How Starshades Work

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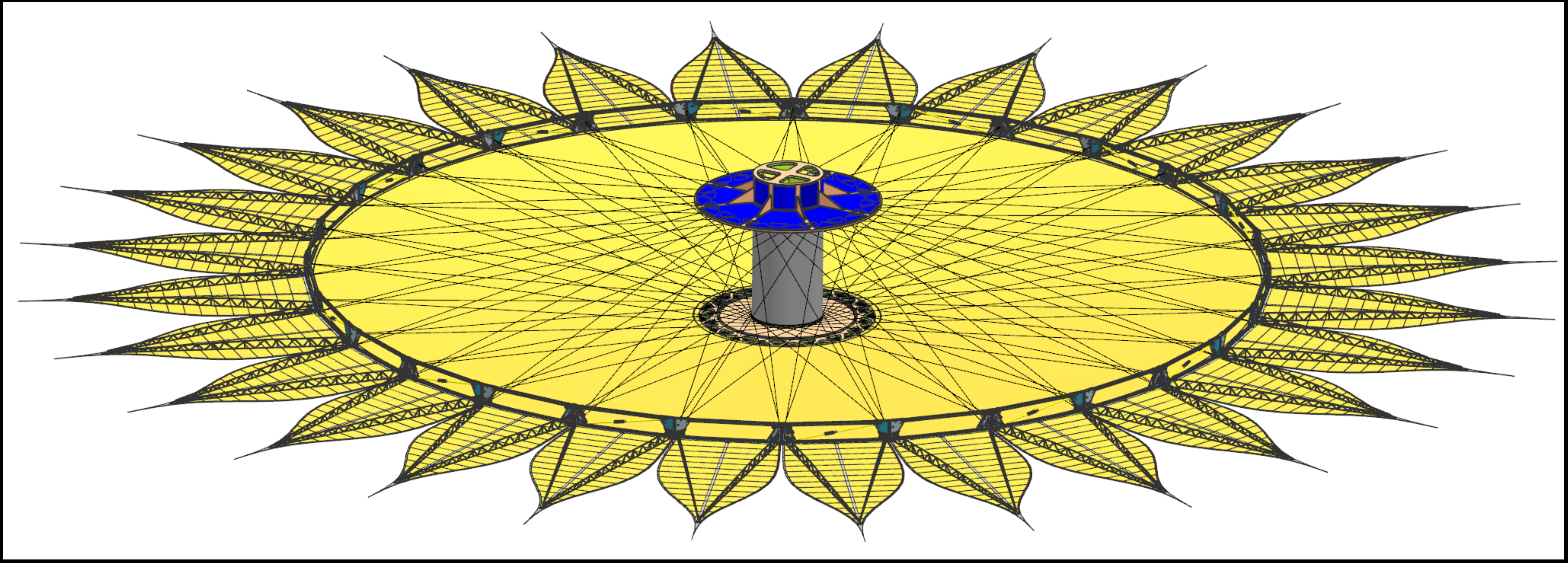
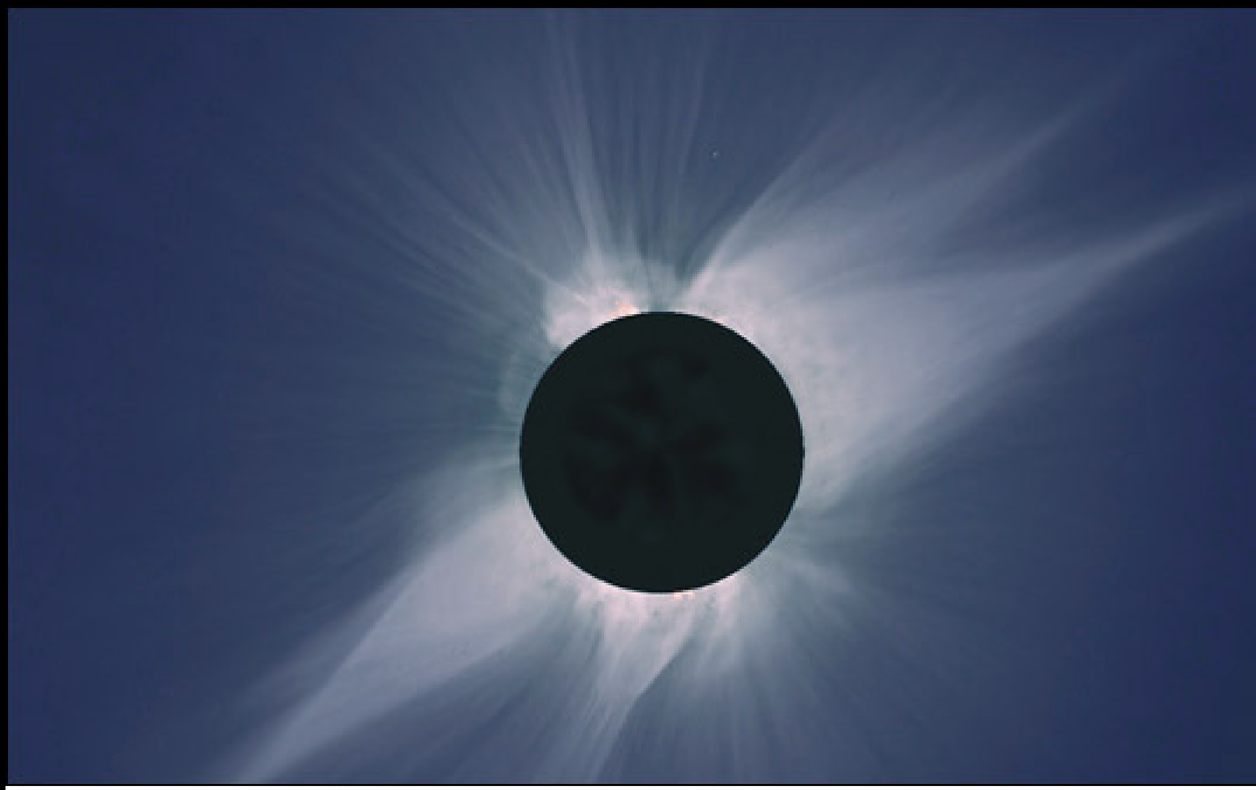


