Thermal transport beyond the quasiparticle paradigm

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Frontiers in Thermal Transport and Energy Conversion @ NAS
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Quasiparticle formulae

- Diffusion invented to describe thermal transport.

"Heat, like gravity, penetrates all substances in the universe."

[Fourier, 1822]

- Quasiparticle picture:

$$D \sim v_{\mathrm{qp}}^2 \tau_{\mathrm{qp}} \sim v_{\mathrm{qp}} \ell_{\mathrm{qp}}$$

[Maxwell, Boltzmann, Einstein …]

- If the same quasiparticles carry charge as well as heat then

$$L \equiv \frac{\kappa}{\sigma T} \sim \frac{k_B^2}{e^2}$$

[Wiedemann-Franz, 1853]

Quasiparticles challenged

- The emergence of well-defined quasiparticles from a quantum many body system is nontrivial (eg. in Fermi Liquid theory) and may not occur.

- Discuss evidence for non-quasiparticle physics:
  — Short mean free paths: $\ell_{\rm qp} \lesssim a, \lambda_{\rm qp}$
  — Strong violation of the WF law: $L \ll L_0$

- Non-quasiparticle physics may potentially lead to e.g. high figures of merit.

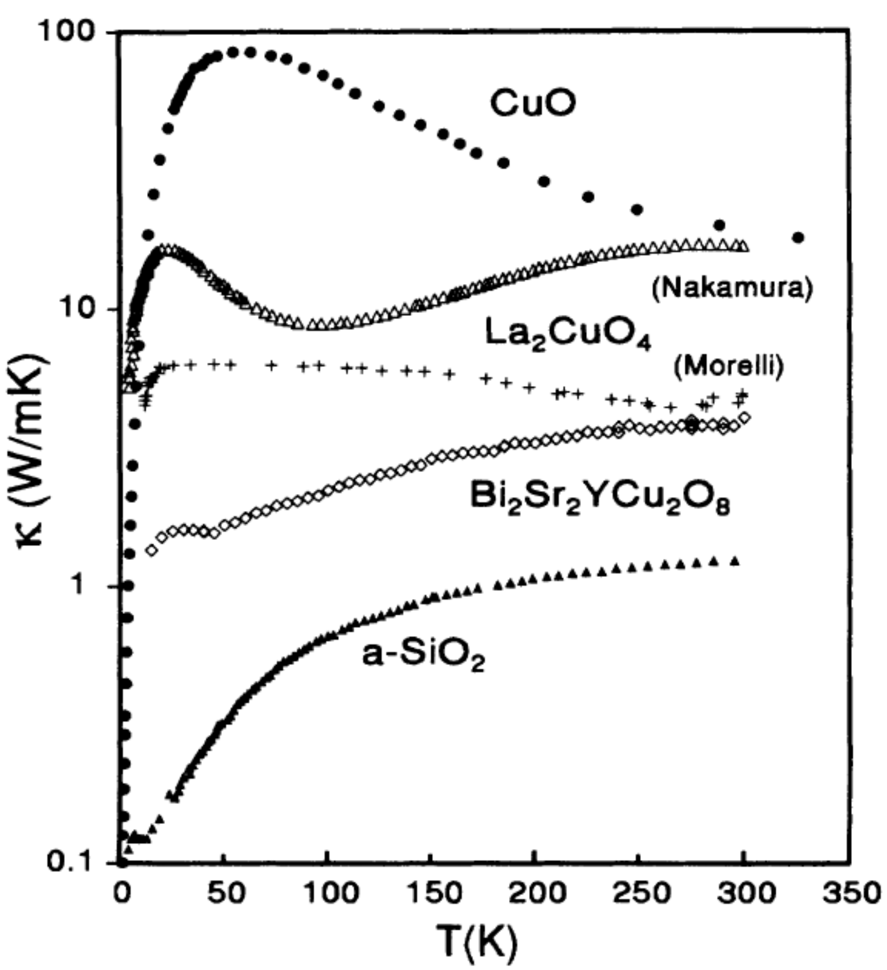
Quasiparticles challenged

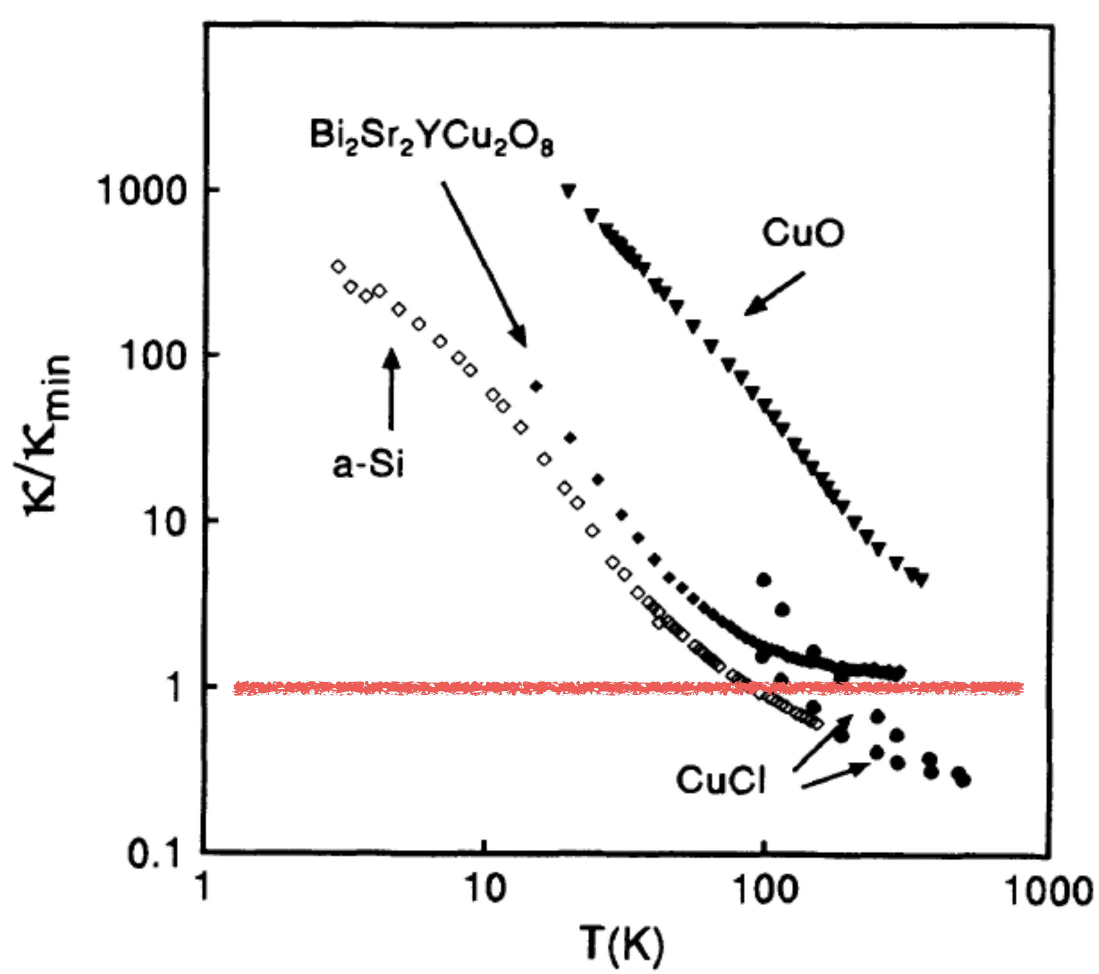
PHYSICAL REVIEW B VOLUME 49, NUMBER 13 1 APRIL 1994-I

