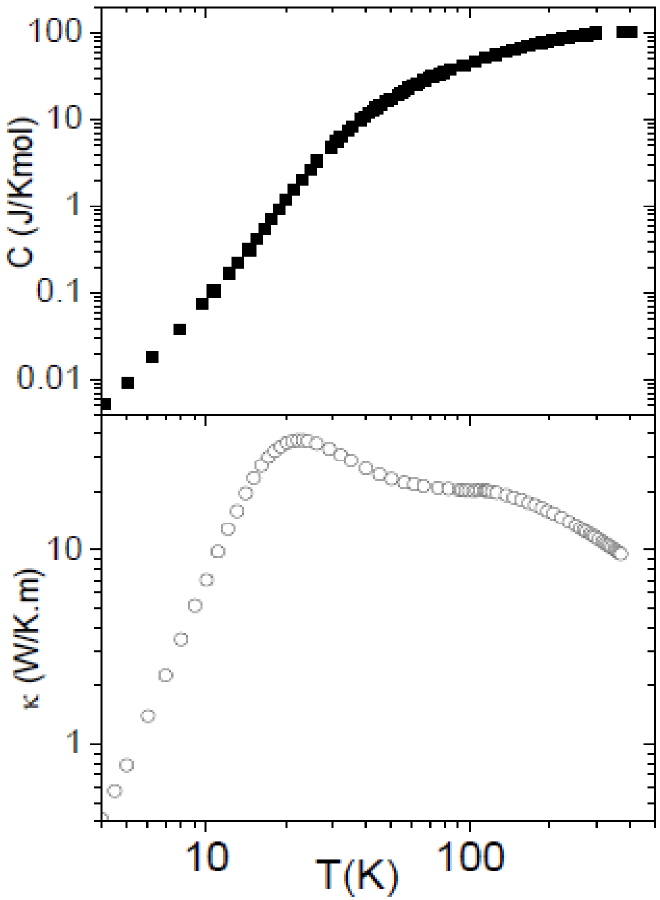
What can amplify them? Proximity to structural instability (STO, Bi, Sb, black P), decoupling of Debye frequencies (graphite), e-h compnsation (Sb & Bi)?

- Nonlinear phonon interaction: what sets the melting temperature of a solid?
- A route towards quantum turbulence. [Enhance the Reynolds number!]

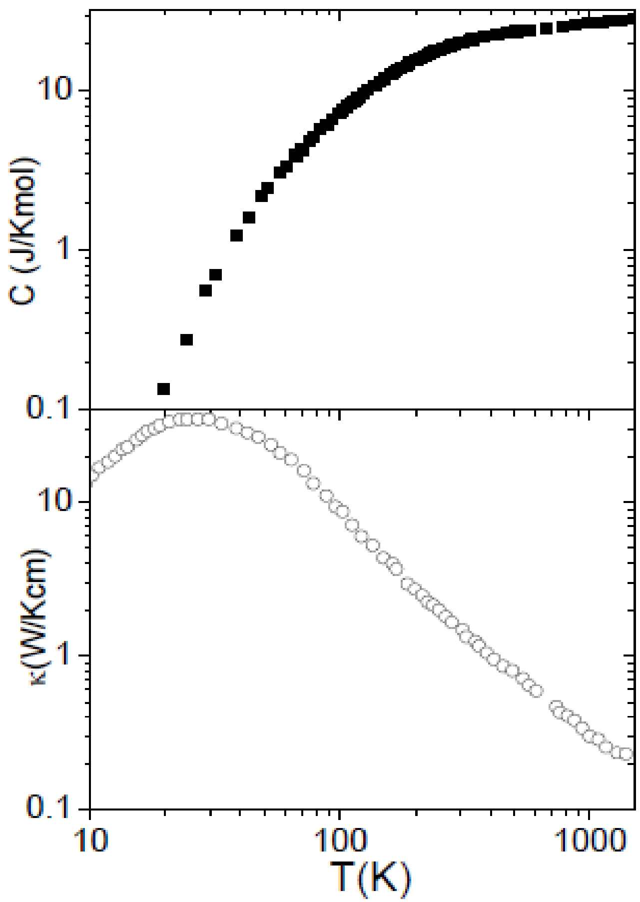
Boundary to thermal diffusivity

High-temperature thermal conductivity

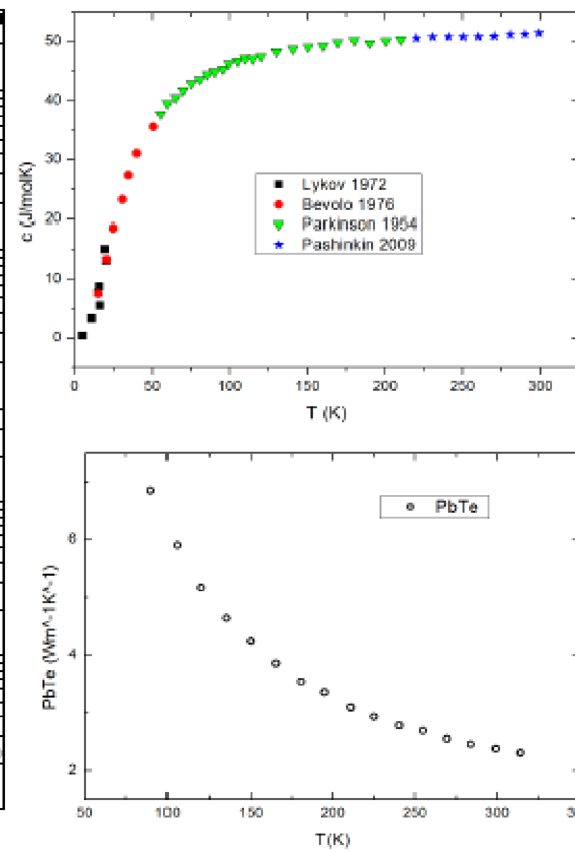
$SrTiO_3$



silicon



PbTe



- The thermal conductivity in an insulator decarease as T^{-1}
- Ascribed to Umklapp scattering

Similarity of Scattering Rates in Metals Showing *T*-Linear Resistivity

J. A. N. Bruin,[1] H. Sakai,[1] R. S. Perry,[2] A. P. Mackenzie[1]

Theory of universal incoherent metallic transport

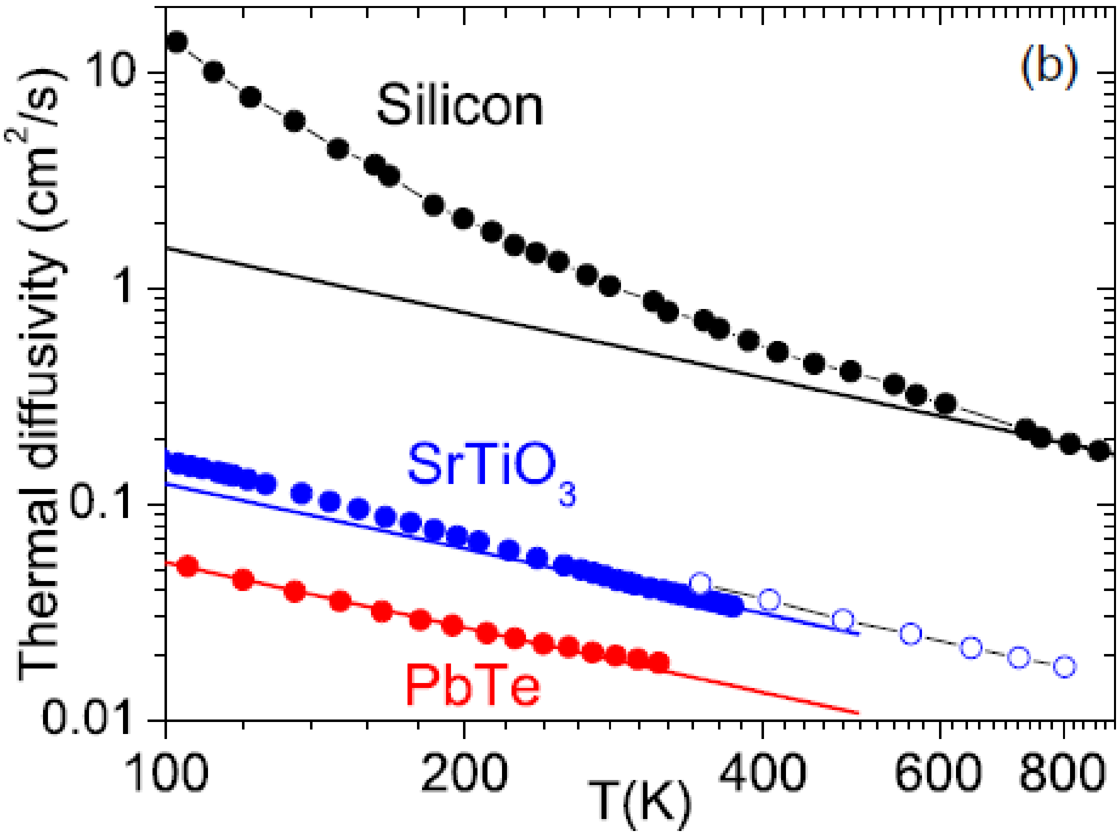
Sean A. Hartnoll

$$\frac{D_{\mathrm{qp}}}{v_{\mathrm{F}}^2} \gtrsim \frac{\hbar}{k_{\mathrm{B}} T}$$

Anomalous thermal diffusivity in underdoped $YBa_2Cu_3O_{6+x}$

Jiecheng Zhang[a,b], Eli M. Levenson-Falk[a,b], B. J. Ramshaw[c], D. A. Bonn[d,e], Ruixing Liang[d,e], W. N. Hardy[d,e], Sean A. Hartnoll[b], and Aharon Kapitulnik[a,b,f,1]

A bound to thermal diffusivity?



$$D = s v_s^2 \tau_p .$$

$$\tau_p = (\hbar / k_B T)$$

System	$D_{300\ K}$ (mm²/s)	v_{sl} (100)(km/s)	s
$SrTiO_3$	4.0	7.87	2.6
PbTe	1.9	3.59	5.9
Si	91	8.43	51