Outline

- Starshade Design
- Making it Work
- Operational Considerations

Nature’s Starshade



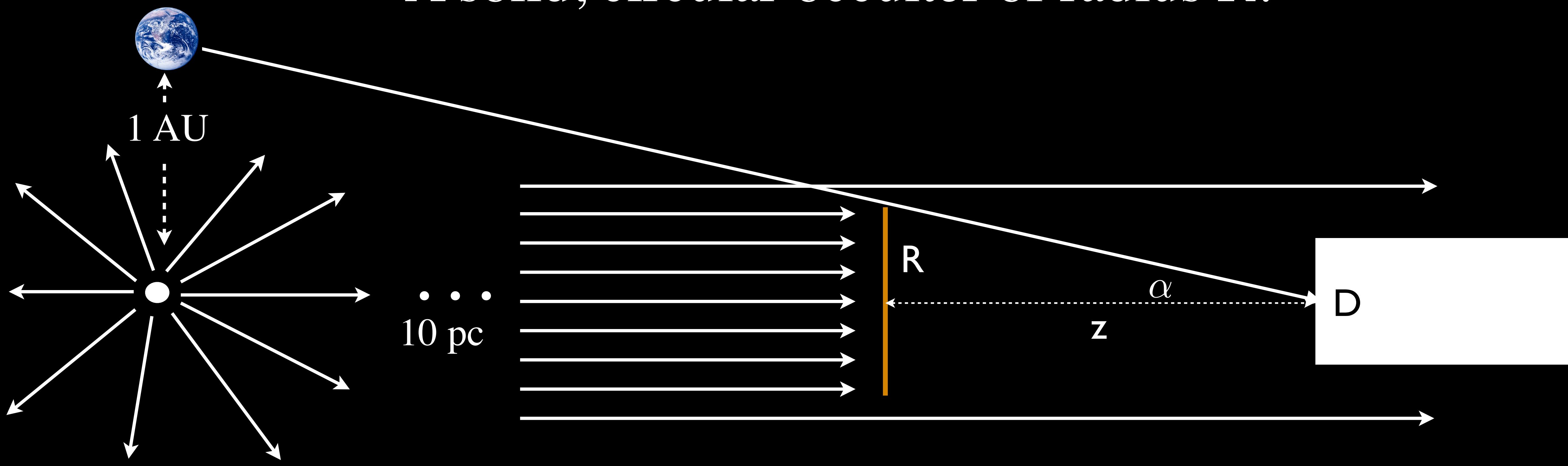
July 11, 1991

August 11, 1999

Create an artificial eclipse to block out sunlight, place telescope in resulting shadow.

Simple Ray Optics Description

A solid, circular occulter of radius R.



IWA = alpha = angle to tip of starshade
= R/Z

A 6m dia. disk at 6,000 km separation gives access to 1AU at 10 parsec

First proposed by Lyman Spitzer in 1962

Why use a starshade?

- Immune to telescope errors
- Operates in broadband
- Maximizes throughput
- No outer working angle limitation
- Inner working angle set by geometry

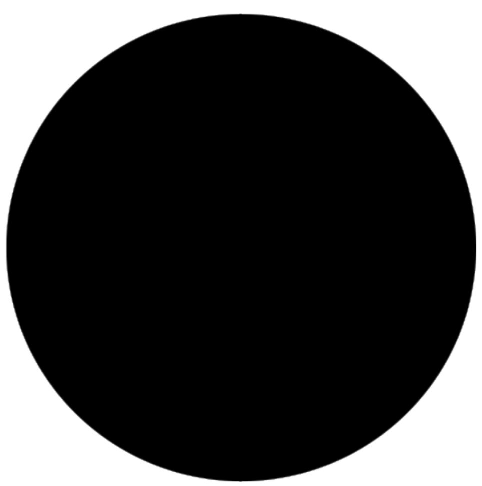
Main limitation is the number of observations, determined by fuel and mission time.

However . . . .

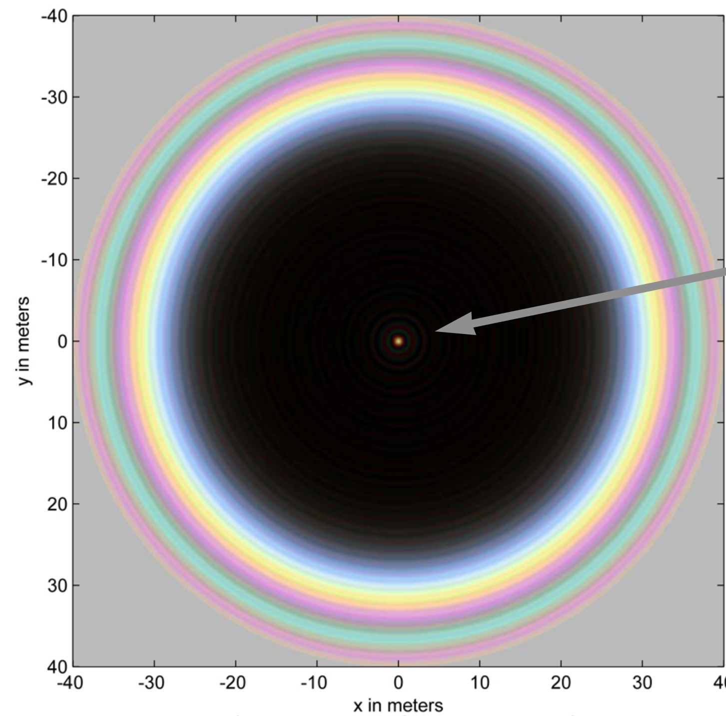
Diffracted field around circular disk

Allowing for diffraction, shadow no darker than 1e-3.