Thermal conductivity of insulating $Bi_2Sr_2YCu_2O_8$ and superconducting $Bi_2Sr_2CaCu_2O_8$: Failure of the phonon-gas picture

Philip B. Allen, Xiaoqun Du, and Laszlo Mihaly
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794-3800

Laszlo Forro*
Department of Physics, Ecole Polytechnique Federale de Lausanne, 1015 Lausanne, Switzerland
(Received 28 October 1993)





Quasiparticles challenged

Anomalous thermal diffusivity in underdoped $YBa_2Cu_3O_{6+x}$

Jiecheng Zhang[a,b], Eli M. Levenson-Falk[a,b], B. J. Ramshaw[c], D. A. Bonn[d,e], Ruixing Liang[d,e], W. N. Hardy[d,e], Sean A. Hartnoll[b], and Aharon Kapitulnik[a,b,f,1]

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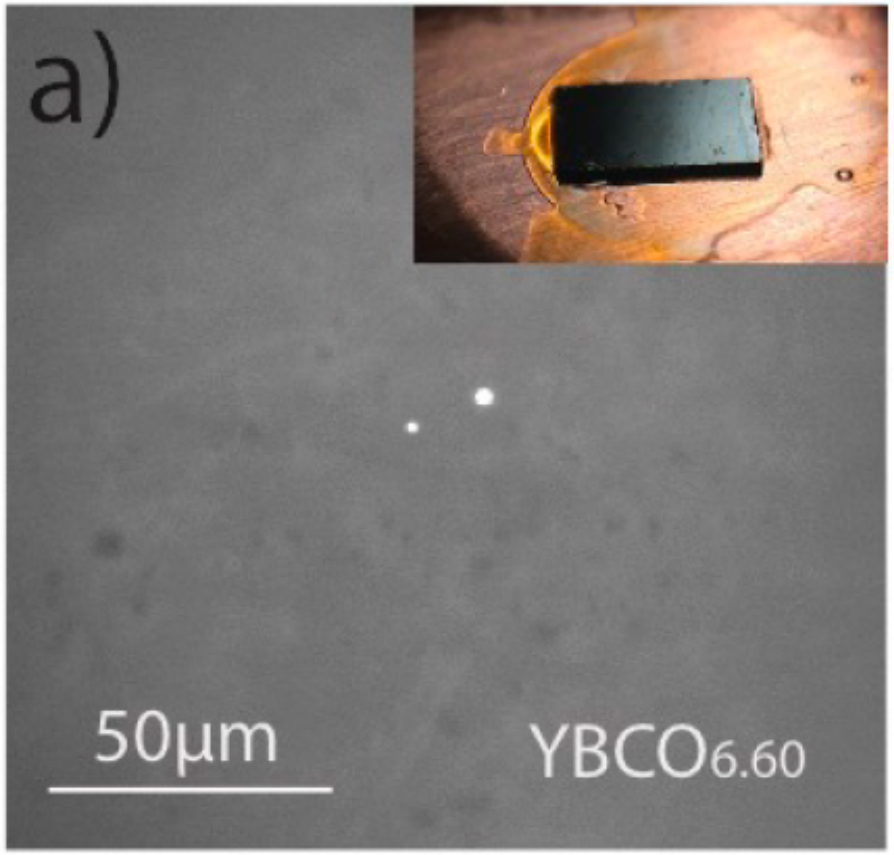