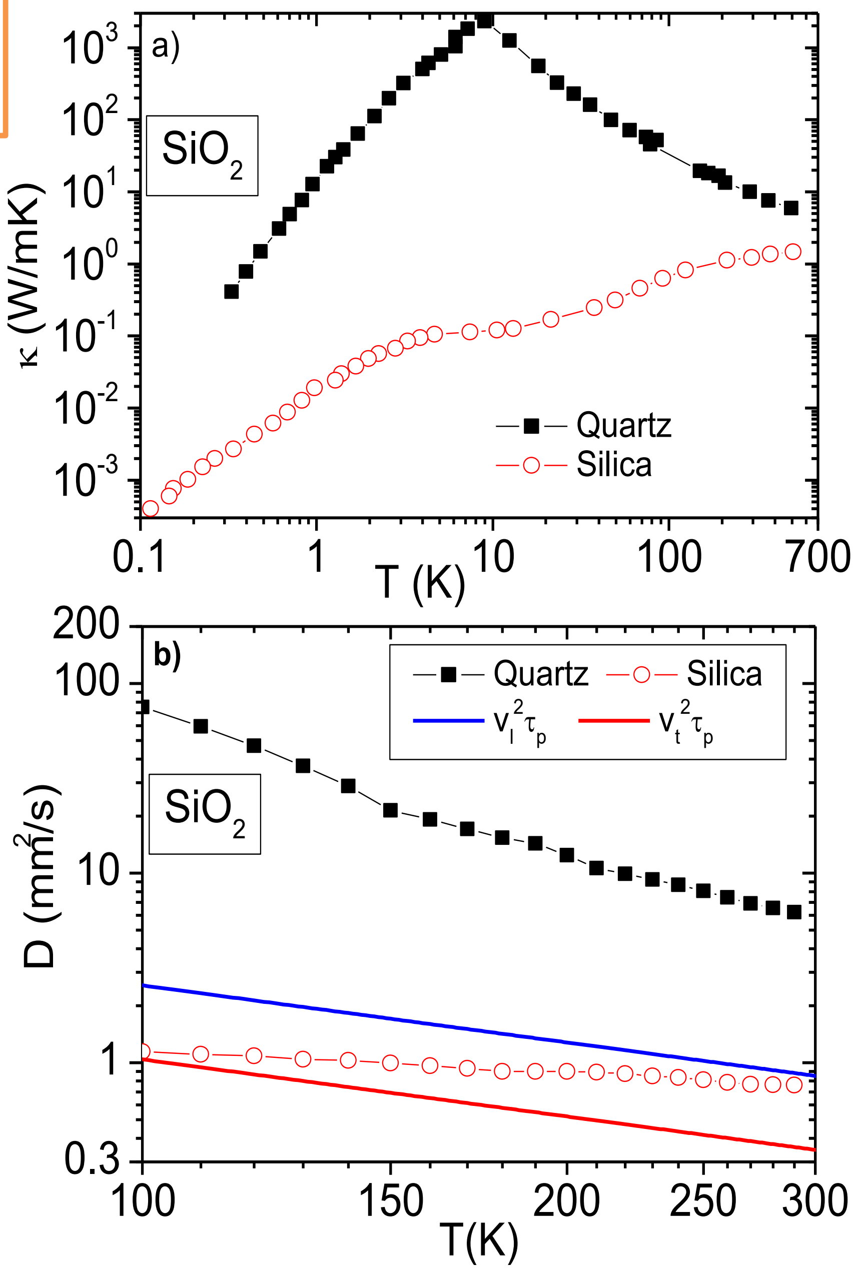
Thermal conductivity in glasses

- Even they appear to respect this inequality

$$D > {v_s}^2 \tau_P$$

D and v_s are both measured experimentally

$$\tau_p = (\hbar / k_B T)$$

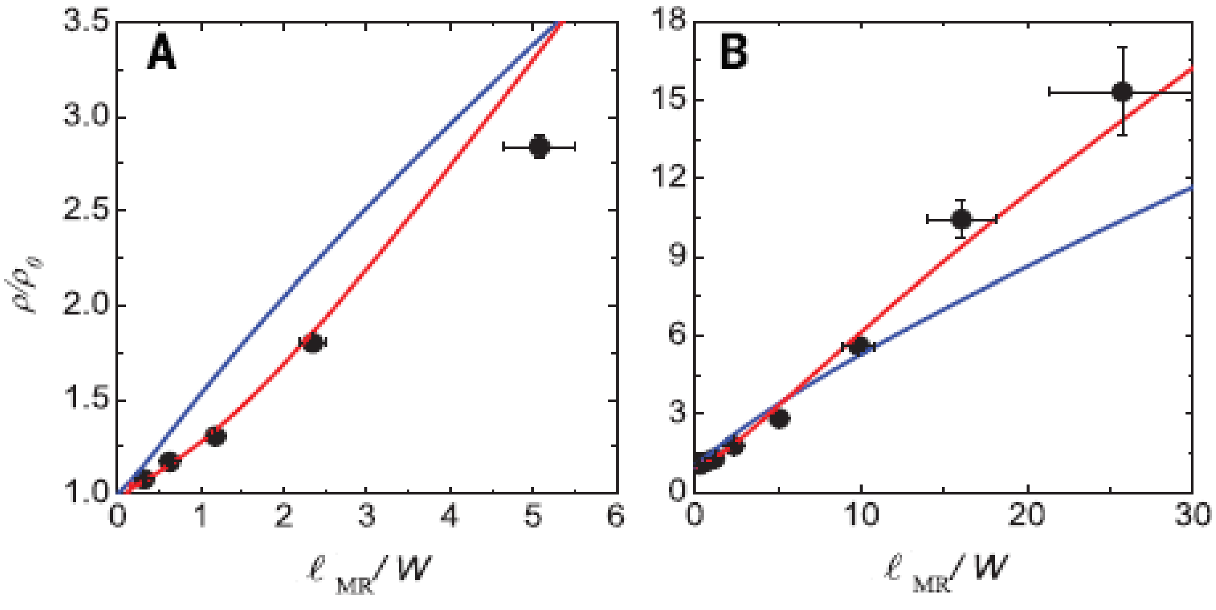


Electron hydrodynamics

ELECTRON TRANSPORT

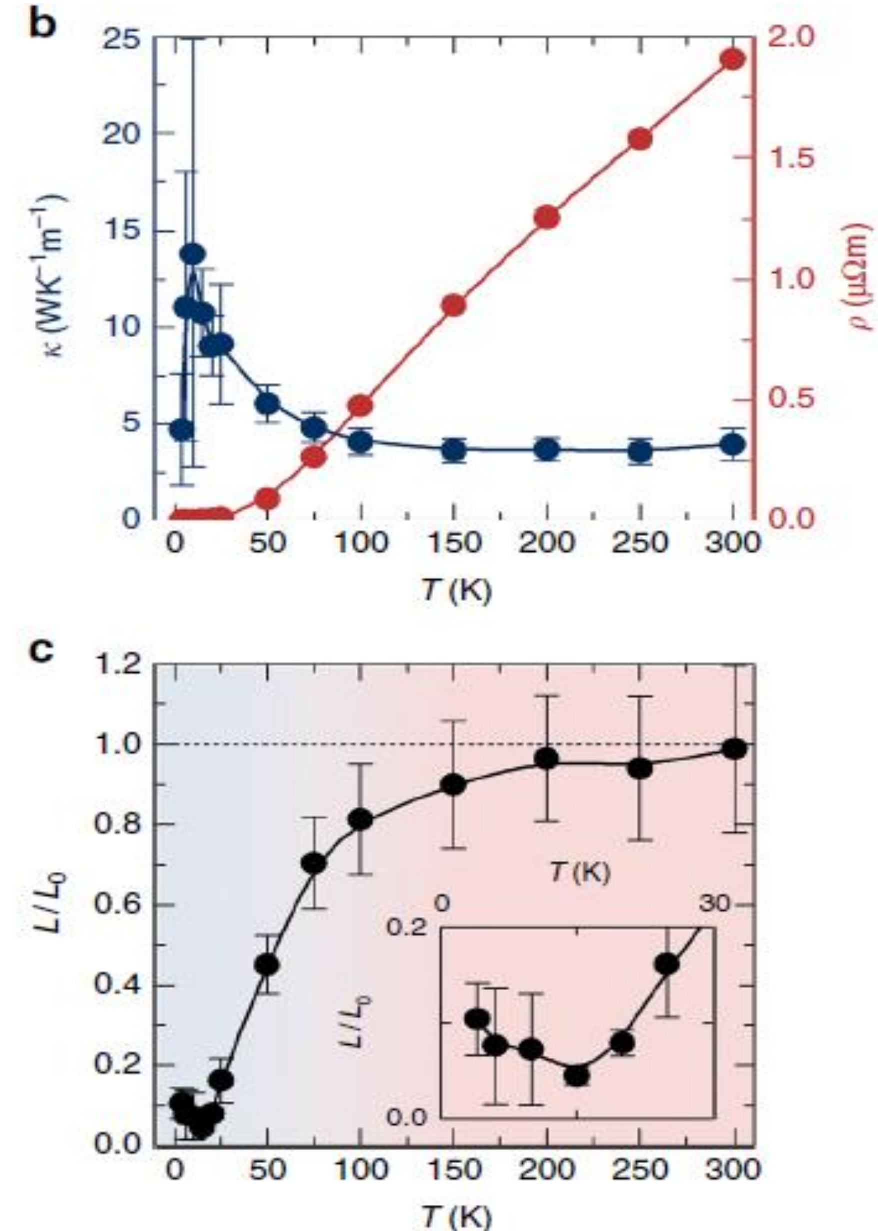
Evidence for hydrodynamic electron flow in $PdCoO_2$

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3]
Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4]*



Thermal and electrical signatures of a hydrodynamic electron fluid in tungsten diphosphide

J. Gooth[1,2], F. Menges[1,4], N. Kumar[2], V. Süβ[2], C. Shekhar[2], C. Felser[2] & B. Gotsmann[1]



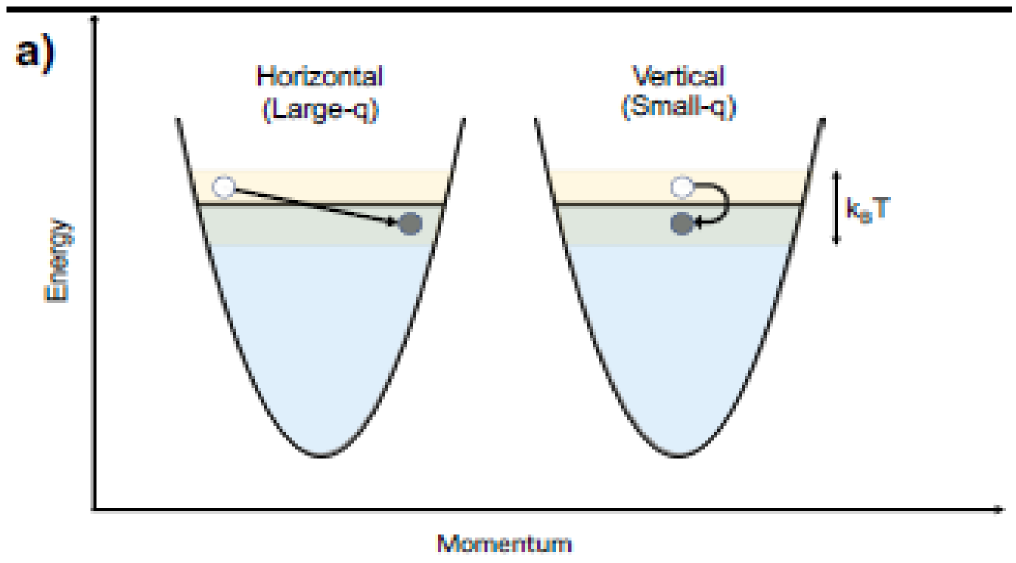
The WF law

$$\mathrm{L} = L_0$$

Lorenz number $$\mathrm{L} = \frac{\kappa}{\sigma T}$$

Sommerfeld value $$L_0 = \frac{\pi^2}{3}\left(\frac{\kappa_B}{e}\right)^2$$ $=2.445\ 10^{-8} \mathrm{W}\ \Omega\ /\ \mathrm{K}^2$

