NON-ABELIAN ANYONS ON SYNTHETIC PLATFORMS

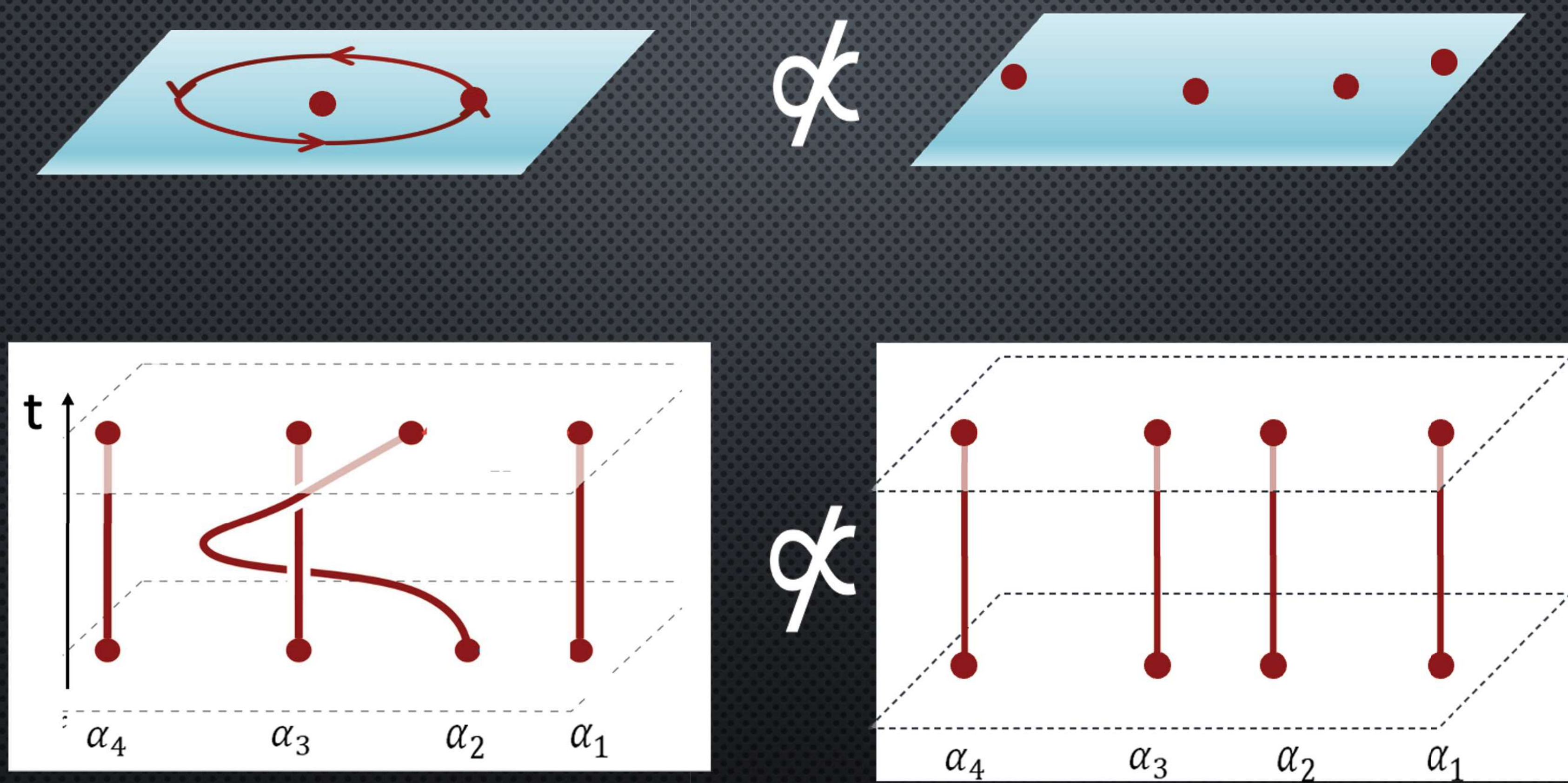
EUN-AH KIM

CORNELL UNIVERSITY

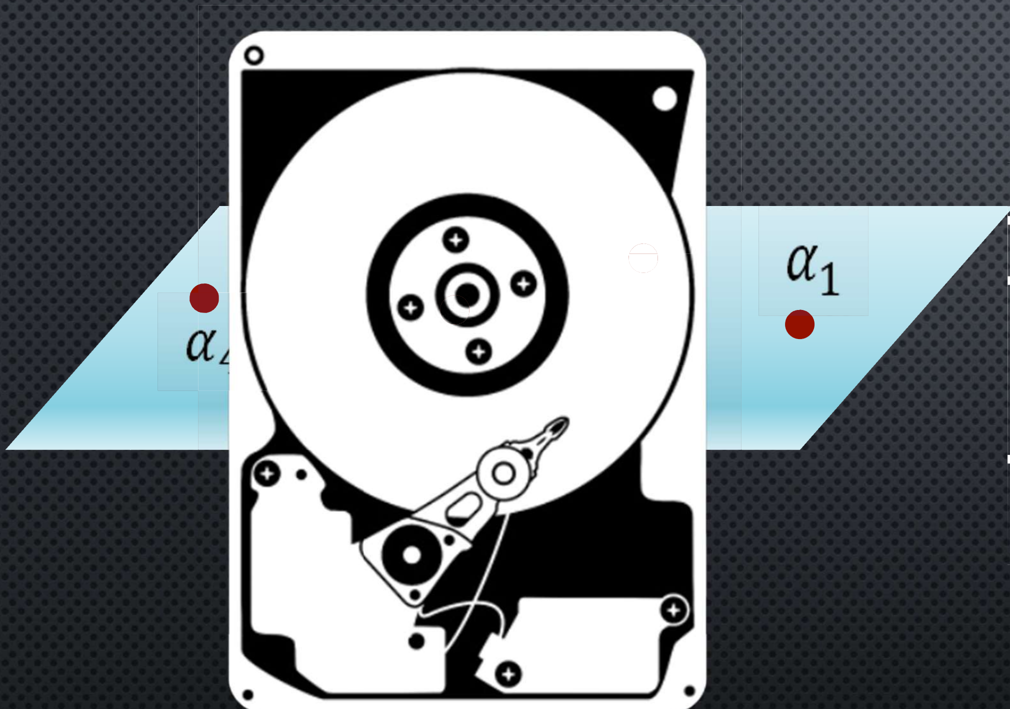
CMMRC WORKSHOP, 10/3/2024

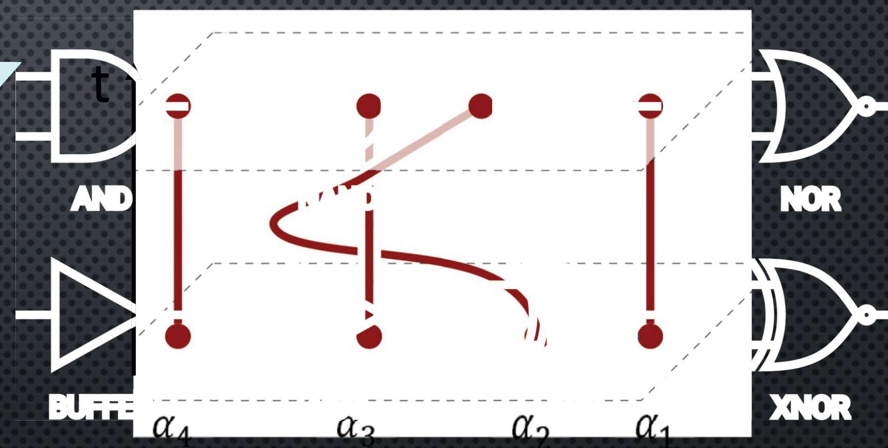
NON-ABELIAN ANYON-BASED QUANTUM COMPUTING

Non-Abelian Anyons

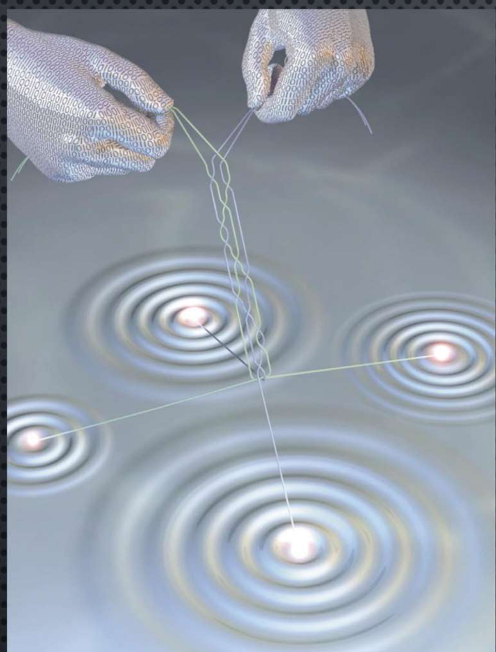


NON-ABELIAN ANYON BASED QUANTUM COMPUTING

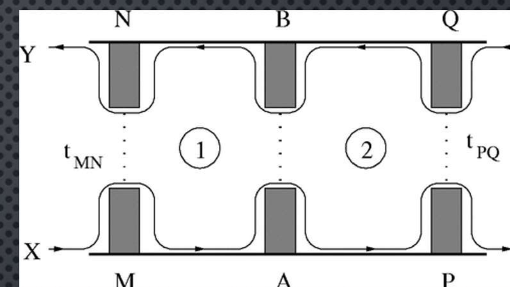




Experimental Realization?



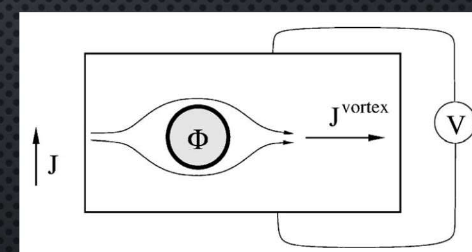