Circular Occulter

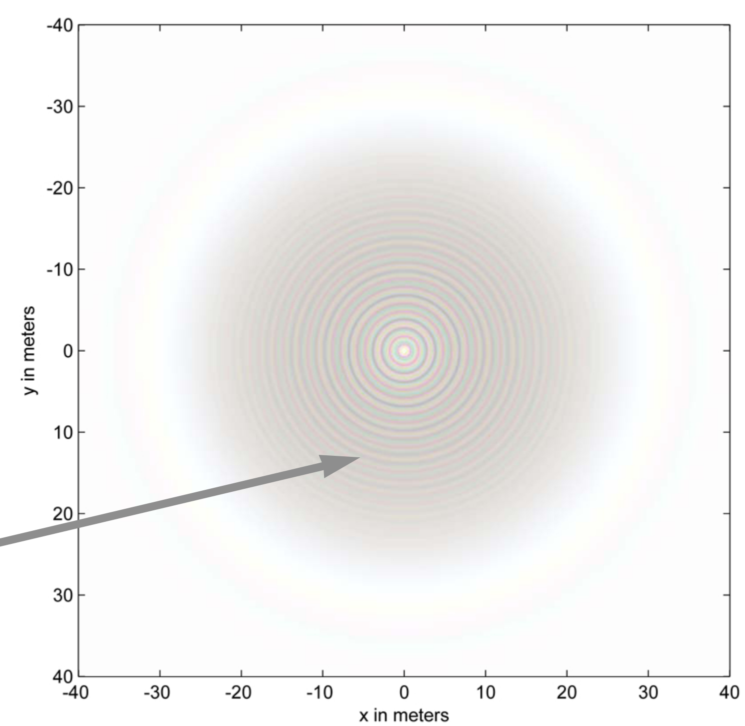


Shadow (linear scale)

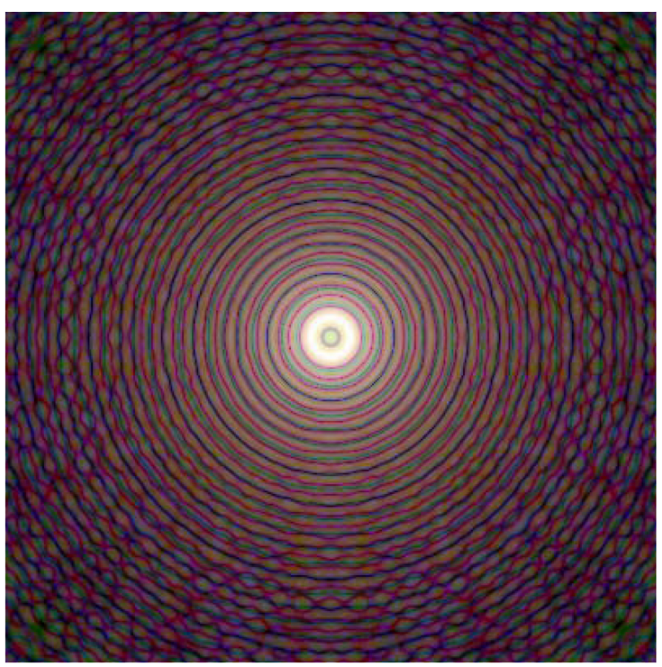


Poisson's Spot!

Shadow (log scale)