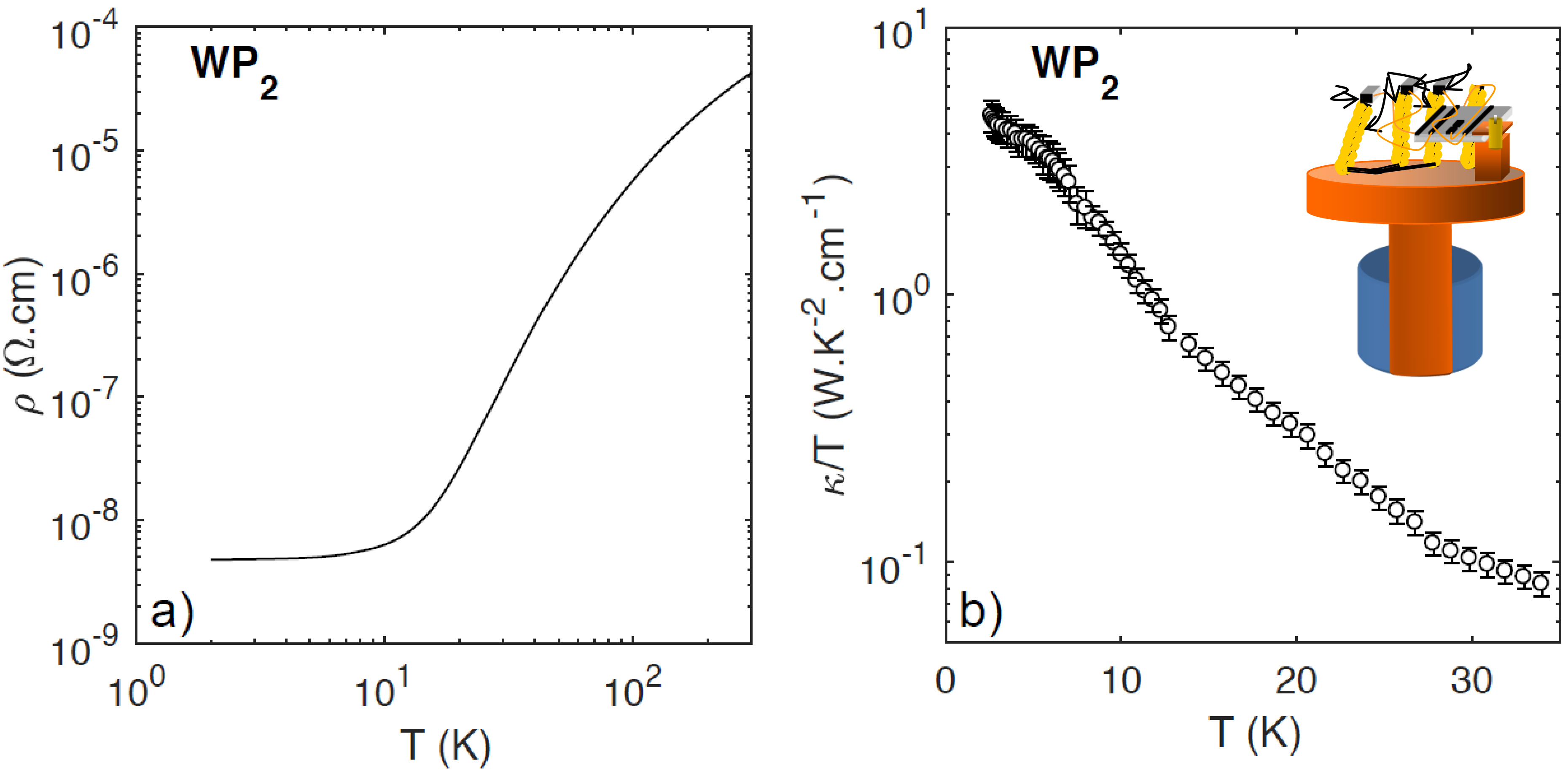
Valid at T=0

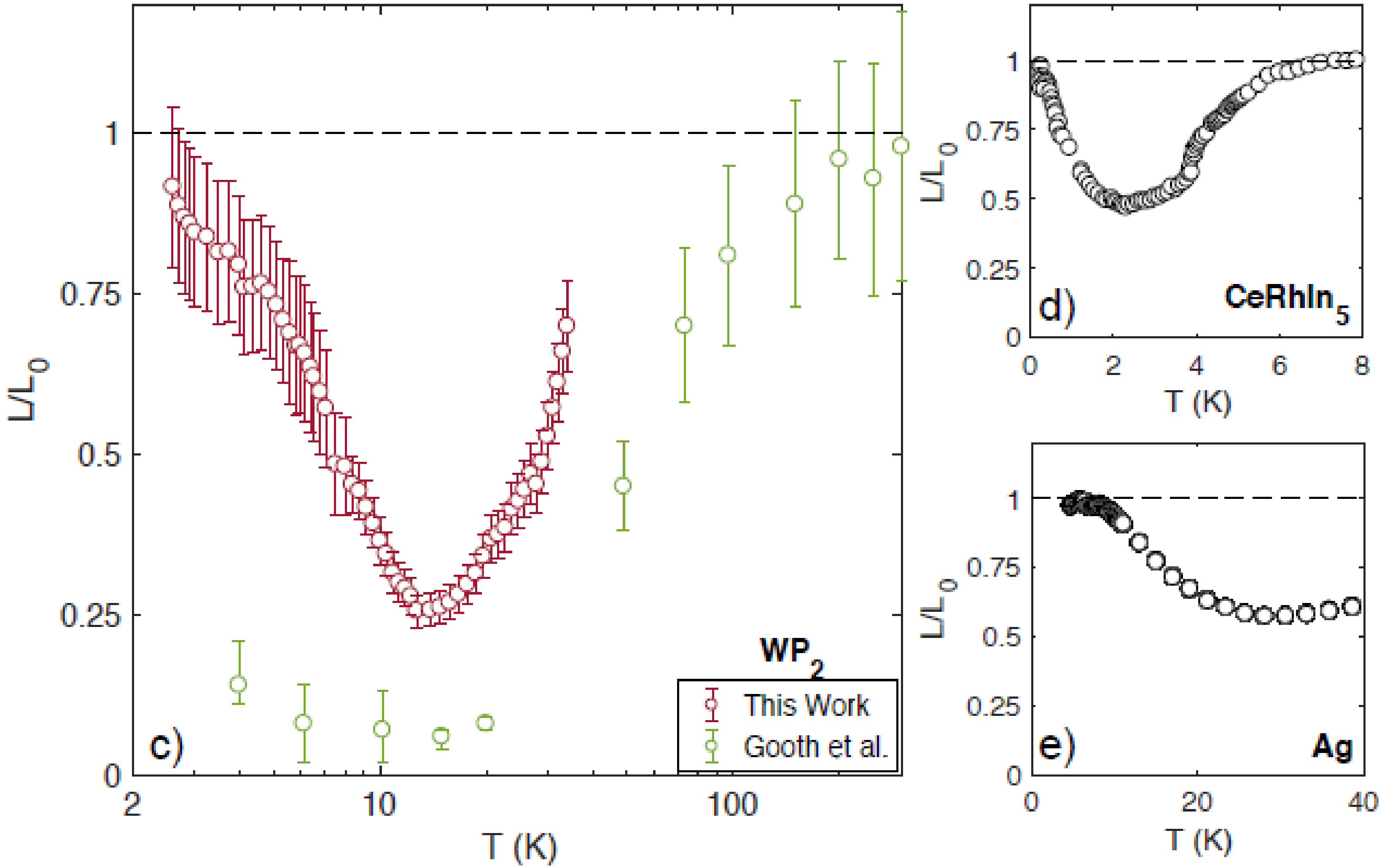


Finite-temperature deviation due to vertical scattering!

Departure from the Wiedemann–Franz law in WP_2 driven by mismatch in *T*-square resistivity prefactors

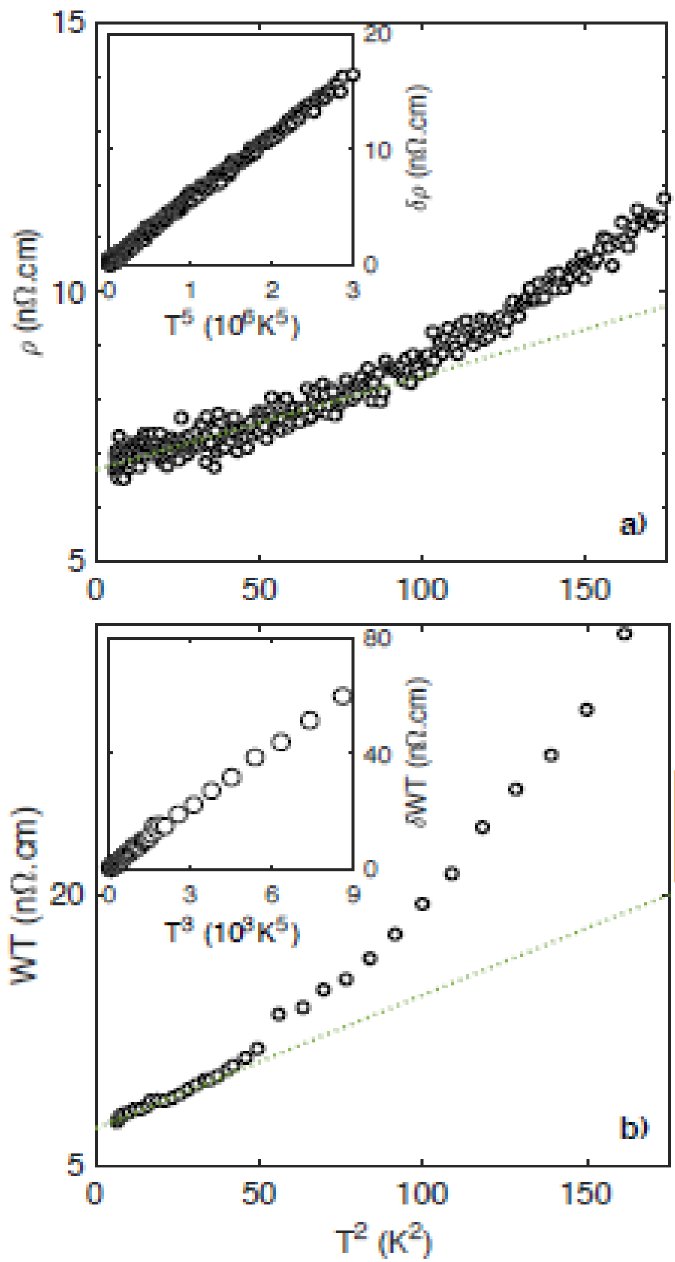
Alexandre Jaoui[1,2], Benoît Fauqué[1,2], Carl Willem Rischau[2,3], Alaska Subedi[4,5], Chenguang Fu[6], Johannes Gooth[6], Nitesh Kumar[6], Vicky Süß[6], Dmitrii L. Maslov[7], Claudia Felser[6] and Kamran Behnia[2,8]





Zero-temperature validity, but a large finite-temperature deviation.