- 5/2 Quantum Hall state
  - Paired state of composite fermions
  - 1/4 charge qp



Das Sarma, Freedman, Nayak, PRL 94, 166802 (2005)

Feldman and Halperin, Rep. Prog. Phys 84, 076501 (2021)

- p+ip Superconductor
  - Theoretical paradigm
  - half quantum vortices



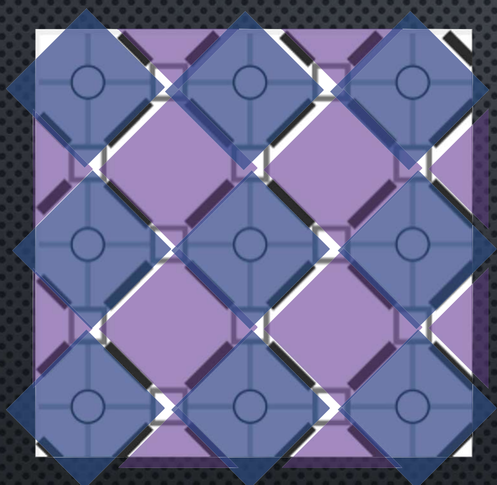
Das Sarma, Nayak, Tewari, PRB 73, 220502 (2006)

If an atomic force microscope can “grab” an anyon in order to measure this force, perhaps it can also be used to drag one around and perform a braid.