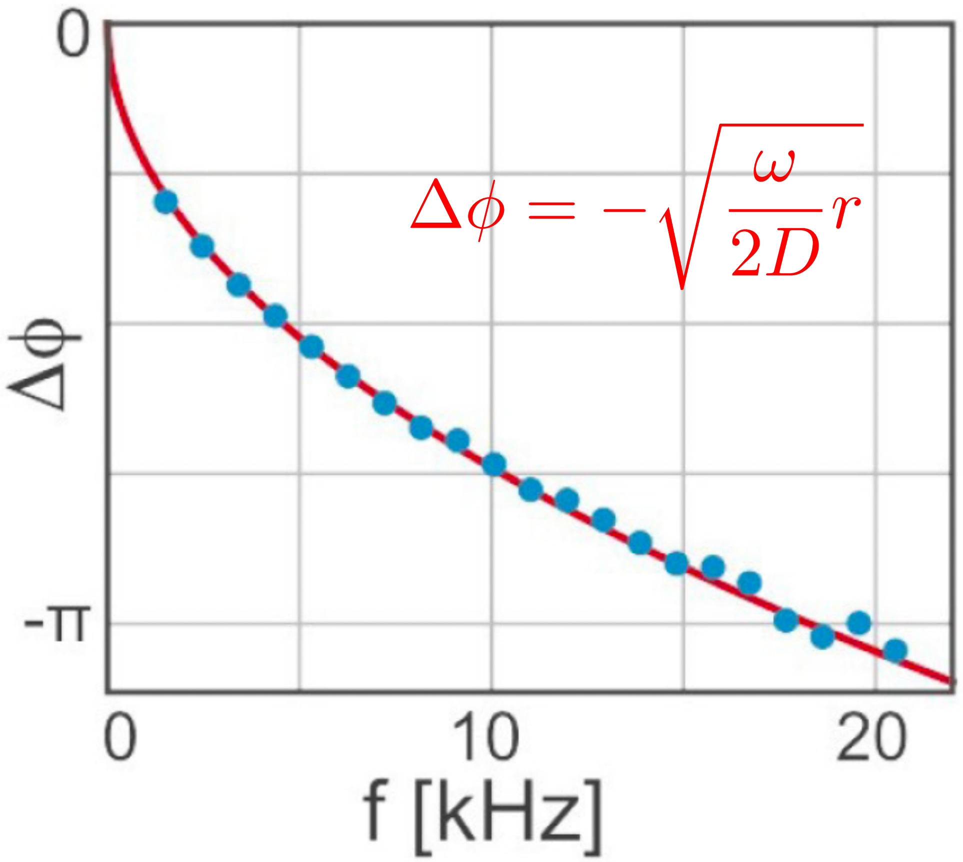
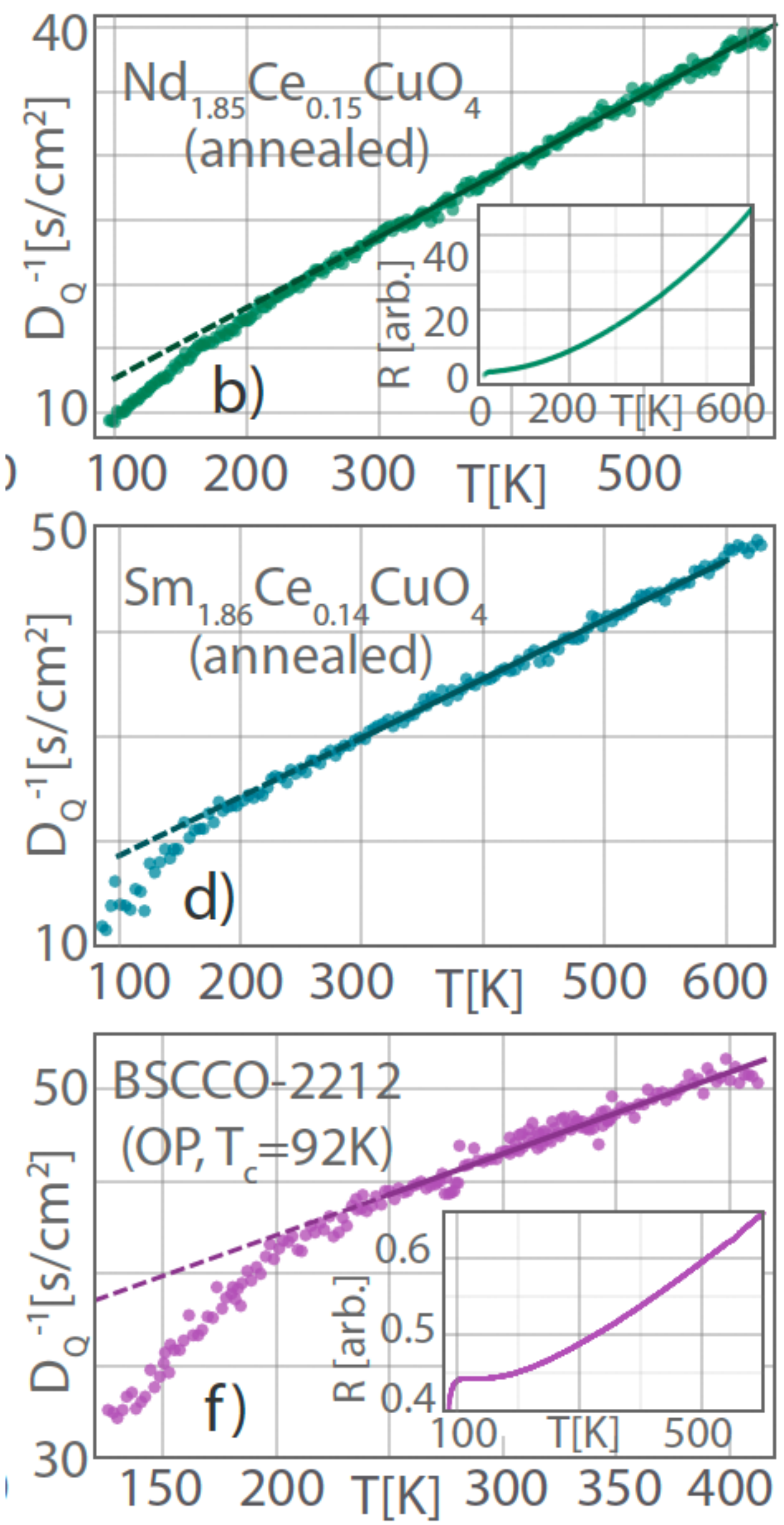
Contributed by Aharon Kapitulnik, April 12, 2017 (sent for review February 28, 2017; reviewed by Kamran Behnia, Andrey Chubukov, and Andrew Mackenzie)





Quasiparticles challenged



Thermal Diffusivity Above the Mott-Ioffe-Regel Limit

Jiecheng Zhang,[1,2,*] Erik D. Kountz,[1,2] Eli M. Levenson-Falk,[3] Dongjoon Song,[4] Richard L. Greene,[5,6] and Aharon Kapitulnik[1,2,7]

arXiv:1808.07564

$$D^{-1} \sim \frac{1}{v_s^2} \frac{k_B T}{\hbar}$$

Strongly reminiscent of widely observed “bad metal” T-linear resistivity:

ARTICLES
PUBLISHED ONLINE: 23 DECEMBER 2014 | DOI: 10.1038/NPHYS3174
nature physics