Quantifying distinct components



$$\rho = \rho_0 + A_2T^2 + A_5T^5 \quad (1)$$

$$WT = WT_0 + B_2T^2 + B_3T^3 \quad (2)$$

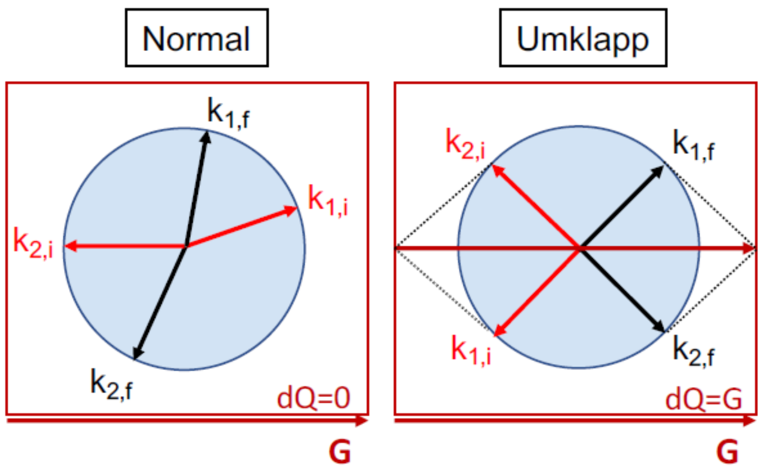
By fitting the data one quantifies :

A_2 and A_5 in ρ(T)

B_2 and B_3 in WT (T)

B_2 is almost FIVE times larger than A_2

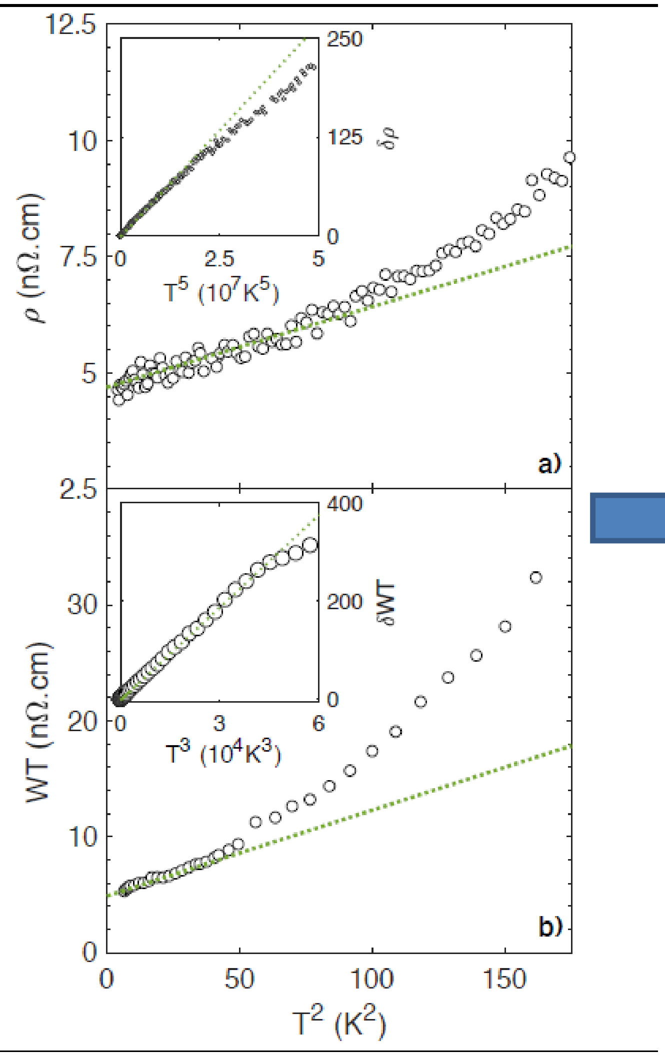
T-square resistivities and hydroynamics

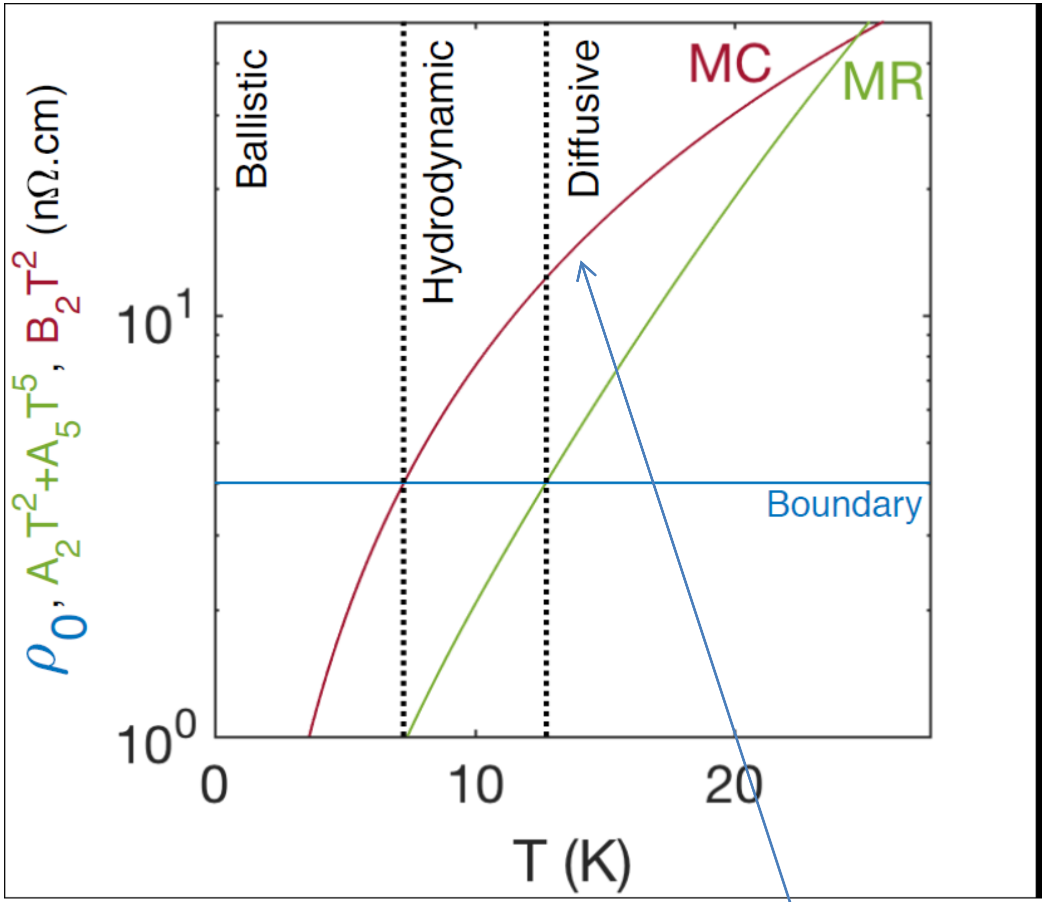


- T-square electrical resistivity (A_2) quantifies momentum-relaxing collisions
- T-square thermal resistivity (B_2) quantifies momentum-conserving collisions

Comparing their ratio , one can see if there is a hydrodynamic window!

Is there a hydrodynamic window for electrons?

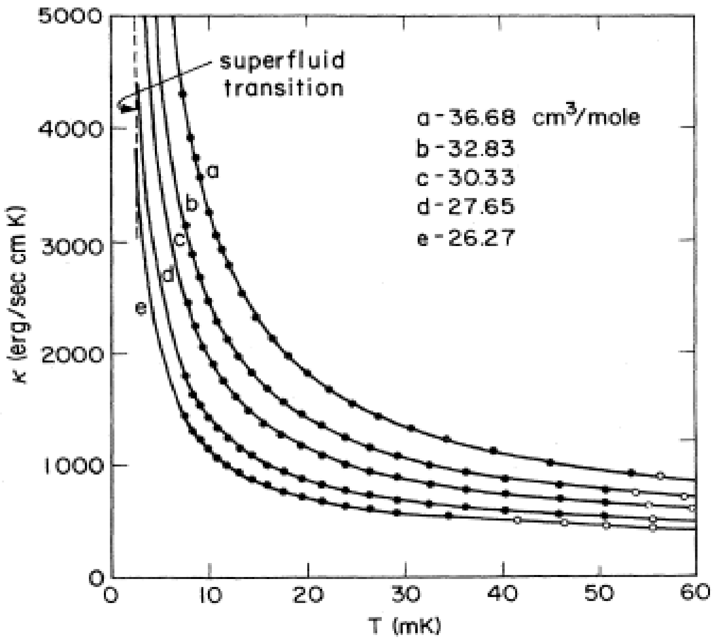


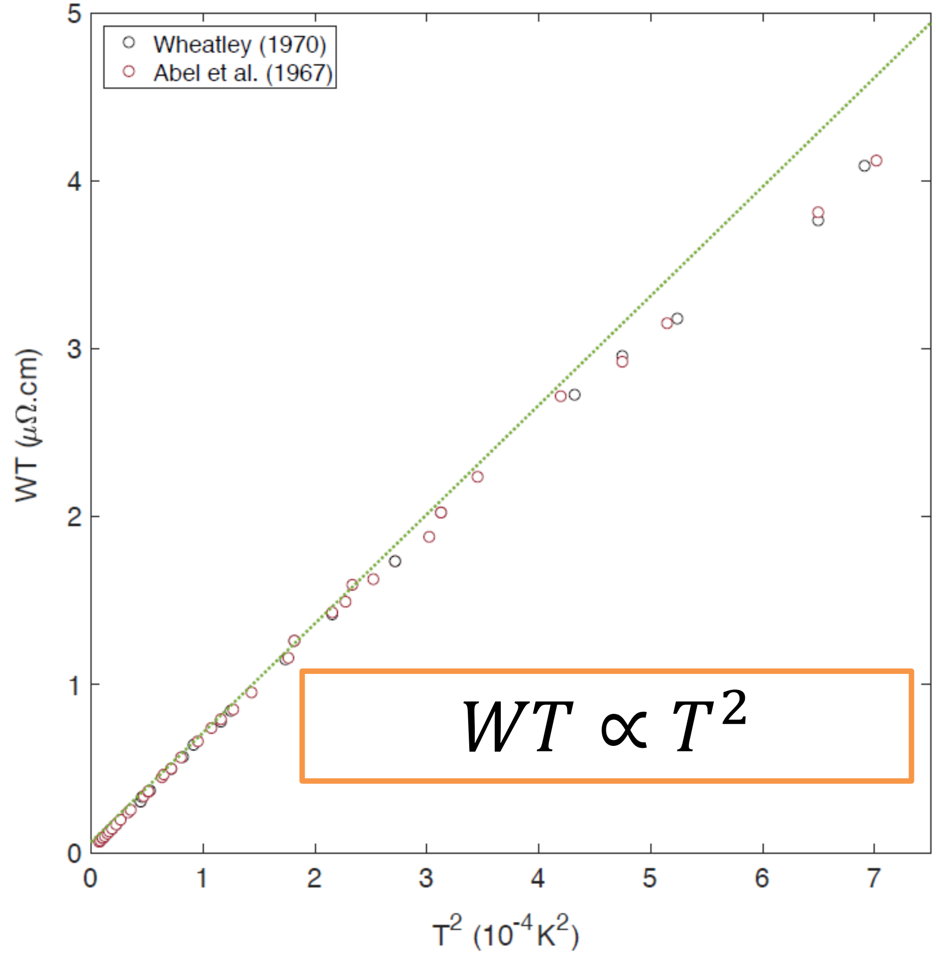


Momentum-conserving collisions can be quantified thanks to B_2T^2!