Nayak, Simon, Stern, Freedman, Das Sarma, RMP (2008)

Stabilizer States

Toric code

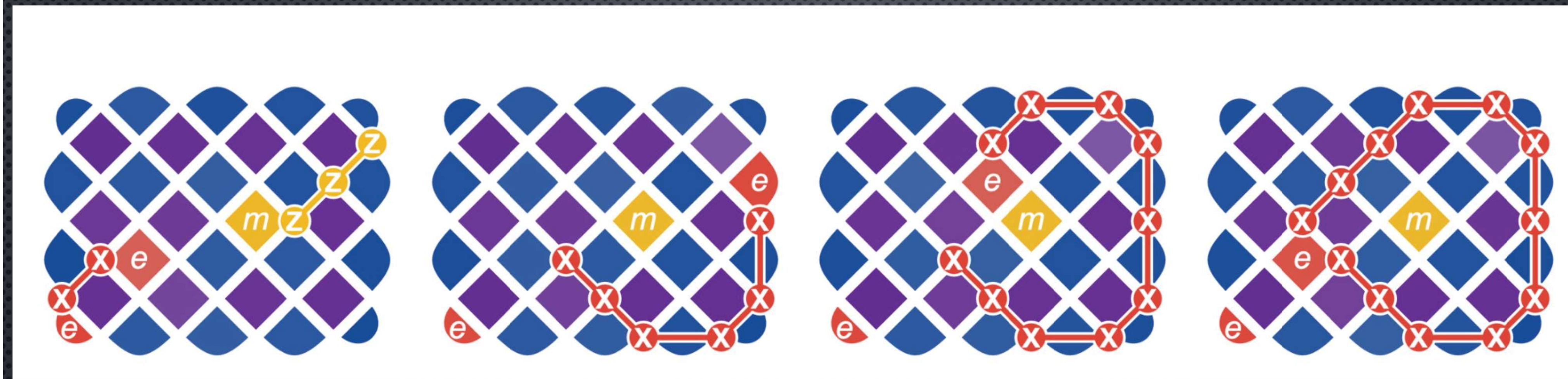


- Toric code $H = -\sum_p B(p)$

$$B(p) = \prod_{j \in p} X_j,\ B(p) = \prod_{j \in p} Z_j$$

- Stabilizers: $\{B_P\}$
- Violations: e & m

Gate-based Dynamics of Abelian Anyon



Satzinger et al, Science 2021

- Abelian anyons
- Braiding checked through interferometry

ISING
NON-ABELIAN ANYON

Collaborators

Yuri Lensky
(Cornell)

Kostya Kechedzhi

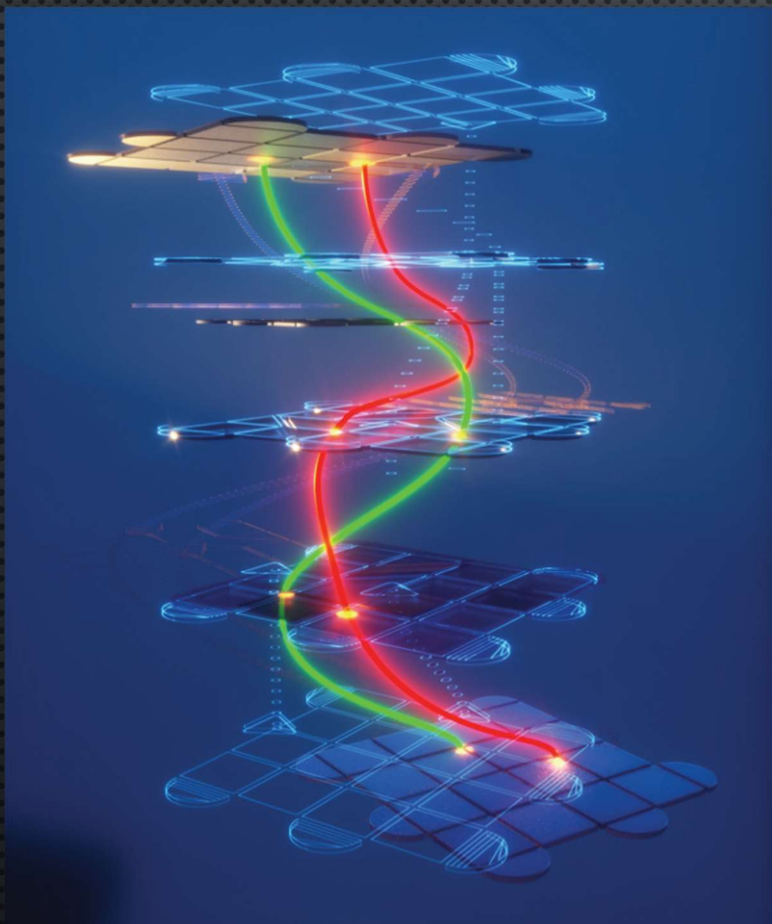
Igor Aleiner

Trond Anderson

Pedram Roushan

(Google Quantum AI)

Ising anyons on a quantum processor



Lensky et al, Annals of Physics 169286(2023), arXiv:2210.09282

Anderson, Lensky et al, Nature **618**, 264(5.11.2023), arXiv:2210.10255

Adding Ising Anyons to Toric Code

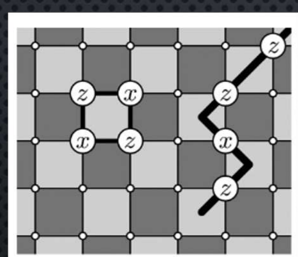
PRL **105,** 030403 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Topological Order with a Twist: Ising Anyons from an Abelian Model

H. Bombin
Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada
(Received 21 April 2010; published 14 July 2010)

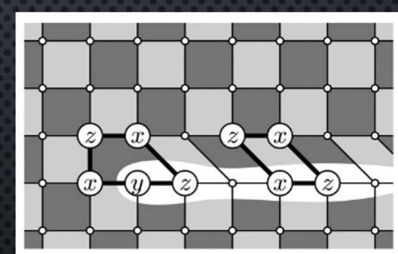
Anyon models can be symmetric under some permutations of their topological charges. One can then conceive topological defects that, under monodromy, transform anyons according to a symmetry. We study the realization of such defects in the toric code model, showing that a process where defects are braided and fused has the same outcome as if they were Ising anyons. These ideas can also be applied in the context of topological codes.

- Toric code on a square lattice



Two types of Abelian anyons: e & m

- Add lattice dislocations



act as Ising anyons

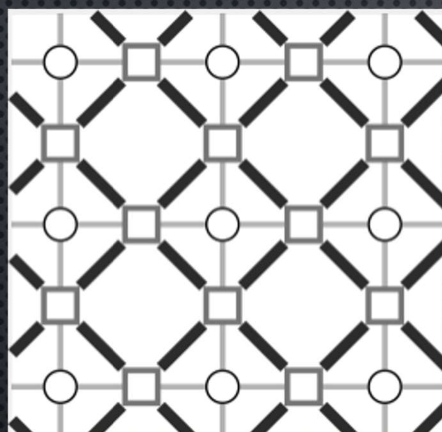
Braiding on a Physical Device?