Theory of universal incoherent metallic transport

Sean A. Hartnoll

$$D^{-1} \sim \frac{1}{v_F^2} \frac{k_B T}{\hbar}$$

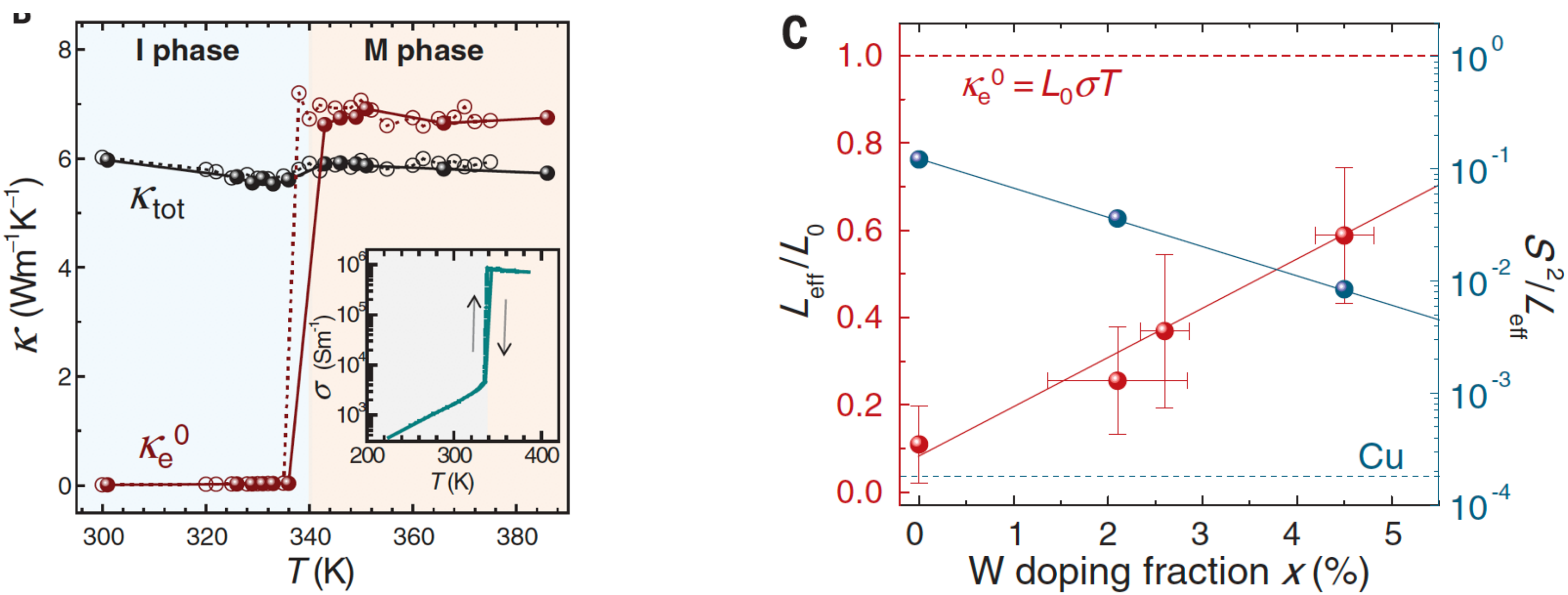
Quasiparticles challenged

SOLID-STATE PHYSICS *Science* **355**, 371–374 (2017)

Anomalously low electronic thermal conductivity in metallic vanadium dioxide

Sangwook Lee,[1,2*] Kedar Hippalgaonkar,[3,4*] Fan Yang,[3,5*] Jiawang Hong,[6,7*] Changhyun Ko,[1] Joonki Suh,[1] Kai Liu,[1,8] Kevin Wang,[1] Jeffrey J. Urban,[5] Xiang Zhang,[3,8,9] Chris Dames,[3,8] Sean A. Hartnoll,[10] Olivier Delaire,[7,11†] Junqiao Wu[1,8†]

$$\frac{L_{\rm el}}{L_0} \sim 0.1, \qquad \frac{S^2}{L_{\rm el}} \sim 0.1$$



Beyond quasiparticles

- What determines the diffusivity in the absence of quasiparticles?
- What concepts replace the quasiparticle velocity, lifetime and mean free path?
- Now discuss:
  — A general bound on diffusivity.
  — A concrete model for non-quasiparticle diffusion.

Bounding diffusion

PRL **119,** 141601 (2017) PHYSICAL REVIEW LETTERS week ending 6 OCTOBER 2017

Upper Bound on Diffusivity

Thomas Hartman,[1] Sean A. Hartnoll,[2] and Raghu Mahajan[2]
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PHYSICAL REVIEW LETTERS **121,** 170601 (2018)

Locality Bound for Dissipative Quantum Transport

Xizhi Han (韩希之) and Sean A. Hartnoll
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A bound can identify fundamental constraints
(cf. bound on efficiency of heat engines)

Key ingredients: Conservation law + locality

Diffusion recap

- Correlations functions of conserved densities about thermal equilibrium are strongly constrained.

[Kadanoff-Martin 63]

- At long times $t > \tau_{\rm th}$, excitations of non-conserved quantities have decayed. Details of their ‘fast’ dynamics control the transport coefficients.

- For example a single conserved density must diffuse at the longest times and distances:

$$\omega = -iDk^2$$

Lieb-Robinson velocity

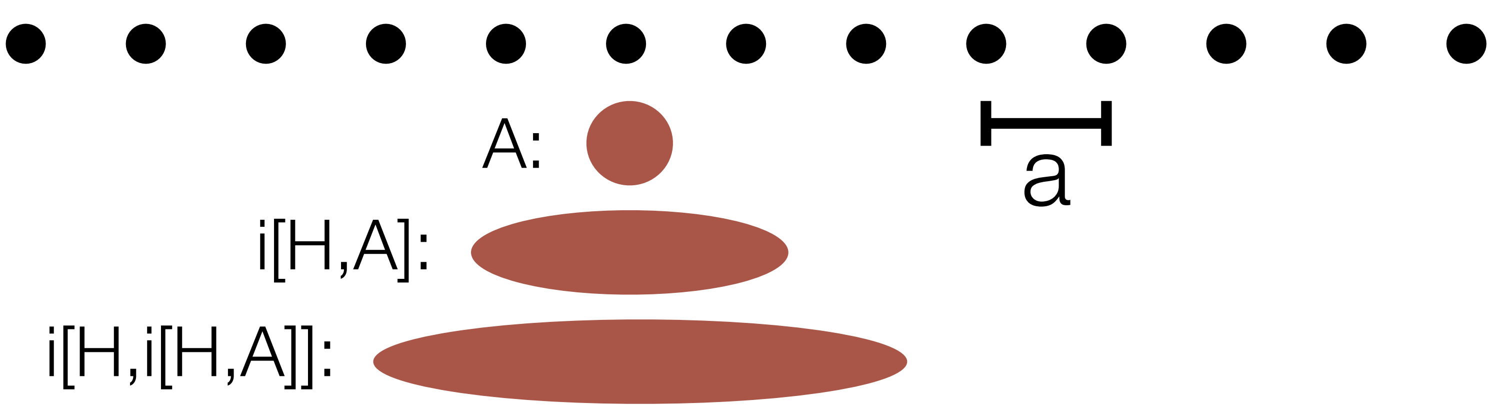
- Even non-relativistic systems have a 'lightcone': bounded propagation of signals from locality.