Back to the original Fermi liquid: ^{3}He

Greywell, 1984



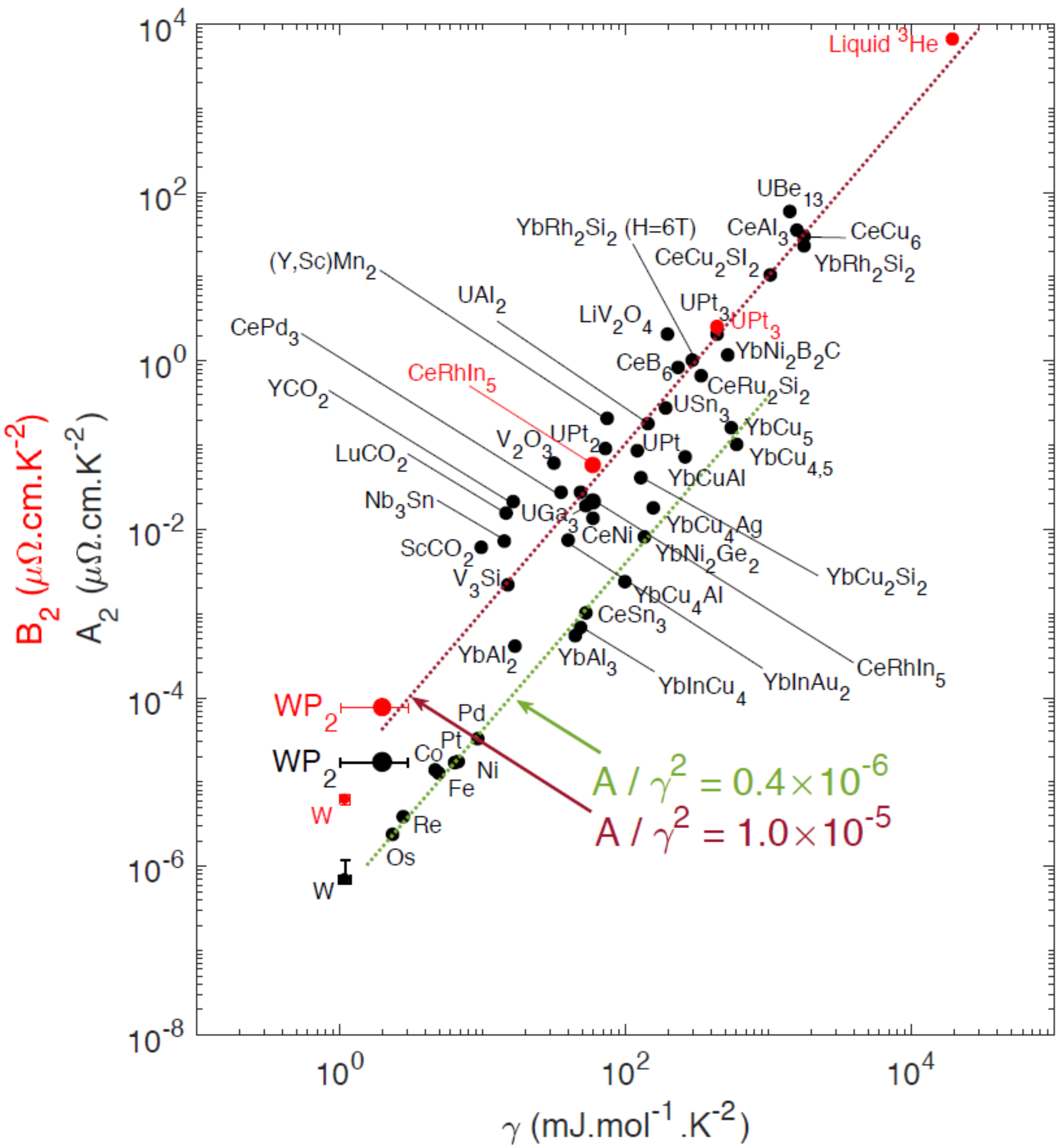


$$k \propto T^{-1}$$

$$WT \propto T^2$$

- Driven by fermion-fermion scattering
- Reflects the temperature dependence of viscosity

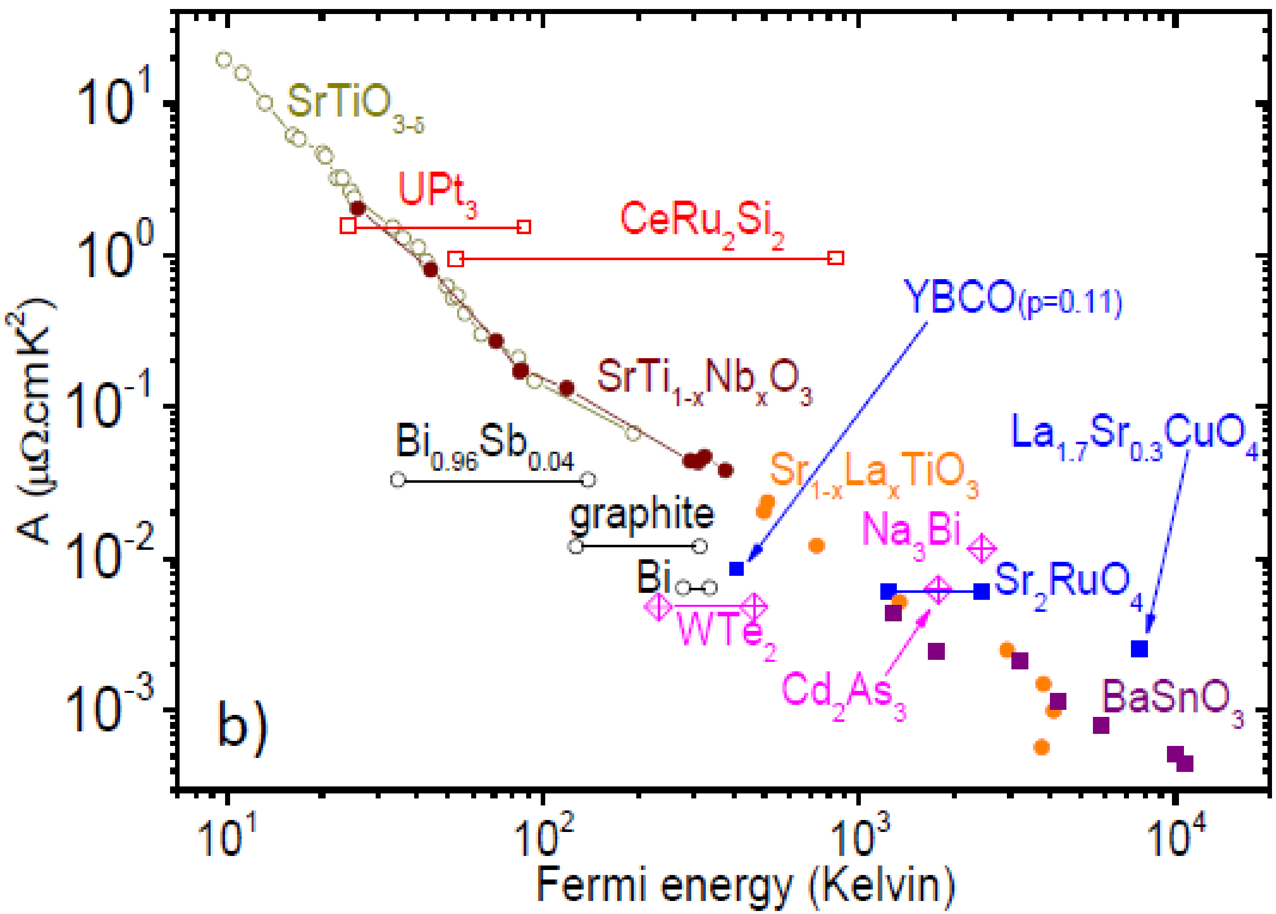
^{3}He and Kadowaki-Woods



- Momentum-relaxing collisions
- Momentum-conserving collisions

Why is this interesting?

- The origin of T-square resistivity in Fermi liquids is a mystery



Even in absence of Umklapp, the lattice taxes any momentum exchange between electrons

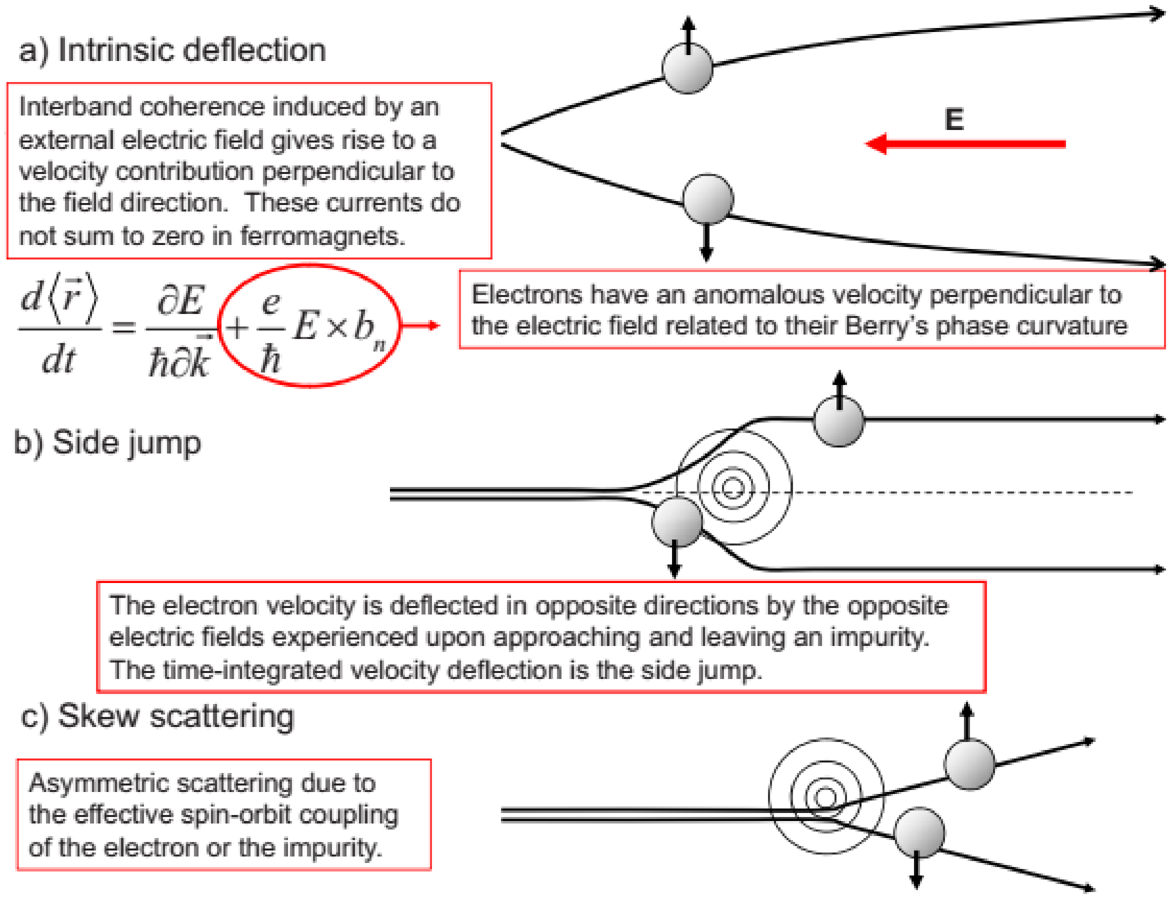
$$A = \frac{\hbar}{e^2}\left(\frac{k_B}{E_F}\right)^2 k_F \sigma_{cs}$$

Summary

- In some solids, in a finite temperature window, phonon flow is strengthened by collisions. This is the Gurzhi hydrodynamic regime.
- Some are close to a structural instability. Such a proximity may enhance normal scattering.
- Electron-hole compensation can boost it.
- A universal lower bound to thermal diffusivity according to the available data.