Immobile Ising Anyon

First EV without rechargeable battery
1832, Robert Anderson

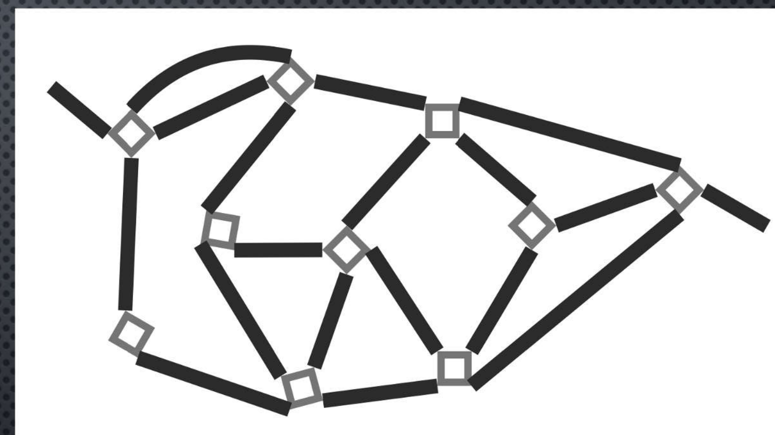
Plaquette Surface Code (PSC)

Surface code



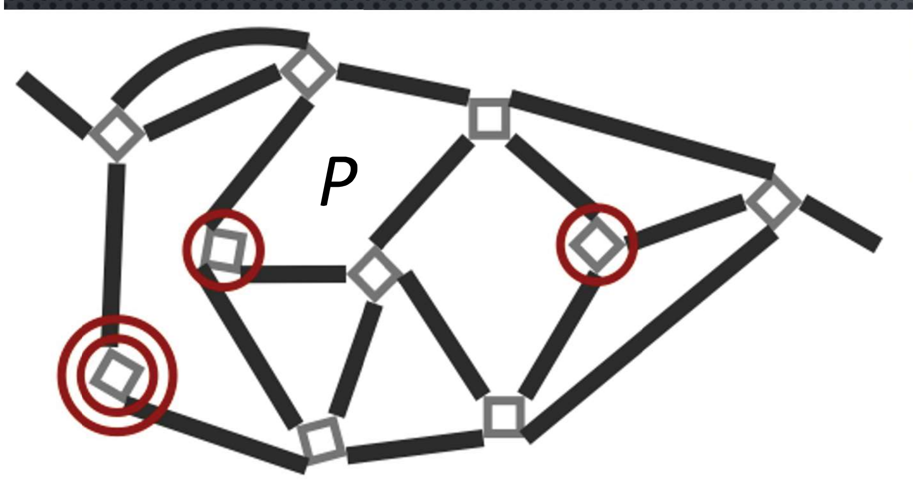
- Stabilizers: $\{B_P\}$
- Violations: e & m
- All D4Vs

PSC



- Stabilizers: $\{B_P\}$
- Violations
- D2V, D3V, D4V

- # Plaquette Surface Code



- Stabilizer code: $B(P)|\psi\rangle = |\psi\rangle$
- Total # of edges per vertex

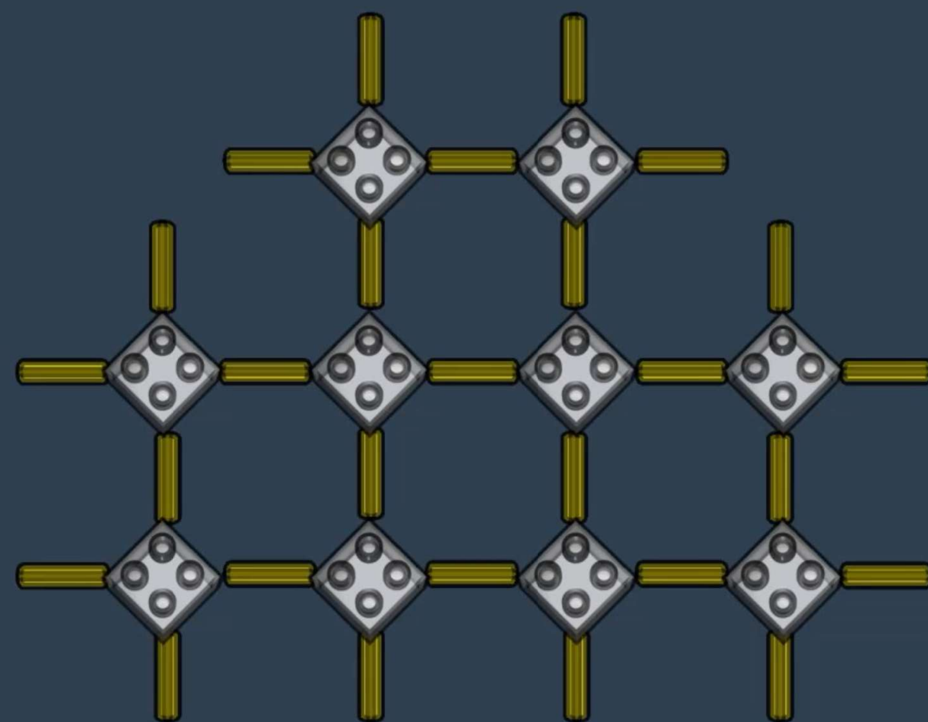
$$2E = 4V - (N_{D3V} + 2N_{D2V})$$
$$E = 2V - \frac{1}{2}N_\sigma,$$