$$||[A(t,x),B(0,0)]|| \lesssim ||A||\,||B||e^{-\mu(|x|-vt)}$$

[Lieb-Robinson 72]

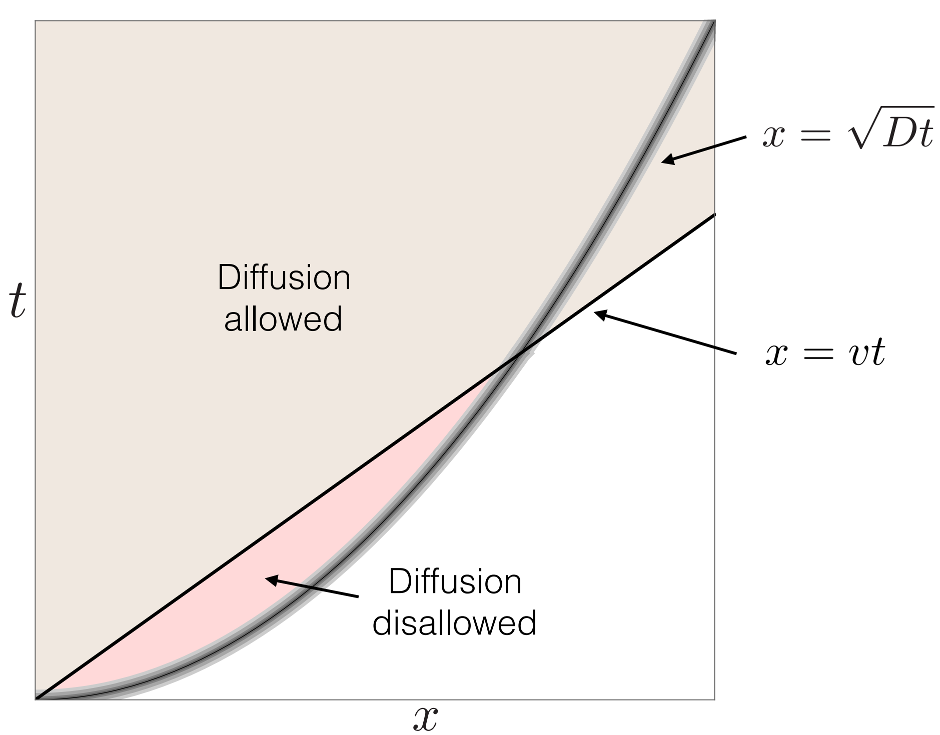
- The "Lieb-Robinson" velocity: $v \sim \frac{J\,a}{\hbar}$

- This is a microscopic, state-independent velocity. It describes the growth of operators under time evolution.



Bounding diffusion

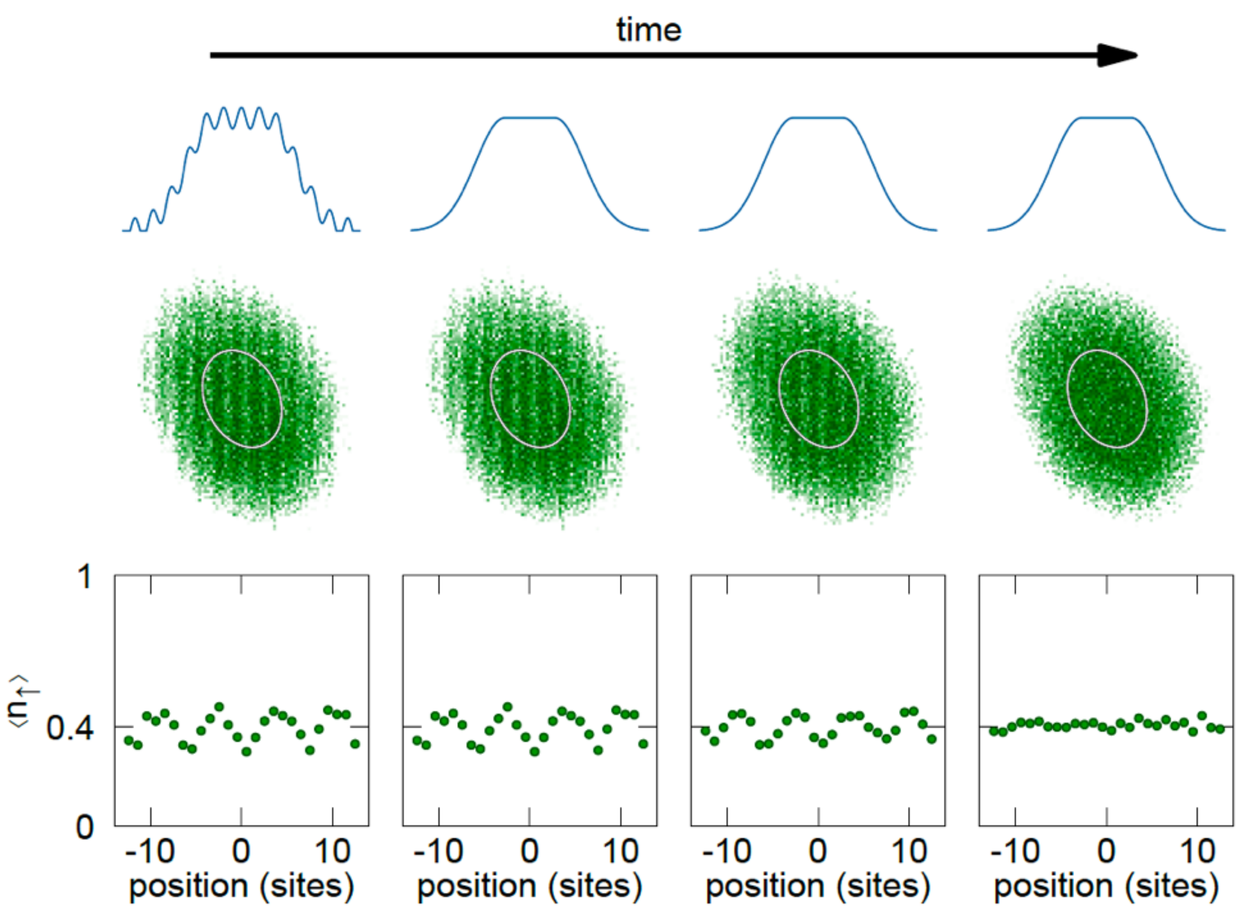
- The LR velocity clearly bounds ballistic transport (e.g. in ordered phases: $v_{\text{spin wave}} < v_{\text{LR}}$).
- It also bounds diffusivity: [Inspired by [Blake PRL 16]]