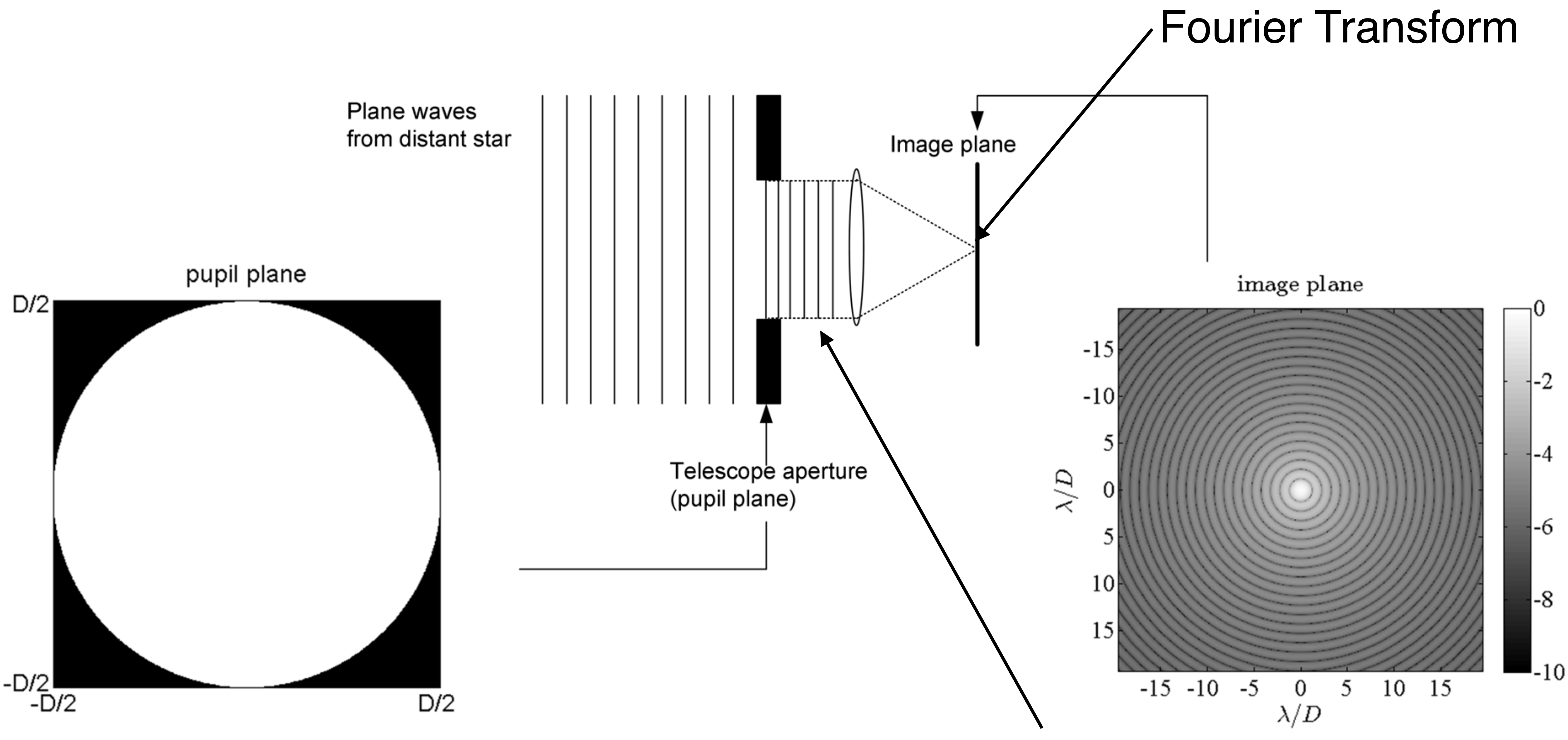


Shadow isn't dark enough



Simulated star/planet image

But we have to consider diffraction



Scalar diffraction through circular opening.