- # of effective (logical) qubits

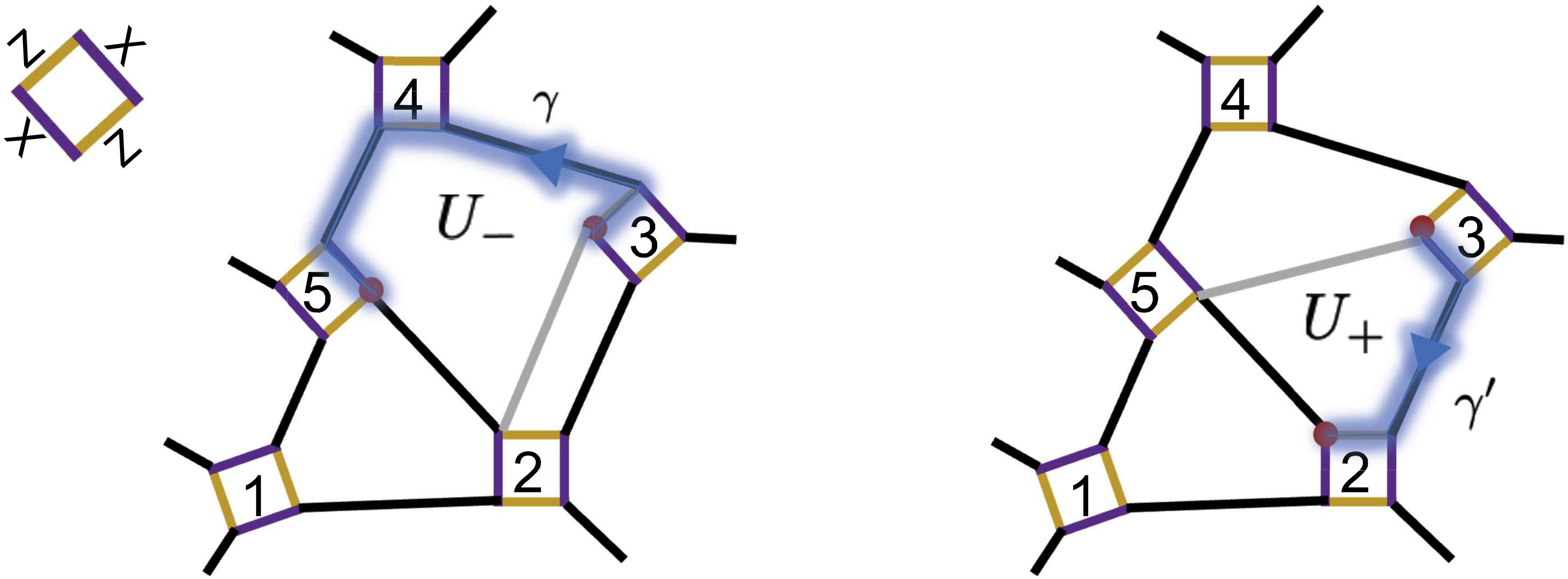
$$N_Q - N_S = \frac{N_\sigma}{2} - \chi(M)$$

Geometric Insight

Braid Generator



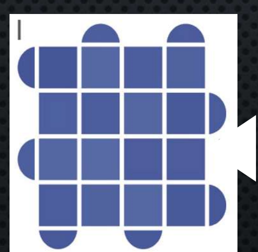
Edge swinging move



$$\mathcal{W}_\gamma = X_5 Z_4 Z_3 \qquad \mathcal{W}_{\gamma'} = Z_2 X_3$$

$$U_\pm = \exp\left(\mp i\frac{\pi}{4}\mathcal{W}_\gamma\right)$$

Ising Anyon Braiding Experiment



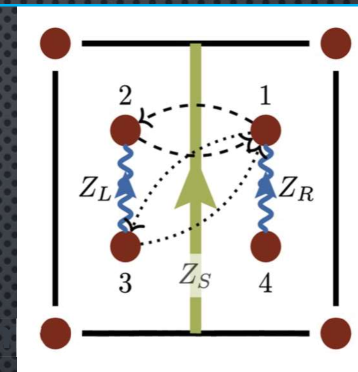
- Two pairs of plaquette violations = two fermions.
- First intentional non-Abelian braiding.
- Logical-X among even parity states

$$\{|0\rangle, \epsilon_1^\dagger|0\rangle, \epsilon_2^\dagger|0\rangle, \epsilon_1^\dagger\epsilon_2^\dagger|0\rangle\}$$

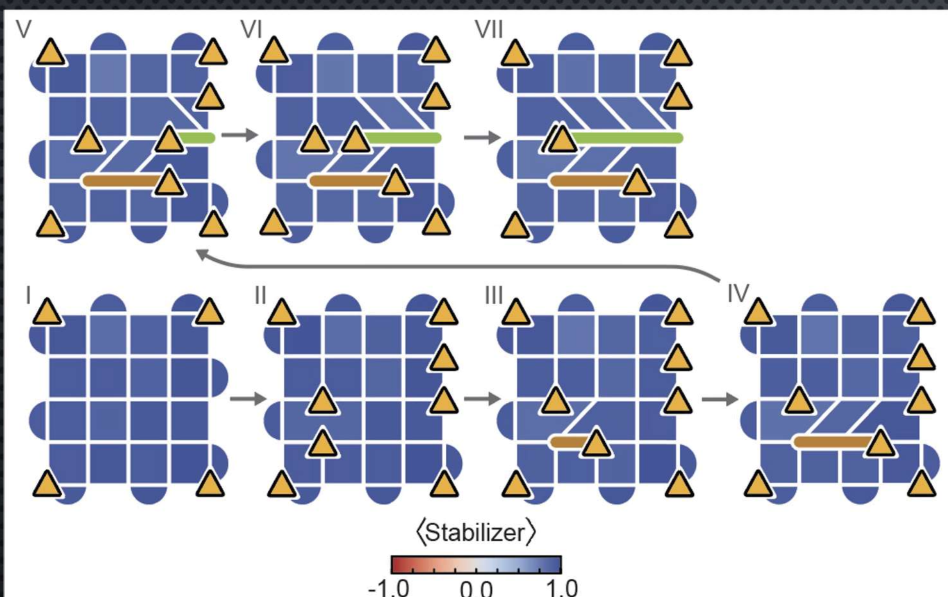
ENTANGLEMENT THROUGH BRAIDING

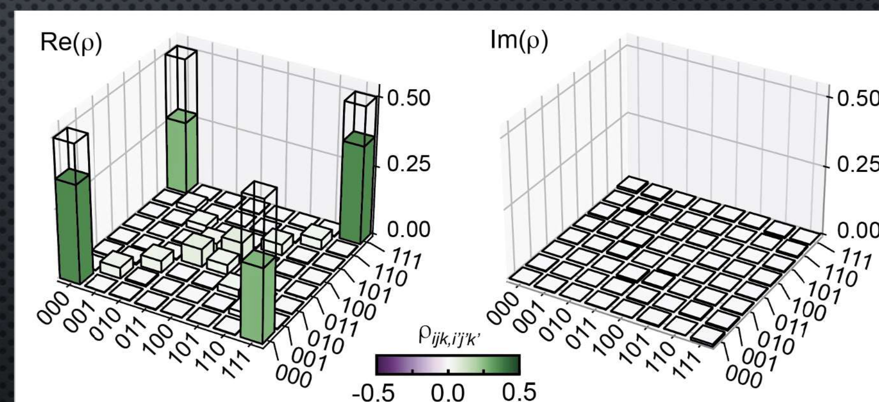
An exchange =√X-gate:

$$|000\rangle \rightarrow (|000\rangle + |111\rangle)/\sqrt{2}$$



Entangling braid:





Fidelity:

$$\langle\psi_{\rm GHZ}|\rho|\psi_{\rm GHZ}\rangle = 0.62$$

Purity:

$$\sqrt{{\rm Tr}\{\rho^2\}} = 0.65$$

FIBONACCI ANYON

Collaborators

Zlatko Minev
(IBM)

Khadijeh Najafi
(IBM)

Swarnadeep
Majumdar
(IBM)

Juven Wang
(Harvard)

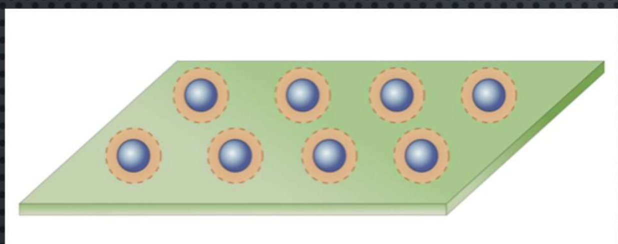
Ady Stern
(Weizmann)

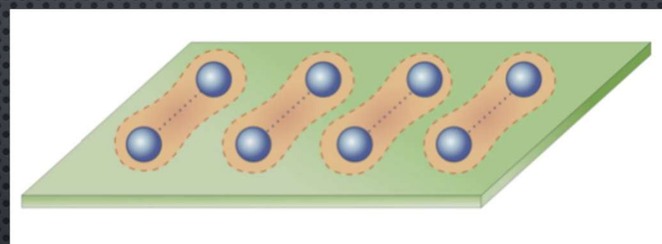
Chaoming Jian
(IBM)

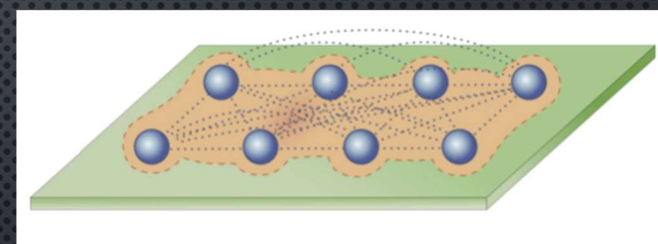
Guanyu Zhu
(IBM)