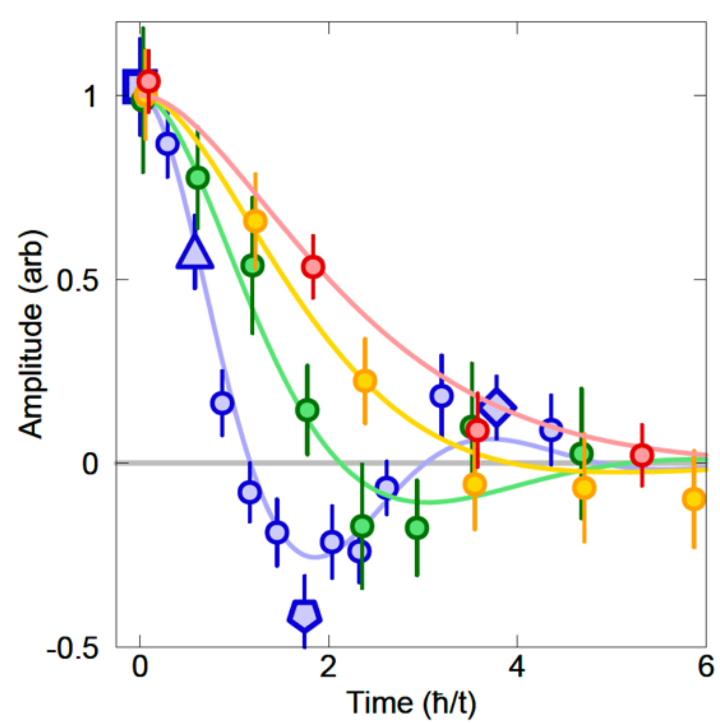


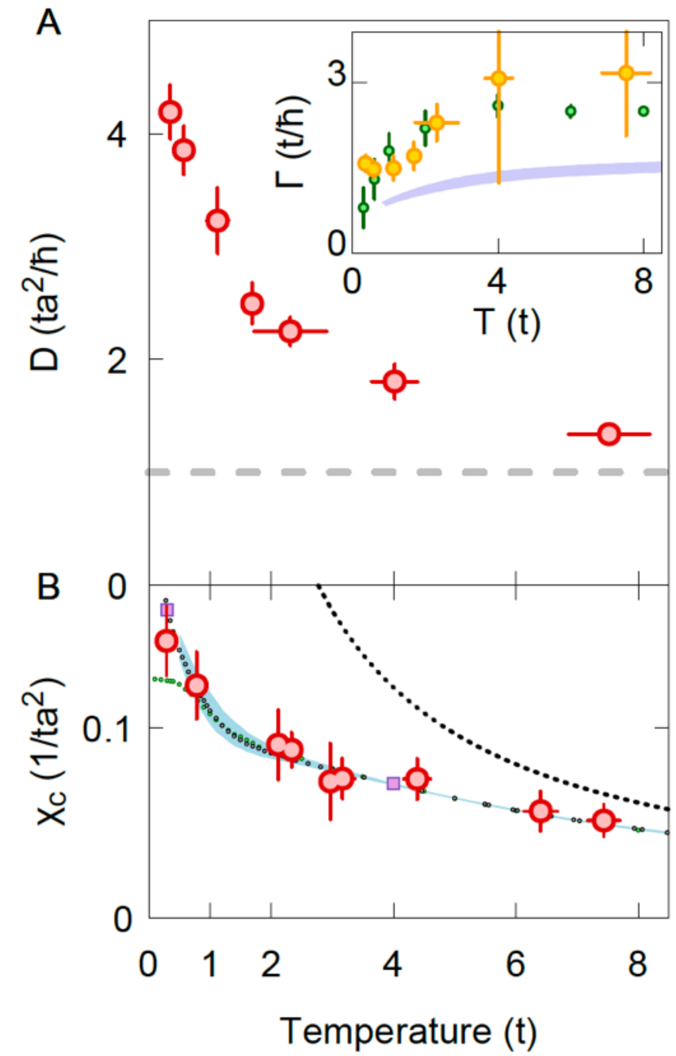
LR causality implies disallowed region must not be diffusive — i.e. must occur before local thermalization, so that:

$$D \lesssim v_{\text{LR}}^2 \tau_{\text{th}}$$

Measuring diffusion







Obeys bound:

$$D \lesssim \frac{(ta)^2}{\hbar^2} \frac{1}{\Gamma}$$

v_{LR}^2 τ_{th}

QUANTUM SIMULATION

Science **363**, 379–382 (2019)

Bad metallic transport in a cold atom Fermi-Hubbard system

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

Beyond quasiparticles

- In systems with a finite on-site Hilbert space (spins, fermions), diffusion is bounded by:
  — The Lieb-Robinson velocity
  — The local thermalization time

- These concepts do not make reference to quasiparticles.

- Next: see this at work in an explicit model. Solvable, realistic model of non-Boltzmann transport.

High T transport

A handle on non-quasiparticle transport: High temperatures in electron systems (e.g. t << T,U in Hubbard model)

[cf. Beni PRB 74, Mukerjee-Oganesyan-Huse PRB 06, Mukerjee-Moore APL 07]

Perturbation in small t in the Hubbard model doesn't work, because the t=0 model is extensively degenerate.