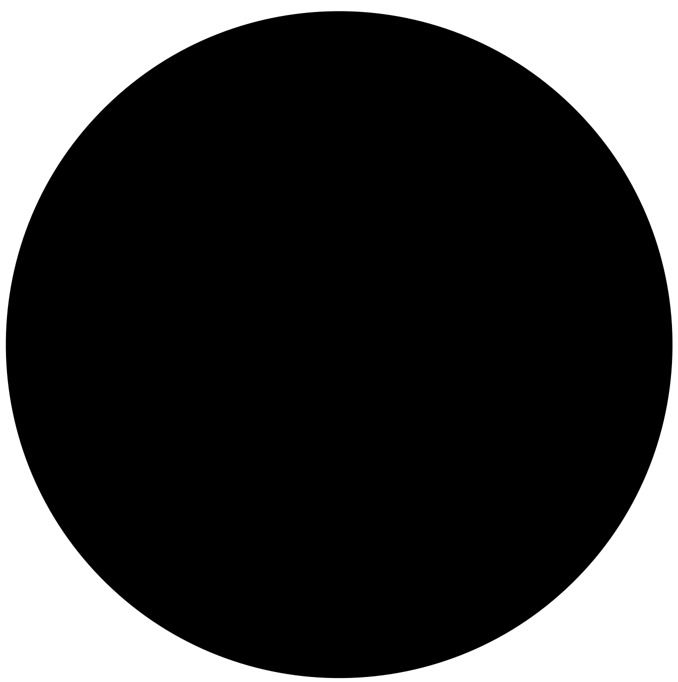
$$E_{hole}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{i\pi}{\lambda z}\rho^2} \int_0^R e^{\frac{i\pi}{\lambda z}r^2} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) r dr$$

Fresnel Transform

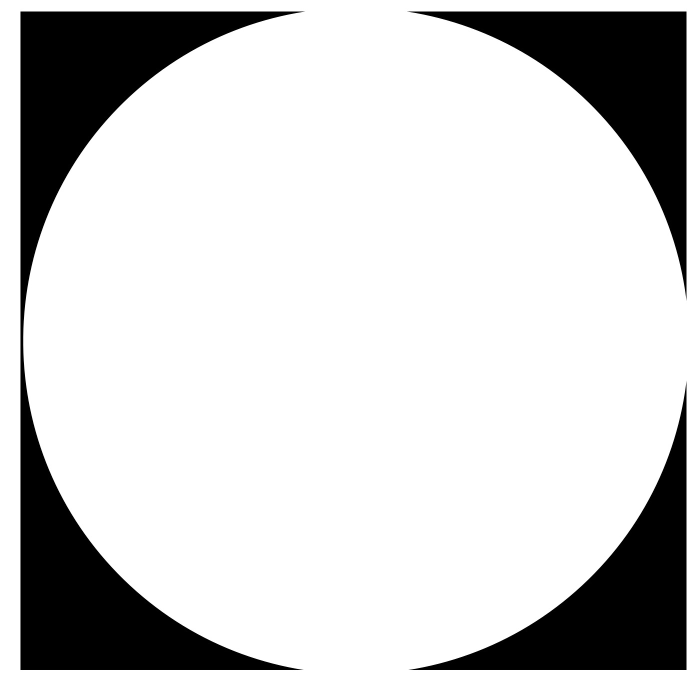
Solving for Diffraction

Babinet's Principle (linearity)

$$E_{\text{starshade}}(r) = 1 - E_{\text{hole}}(r)$$



= 1 -



$$E_{hole}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{i\pi}{\lambda z}\rho^2} \int_0^R e^{\frac{i\pi}{\lambda z} r^2} J_0\left(\frac{2\pi r \rho}{\lambda z}\right) r dr$$

To achieve 10^{-10} suppression, a circular occulter would need to be roughly 750 times larger and 750 times further away than ray optics solution to control diffraction.

So, the question becomes how to design a star shade that is smaller and closer while achieving the same high suppression and small inner working angle.

Babinet and Fresnel integral for circular occulter:

$$E(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 - \frac{2\pi}{i\lambda z} e^{\frac{i\pi}{\lambda z}\rho^2} \int_0^R e^{\frac{i\pi}{\lambda z} r^2} J_0\left(\frac{2\pi r \rho}{\lambda z}\right) r dr \right)$$