Thermal transport and Berry curvature

Anomalous Hall Effect



Nagaosa, Sinova, Onoda, MacDonald and Ong, RMP 2010

Berry Curvature on the Fermi Surface: Anomalous Hall Effect as a Topological Fermi-Liquid Property

F. D. M. Haldane

Department of Physics, Princeton University, Princeton New Jersey 08544-0708, USA

(Received 28 June 2004; revised manuscript received 20 October 2004; published 11 November 2004)

The intrinsic anomalous Hall effect in metallic ferromagnets is shown to be controlled by Berry phases accumulated by adiabatic motion of quasiparticles on the Fermi surface, and is purely a Fermi-liquid property, not a bulk Fermi sea property like Landau diamagnetism, as has been previously supposed. Berry phases are a new topological ingredient that must be added to Landau Fermi-liquid theory in the presence of broken inversion or time-reversal symmetry.

DOI: 10.1103/PhysRevLett.93.206602 PACS numbers: 72.15.–v, 73.43.–f

… a purely Fermi-liquid property not a bulk Fermi sea property like Landau diamagnetism.

$$\kappa_0^{ab}(\mu) = \frac{\pi^2}{3}\frac{k_B^2 T}{e^2}\sigma_0^{ab}(\mu), \qquad \alpha_0^{ab}(\mu) = e\frac{\partial \kappa_0^{ab}(\mu)}{\partial \mu}.$$

The Fermi-surface vs. Fermi sea debate

$$\sigma_{ij}^{A} = \frac{-e^2}{\hbar} \sum_n \int_{BZ} \frac{d^3k}{(2\pi)^3} f_n(k) \Omega_n^k(k)$$

FD distribution

$$\sigma_{ij}^{A} = \frac{-e^2}{\hbar} \sum_n \int_{S_n} \frac{d^2k}{(2\pi)^2} [\Omega_n^k(k).\hat{n}(k)]\mathbf{k}$$

Unit vector normal to the FS

Do Weyl nodes operate deep below the Fermi sea?

Yes!

"… the common belief that (the nonquantized part of) the intrinsic anomalous Hall conductivity of a ferromagnetic metal is entirely a Fermi-surface property, is incorrect.''
Chen, Bergman & Burkov, Phys. Rev. B **88**, 120110 (2013)

No!

''…the nonquantized part of the intrinsic anomalous Hall conductivity can be expressed as a Fermi-surface property even when Weyl points are present in the band structure.''
Vanderblit, Souza & Haldane, Phys. Rev. B **92**, 1117101 (2014)

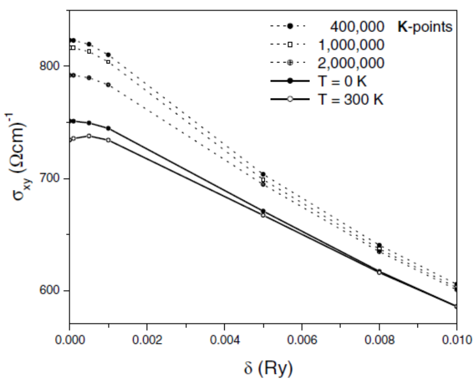
The case of BCC iron

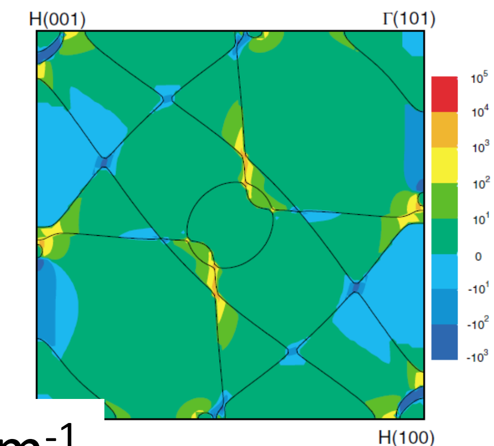
VOLUME 92, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 23 JANUARY 2004

First Principles Calculation of Anomalous Hall Conductivity in Ferromagnetic bcc Fe

Yugui Yao,[1,2,3] Leonard Kleinman,[1] A. H. MacDonald,[1] Jairo Sinova,[4,1] T. Jungwirth,[5,1] Ding-sheng Wang,[3] Enge Wang,[2,3] and Qian Niu[1]

VOLUME 92, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 23 JANUARY 2004





σ^{A}_{xy} (Theory) ~750 Ωcm^{-1}

The solid lines are obtained by an adaptive mesh refinement method.

r). Fermi surface in (010) plane (solid lines) and Berry curvature $-\Omega^z(\mathbf{k})$ in atomic units (color map).

Fermi sea

Band n	$\mathcal{K}^{(\Omega)}_{n3}$	$\mathcal{K}^{(\chi)}_{n3}$	$\mathcal{K}_{n3}$	AHC (S/cm)
1	2	0.51	2.51	−3394
2	− 6	3.03	− 2.97	4018
3	2	1.96	3.96	−5345
4	6	− 8.85	− 2.85	3840
5	− 8.01	6.22	− 1.79	2413
6	− 7.80	3.27	− 4.53	6111
7	14.12	− 6.44	7.68	−10 368
8	− 3.17	− 0.31	− 3.48	4702
9	− 0.53	1.33	0.80	−1076
10	0.83	− 0.72	0.11	−146
Total	− 0.56	0	− 0.56	755

Fermi surface

same band is also indicated.

Band n	Sheet a	Group label	Distance to a PN $(2\pi/a)$	AHC (S/cm)	
5	1	IV	0.30	9	
6	1	III	0.02	–274	
7	1	V	0.06	459	
7	2,3,4,5	VIII(a)	0.01	–203	×4
7	6,7	VIII(b)	0.09	100	×2
8	1	II	0.03	242	
9	1	I	0.02	714	
10	1	VI	0.10	58	
10	2,3,4,5	VII(a)	0.31	–1	×4
10	6,7	VII(b)	0.01	167	
Total				759	

PHYSICAL REVIEW B **92**, 085138 (2015)

Chiral degeneracies and Fermi-surface Chern numbers in bcc Fe

Daniel Gosálbez-Martínez,[1,2] Ivo Souza,[1,3] and David Vanderbilt[4]
[1]Centro de Física de Materiales, Universidad del País Vasco, 20018 San Sebastián, Spain
[2]Donostia International Physics Center, 20018 San Sebastián, Spain
[3]Ikerbasque Foundation, 48013 Bilbao, Spain
[4]Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854-8019, USA

The Wiedemann-Franz law and the surface-sea debate!

$$L_{xy}^{A} = \frac{\kappa_{xy}^{A}}{T\sigma_{xy}^{A}} = \frac{\pi^2}{3}(\frac{k_B}{e})^2$$

- In the Fermi-sea picture, an accident!
- In the Fermi-surface picture, unavoidable!

Entropy flow is restricted to the surface of the Fermi sea!