Bad metallic transport in a modified Hubbard model

Connie H. Mousatov, Ilya Esterlis, Sean A. Hartnoll

arXiv:1803.08054

Restore exponential interactions:

$$H = t \sum_{\langle ij \rangle, s} c_{is}^{\dagger} c_{js} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \frac{V}{2} \sum_{i \neq j} e^{-|\vec{x}_i - \vec{x}_j|/\ell} \, n_i n_j \, .$$

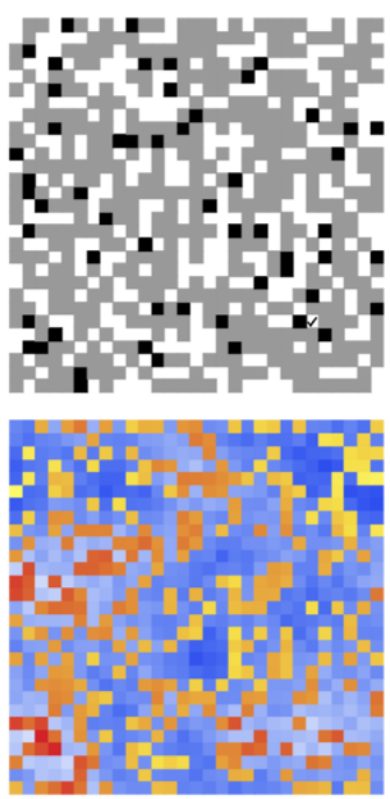
$$\ell > \ell_{\star} \approx 1.76a$$

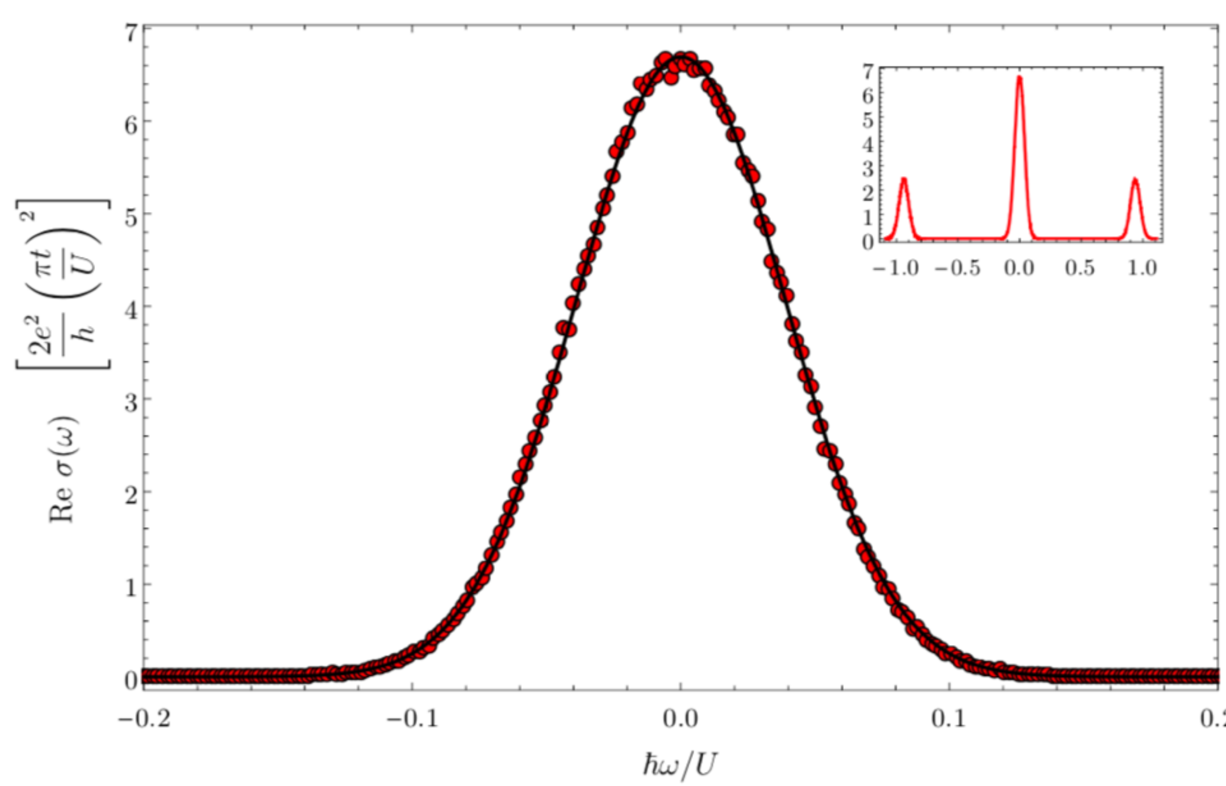
Beyond quasiparticles

Transport: Hopping in an emergent disordered landscape:

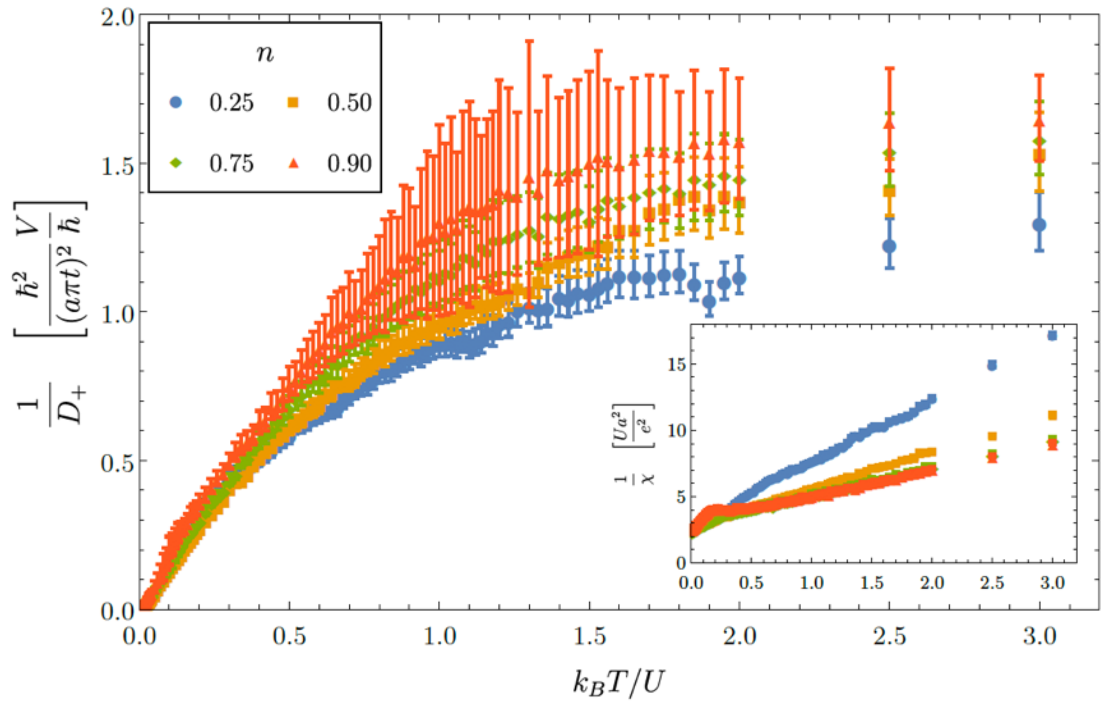
$$\mathrm{Re}\,\sigma(\omega) = \frac{2e^2}{h}\frac{(\pi t)^2}{\hbar}\frac{1-e^{-\beta\hbar\omega}}{\hbar\omega}\sum_{\{n\}}\frac{e^{-\beta(E_{\{n\}}-\mu N_{\{n\}})}}{\mathcal{Z}}\frac{a^2}{\mathrm{vol}}\sum_{i,s}\Delta_{is}(\omega)$$