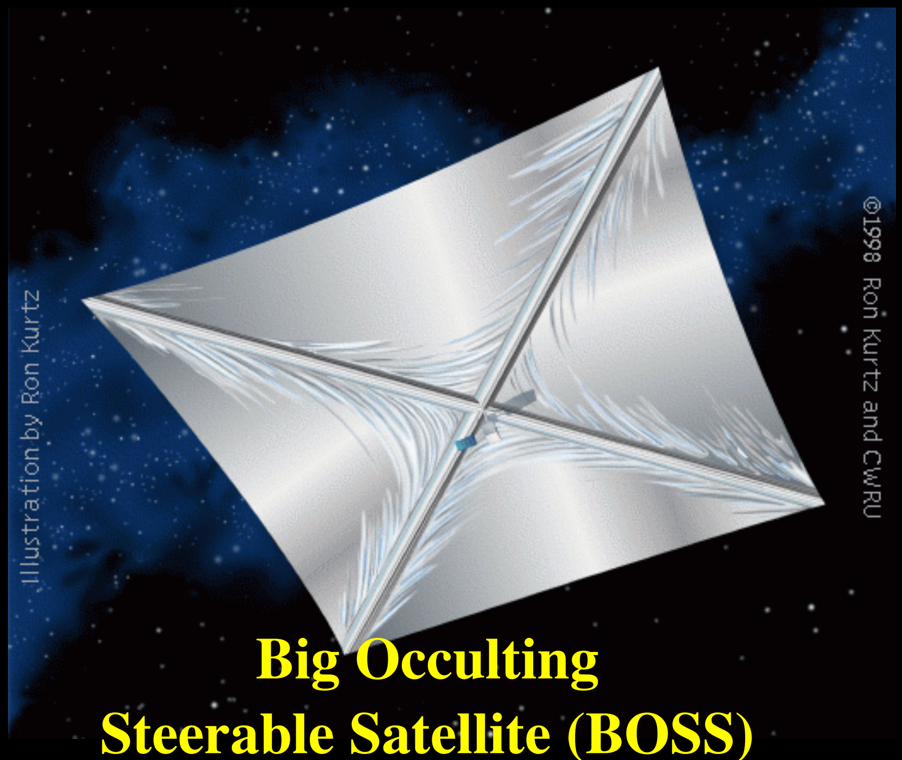
which is a Fourier (Hankel) Transform of $e^{\frac{i\pi}{\lambda z} r^2}$

Ray Optics Solution:

Functions can’t be band limited and space limited at once without violating the uncertainty principle.

Apodize the Occulter

It has been known since 1962 (Spitzer) that an apodized occulter can produce the needed shadow.



Copi & Starkman (2000)

Schultz (2003)

$$E(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 - \frac{2\pi}{i\lambda z} e^{\frac{i\pi}{\lambda z}\rho^2} \int_0^R A(r) e^{\frac{i\pi}{\lambda z} r^2} J_0\left(\frac{2\pi r \rho}{\lambda z}\right) r dr \right)$$

Smoothly vary transmission by *A(r)*

How do you find the apodization?

$$E(\hat\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 + 2\pi i e^{i\pi\hat\rho^2} \int_0^{\alpha\sqrt{\frac{z}{\lambda}}} A(\hat r) e^{i\pi\hat r^2} J_0(2\pi\hat r\hat\rho)\hat r d\hat r\right)$$

Scale set by Fresnel # $\dfrac{R^2}{\lambda z}$

Given a shadow radius S and desired suppression, solution for $A(r)$ set by two parameters: Fresnel number and inner disc size. For given inner working angle, z found from Fresnel number.

There is no closed form solution to this integral in terms of elementary functions for A(r).

There have been several numerical or approximate approaches.

Semi-Analytical Approach I

Copi & Starkman (2000) solved for the electric field at the center of telescope (rho = 0) only:

$$E(0) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 + 2\pi i \int_0^{\alpha\sqrt{\frac{z}{\lambda}}} A(\hat{r}) e^{i\pi \hat{r}^2} \hat{r} d\hat{r} \right)$$

They found closed form solutions with *A(r)* a series in Chebychev polynomials. Coefficients then chosen to get very high contrast in center and best possible contrast over shadow and wavelength.

Occulter size, R, is tuned through iteration to achieve desired contrast across shadow and bandwidth.