arXiv:2406.12820

Different non-Abelian anyons







Fermions

2^n

Majoranas

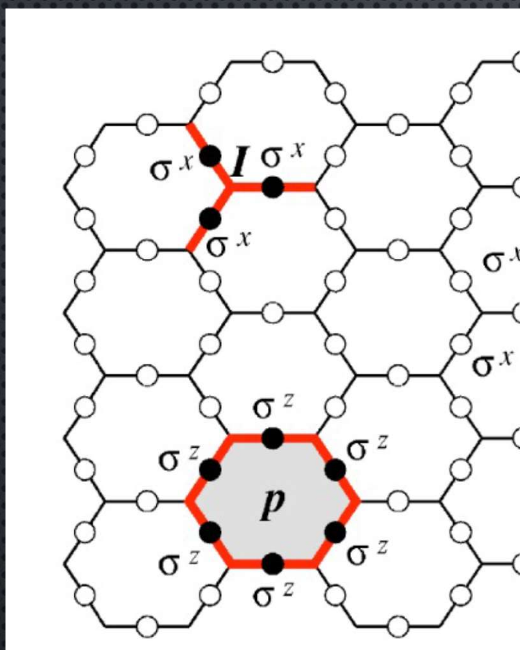
$2^{n/2}$

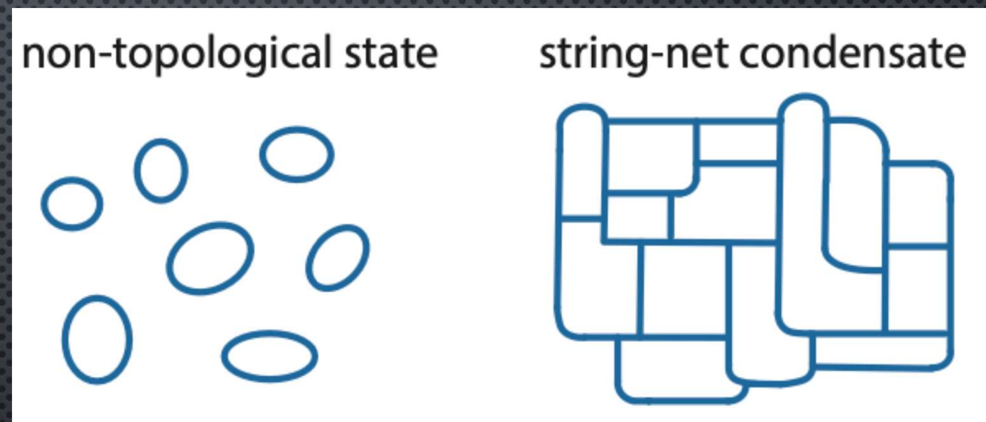
Fibonacci

$\phi^{n\dots}$

$$\sigma \times \sigma = 1 + \psi \quad \tau \times \tau = 1 + \tau$$

Levin-Wen Stringnet condensates



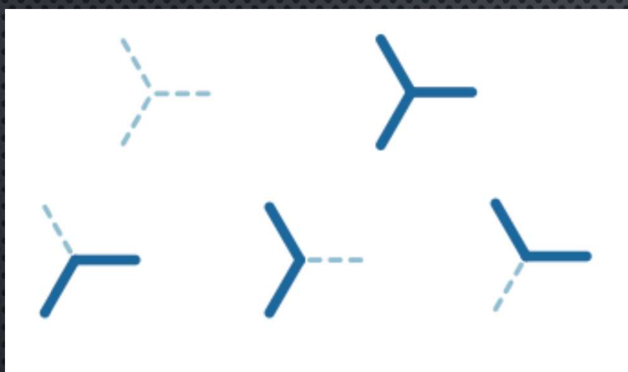


$$H = -\sum_v Q_v - \sum_p B_p$$

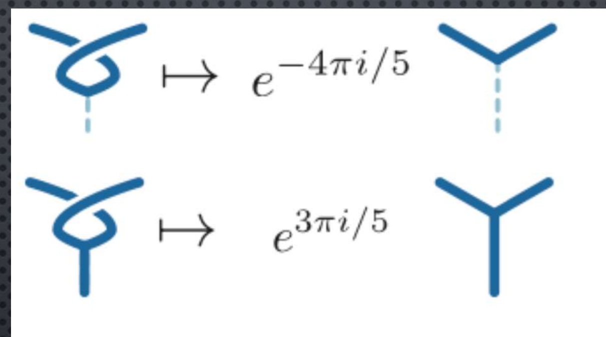
Can we realize this state?

Rules and Goals

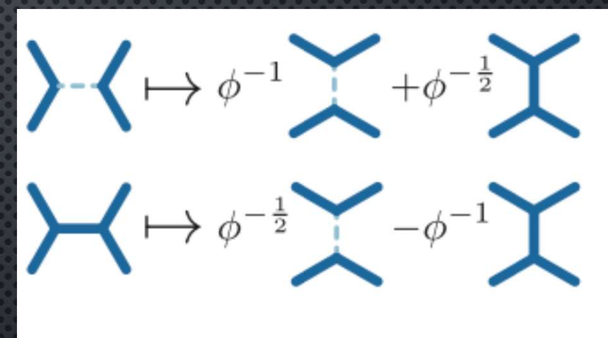
- The rules are local!



Branching rule



R-move

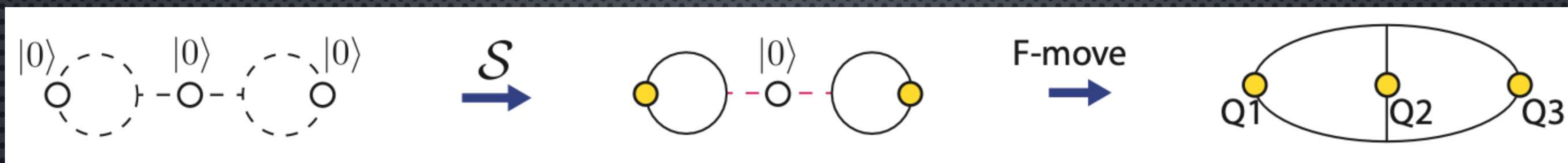


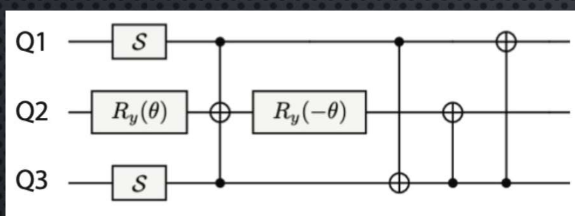
F-move

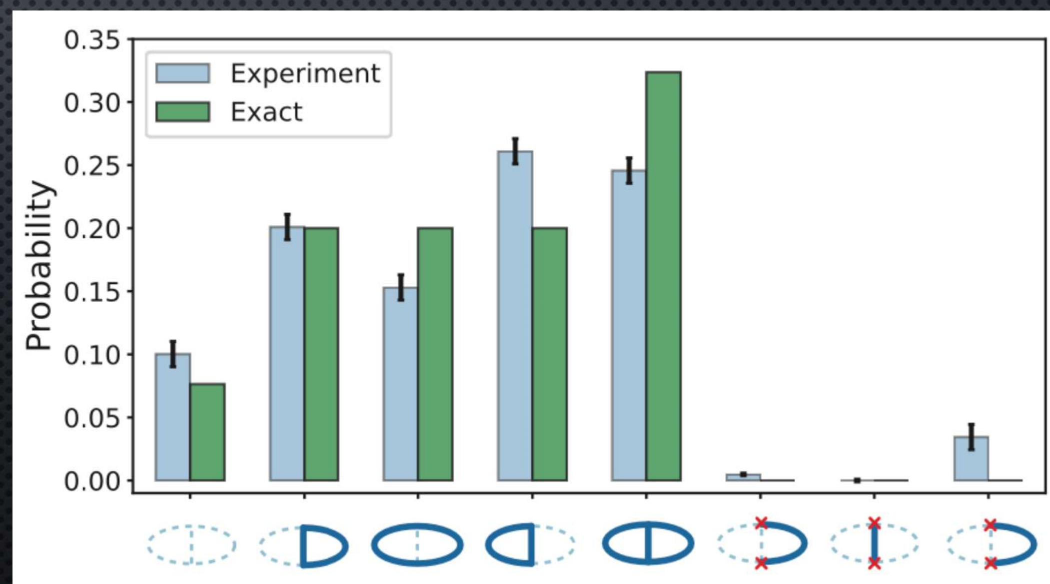
- SN condensate: sample chromatic polynomial
- Anyons: universal gates

Dynamical String Net Preparation (DSNP)