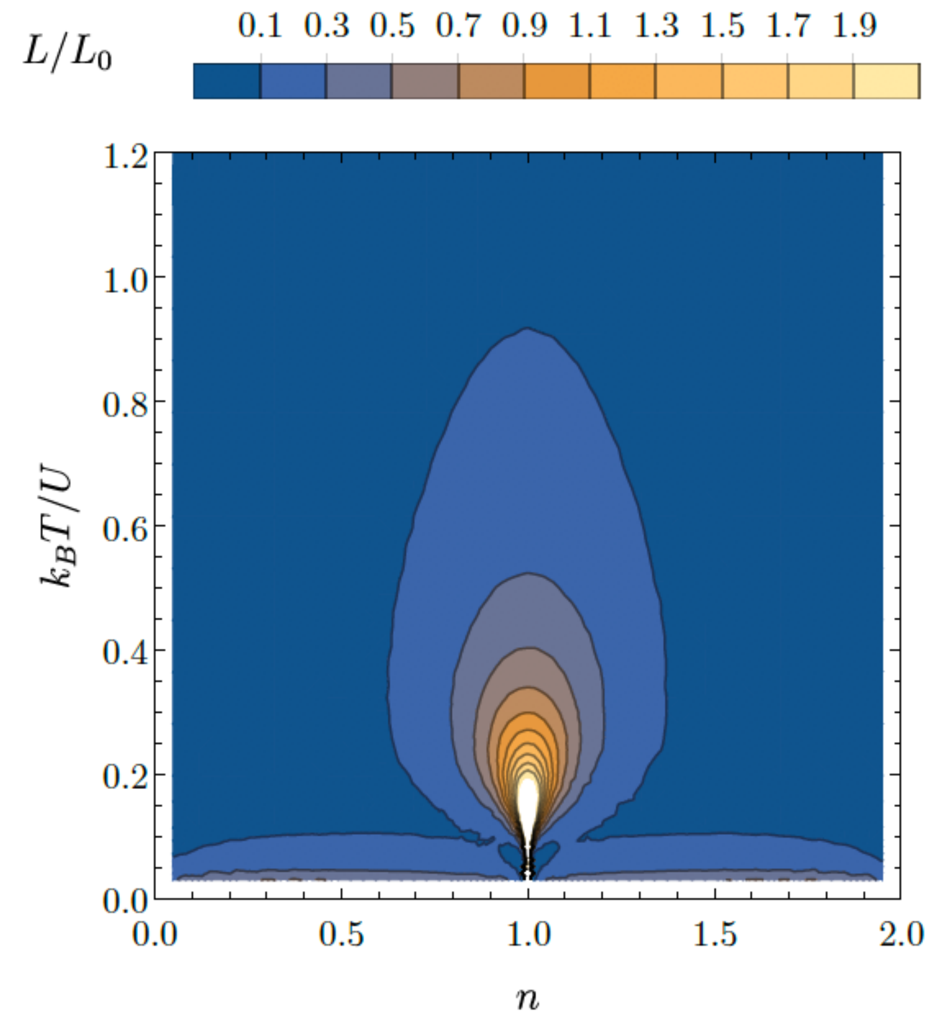
Classical Monte-Carlo: [cf. Beni 74]





Beyond quasiparticles





At high T: $D \sim \frac{(at)^2}{\hbar^2}\frac{\hbar}{V}$

Reason for $L \ll L_0$: at high T, chemical potential $\mu \sim T$

$$\Rightarrow \vec{Q} = \vec{J}_E - \mu\vec{J} \sim T\vec{J}$$

Heat and charge currents proportional: $\kappa = \overline{\kappa} - \frac{\alpha^2 T}{\sigma} \ll \overline{\kappa}$

Future challenges

- Phonons absent from theoretical discussion above:
  — Lieb-Robinson velocity not defined for infinite
  on-site Hilbert-space. Use butterfly velocity
  instead? (connection to quantum chaos).

  — High T expansion for transport more complicated.

- The bound does not tell you what the thermalization time $\tau_{\rm th}$ actually is. Is the value $\tau_{\rm th} \sim \hbar/k_B T$ special?

Two remarks

1. The cancellation $\kappa = \overline{\kappa} - \frac{\alpha^2 T}{\sigma} \ll \overline{\kappa}$, leading to dramatic violation of the WF law, occurs whenever the electric and heat currents become proportional.

A further circumstance where this occurs naturally is deep in a hydrodynamic regime, where both electric and heat currents are proportional to a collective velocity field:

$$\vec{j} = n\vec{v} + \cdots \,, \qquad \vec{q} = s\vec{v} + \cdots$$

PHYSICAL REVIEW B **88**, 125107 (2013)

Non-Fermi liquids and the Wiedemann-Franz law

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Two remarks

2. There exist fluctuation corrections to classical diffusion.
Nonlinear because thermal diffusivity D itself is a function of T.

Full consistent theory of these fluctuations:

PHYSICAL REVIEW LETTERS **122,** 091602 (2019)

Theory of Diffusive Fluctuations

Xinyi Chen-Lin, Luca V. Delacrétaz, and Sean A. Hartnoll
Department of Physics, Stanford University, Stanford, California 94305-4060, USA

The magnitude of the fluctuations is controlled by $1/(c\ell_{\rm th}^d)$

In non-quasiparticle diffusion, $\ell_{\rm th} \to a$ or possibly smaller

Fluctuation effects may become significant.

Conclusions

- Non-quasiparticle concepts are likely necessary to get a handle on quantum materials, and suggest the possibility of exotic material properties.
- The Lieb-Robinson velocity and the local thermalization time constrain diffusion (with or without quasiparticles).
- High temperature expansions can give an intuitive, tractable model of non-Boltzmann transport.
- Need to extend these ideas to phonons and understand constraints on thermalization time.