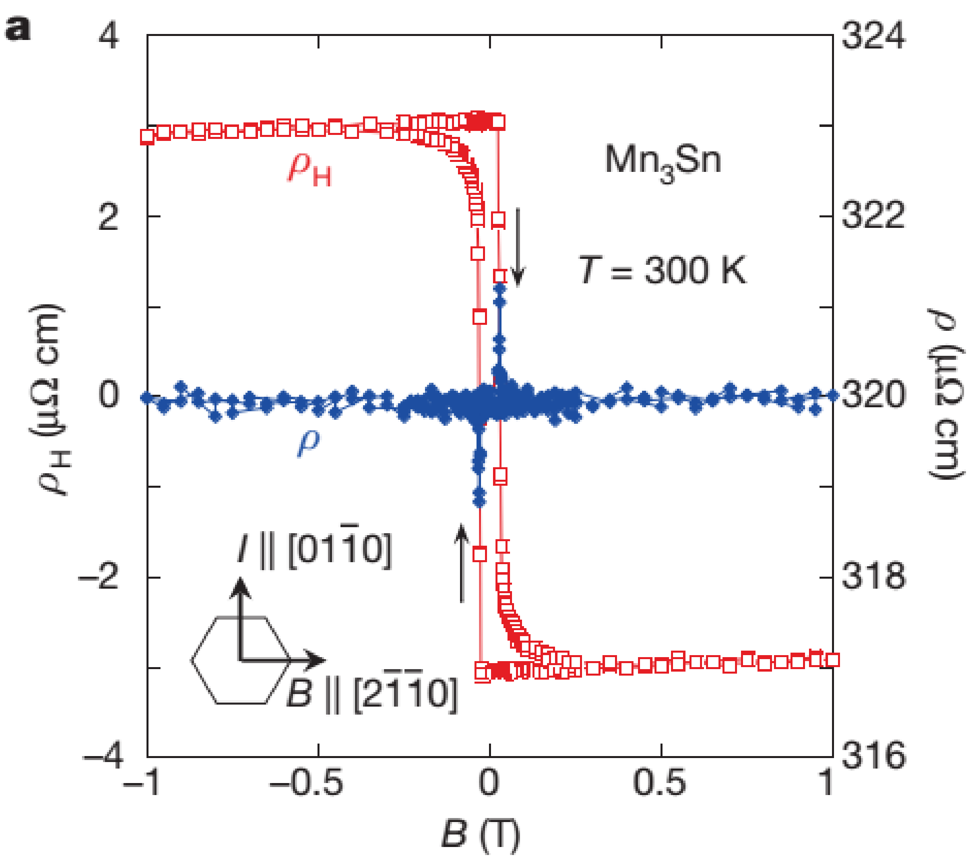
LETTER

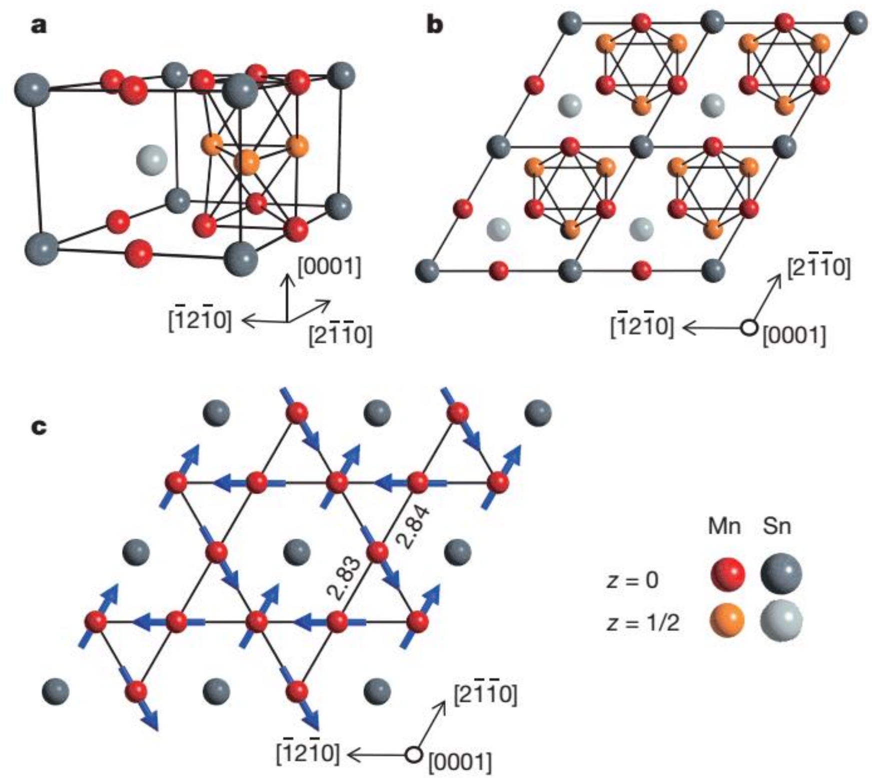
doi:10.1038/nature15723

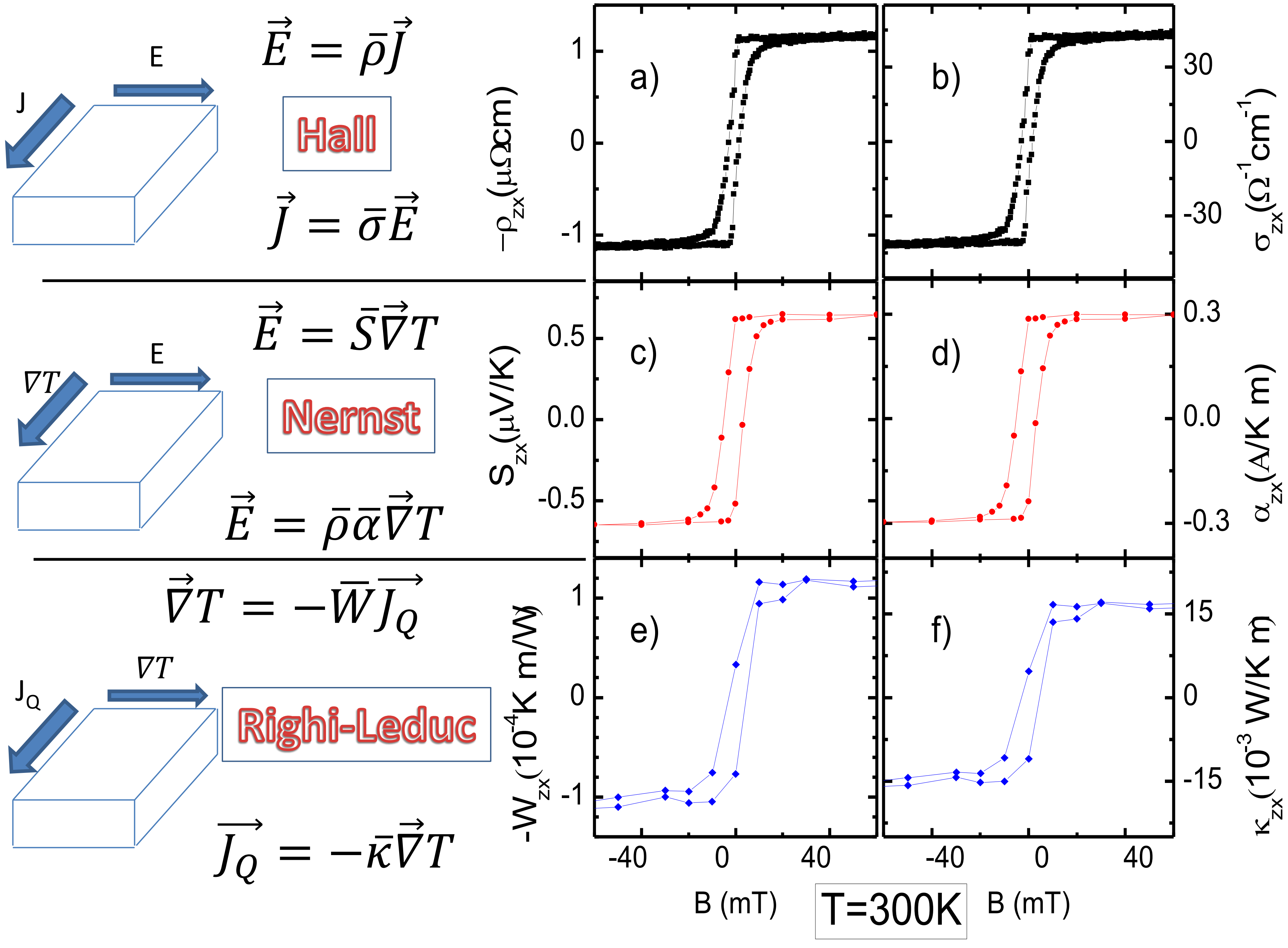
Large anomalous Hall effect in a non-collinear antiferromagnet at room temperature

Satoru Nakatsuji[1,2], Naoki Kiyohara[1] & Tomoya Higo[1]

212 | NATURE | VOL 527 | 12 NOVEMBER 2015

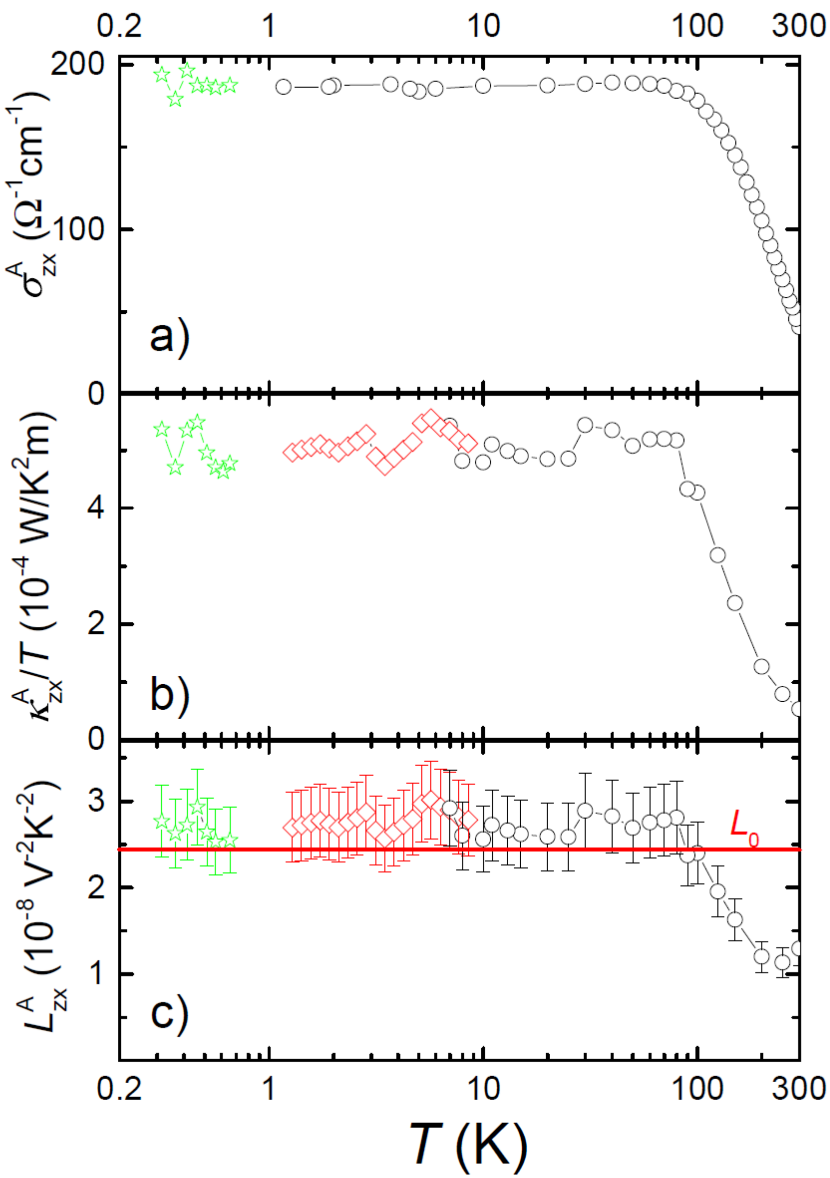






$\vec{E} = \bar{\rho}\vec{J}$
Hall
$\vec{J} = \bar{\sigma}\vec{E}$
J
E
$\vec{E} = \bar{S}\vec{\nabla}T$
Nernst
$\vec{E} = \bar{\rho}\bar{\alpha}\vec{\nabla}T$
∇T
E
$\vec{\nabla}T = -\bar{W}\overrightarrow{J_Q}$
Righi-Leduc
$\overrightarrow{J_Q} = -\bar{\kappa}\vec{\nabla}T$
J_Q
∇T
a)
b)
c)
d)
e)
f)
$-\rho_{zx}$ (μΩcm)
σ_{zx} ($\Omega^{-1}cm^{-1}$)
S_{zx} (μV/K)
α_{zx} (A/K m)
$-W_{zx}$ (10^{-4}K m/W)
κ_{zx} (10^{-3} W/K m)
B (mT)
B (mT)
T=300K

The Anomalous transverse WF law in Mn_3Ge

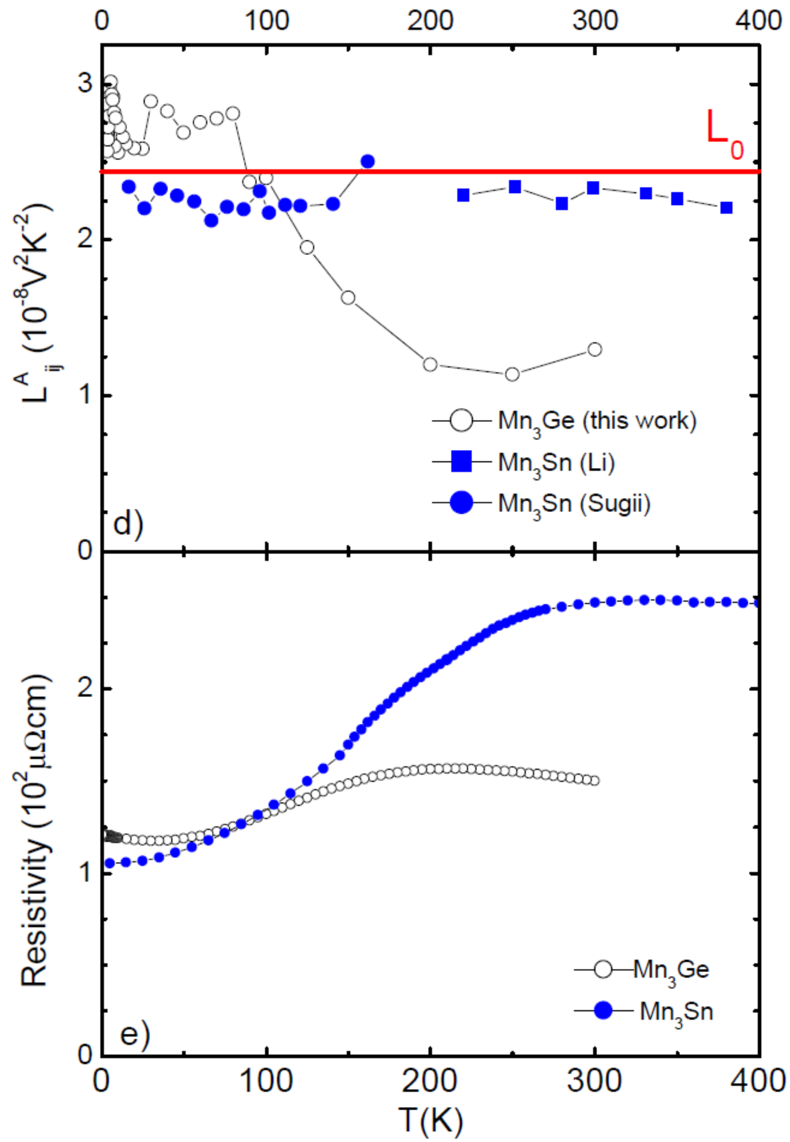


$$L^A_{xy} = \frac{\kappa^A_{xy}}{T\sigma^A_{xy}} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

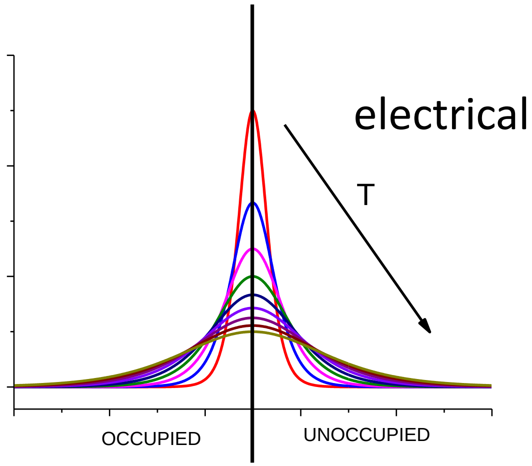
- Holds at T→0 K!
- But not above 100 K!