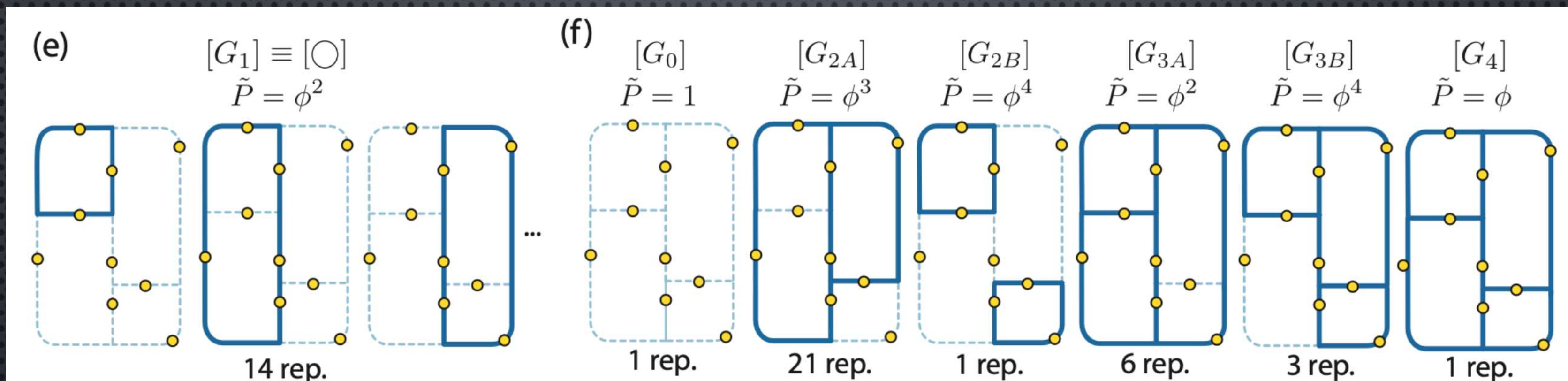
- Baby stringnet



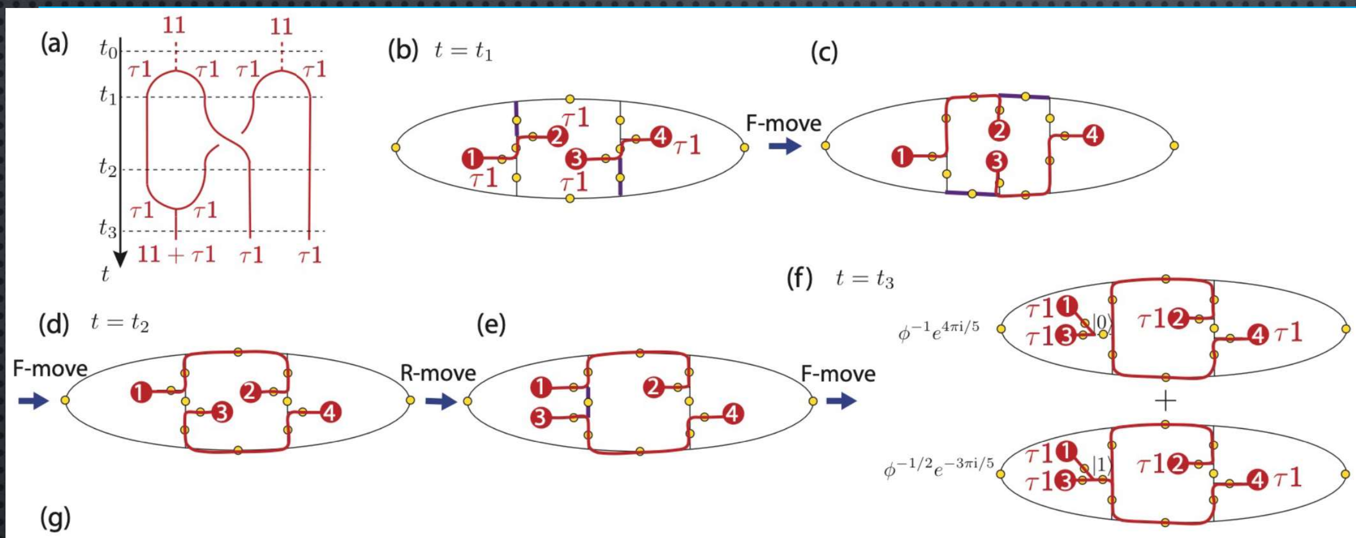


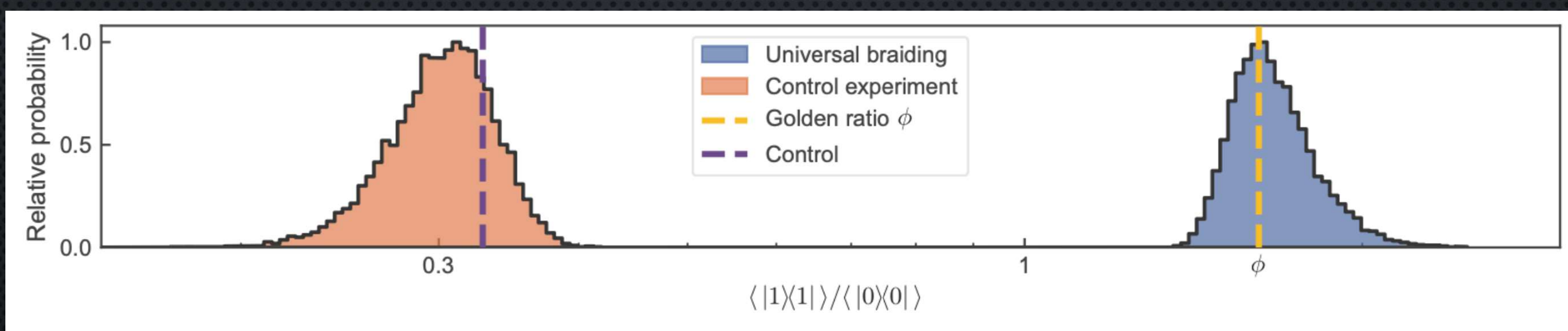


Sampling chromatic polynomials



Braiding Fib and golden ratio





Non-Abelian anyons on synthetic platforms

- Braiding Ising anyons on a quantum processor
  - Prepared and braided Ising anyons
  - Scalable and protectable logical gates
- Realizing Fibonacci String net condensate
  - Prepare SNC and sample chromatic polynomial
  - Prepared and braided Fibonacci anyons
  - Universal single qubit gates