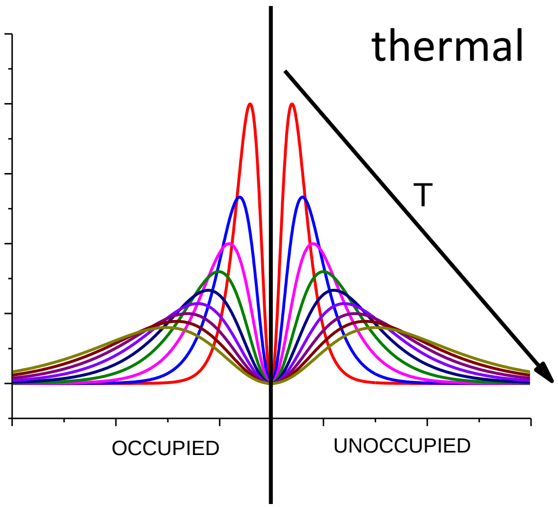
Validity and violation of the WF law

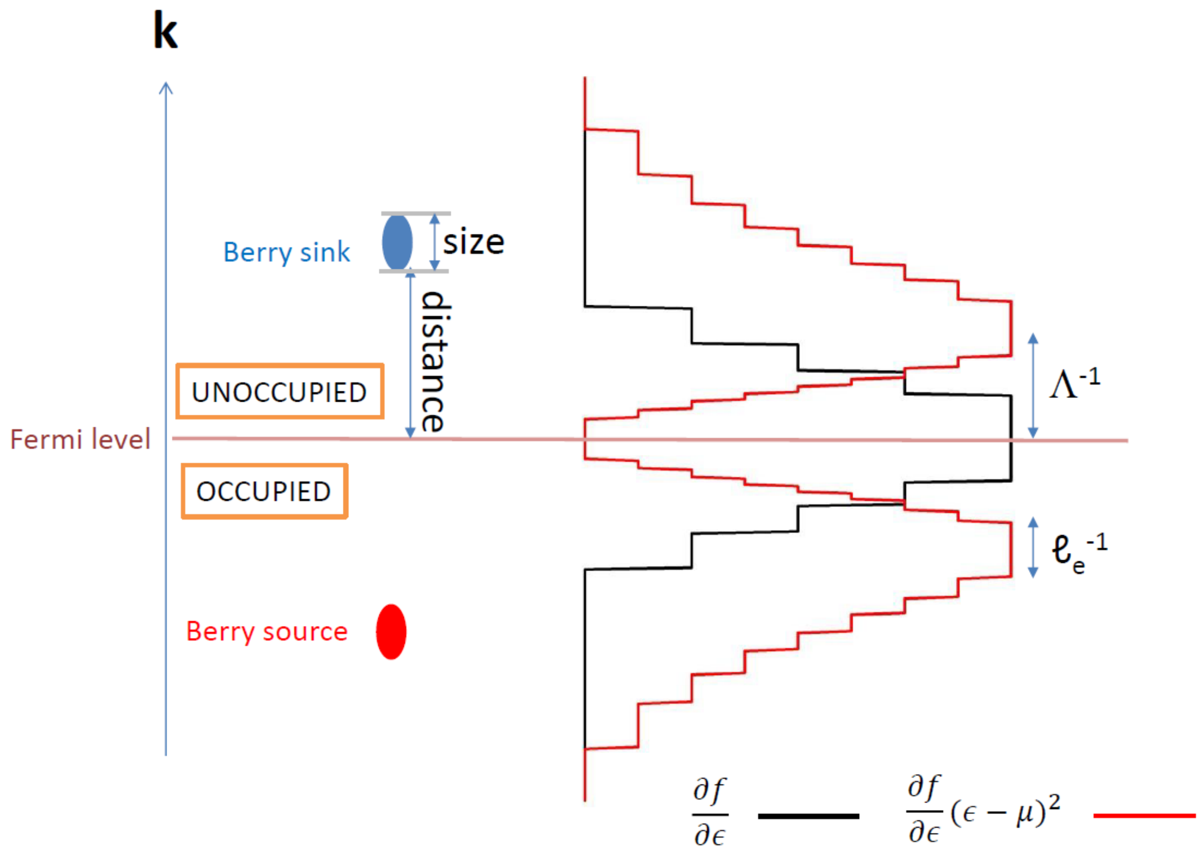
- Finite-temperature violation in Mn_3Ge and validity in Mn_3Sn!
- Similar inelastic scattering!



Electrical and thermal summations



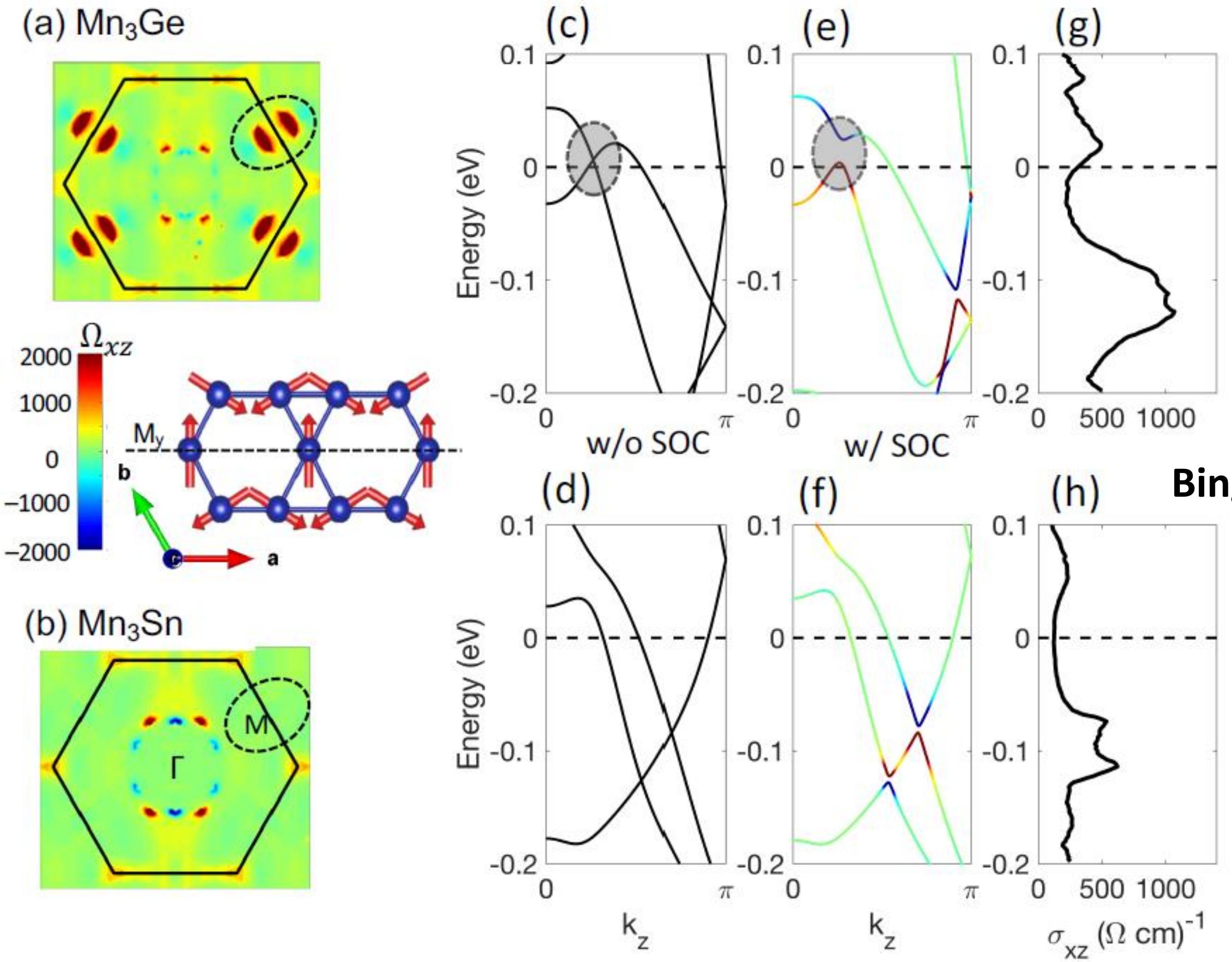




They mismatch in summing up Berry curvature!

Ab initio theory

- A 10 meV gap generating a large Berry curvature in Mn_3Ge and absent in Mn_3Sn.



Binghai Yan

Summary

- The anomalous transverse WF law is valid at T=0 implying that AHE is a Fermi surface property.

- The finite temperature violation can occur by a difference in thermal and electrical summations of the Berry curvature over the Fermi surface.

- It reveals an energy scale in the Berry spectrum, an information unavailable with charge transport alone.

- We need a theory taking care of the role of disorder and experiments quantifying it.



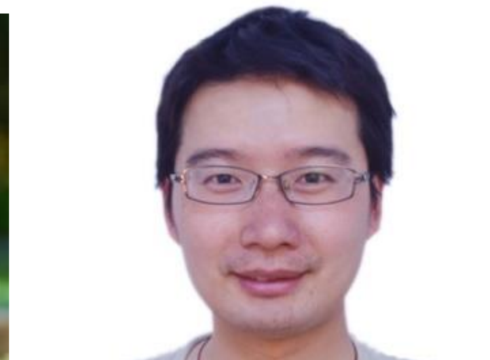

Benoît Fauqué, Alexandre Jaoui, Clément Collignon & XiaoKang Li
Paris

Willem Rischau
Geneva

Xiao Lin
Hangzhou

Valentina Martelli
Sao Paolo

Binghai Yan
Tel Aviv



Zengwei Zhu, Wuhan

Liangcai Xu , Wuhan

Yo Machida, Tokyo