Semi-Analytical Approach II

Cash (2006) also solved for the electric field at the center of telescope (rho = 0) while also approximating the integral by extending limit to infinity:

$$E(0) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 + 2\pi i \int_0^\infty A(\hat{r}) e^{i\pi \hat{r}^2} \hat{r} d\hat{r} \right)$$

He found closed form solution with A(r) a hypergaussian function. The two parameters are chosen to get broad shadow. Width is typically quoted as "*1*/e" value. Function is truncated at distance with minimal impact on shadow, setting *R*. Iterated to get desired inner working angle.

Optimal Approach

Vanderbei, et al. (2007) solved a linear program to find apodization at discrete points along radius using exact, scalar integral.

* Electric field suppression
* Shadow diameter
* Inner Working Angle
* Shortest wavelength of bandpass
* Longest wavelength of bandpass
* Smoothness
* Engineering features (gaps and tip widths)

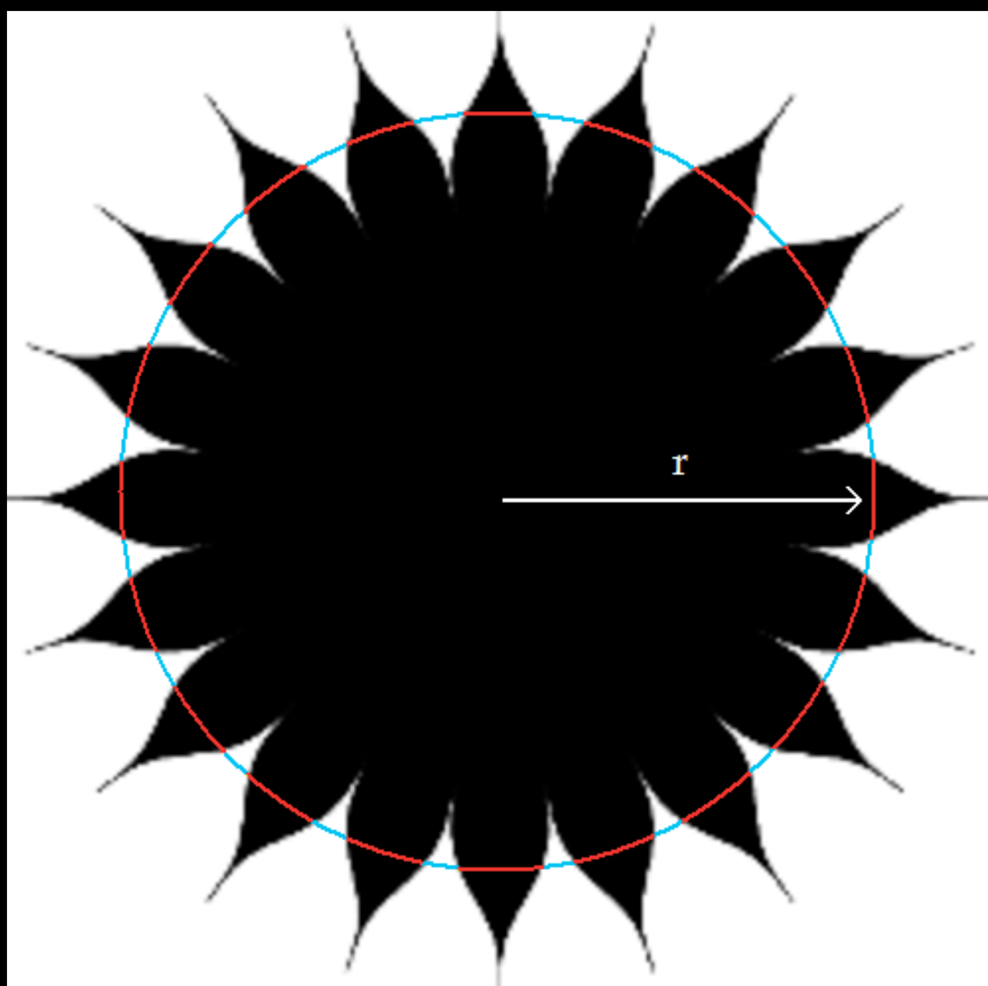
→ Global minimum establishes size, distance, shape of occulter

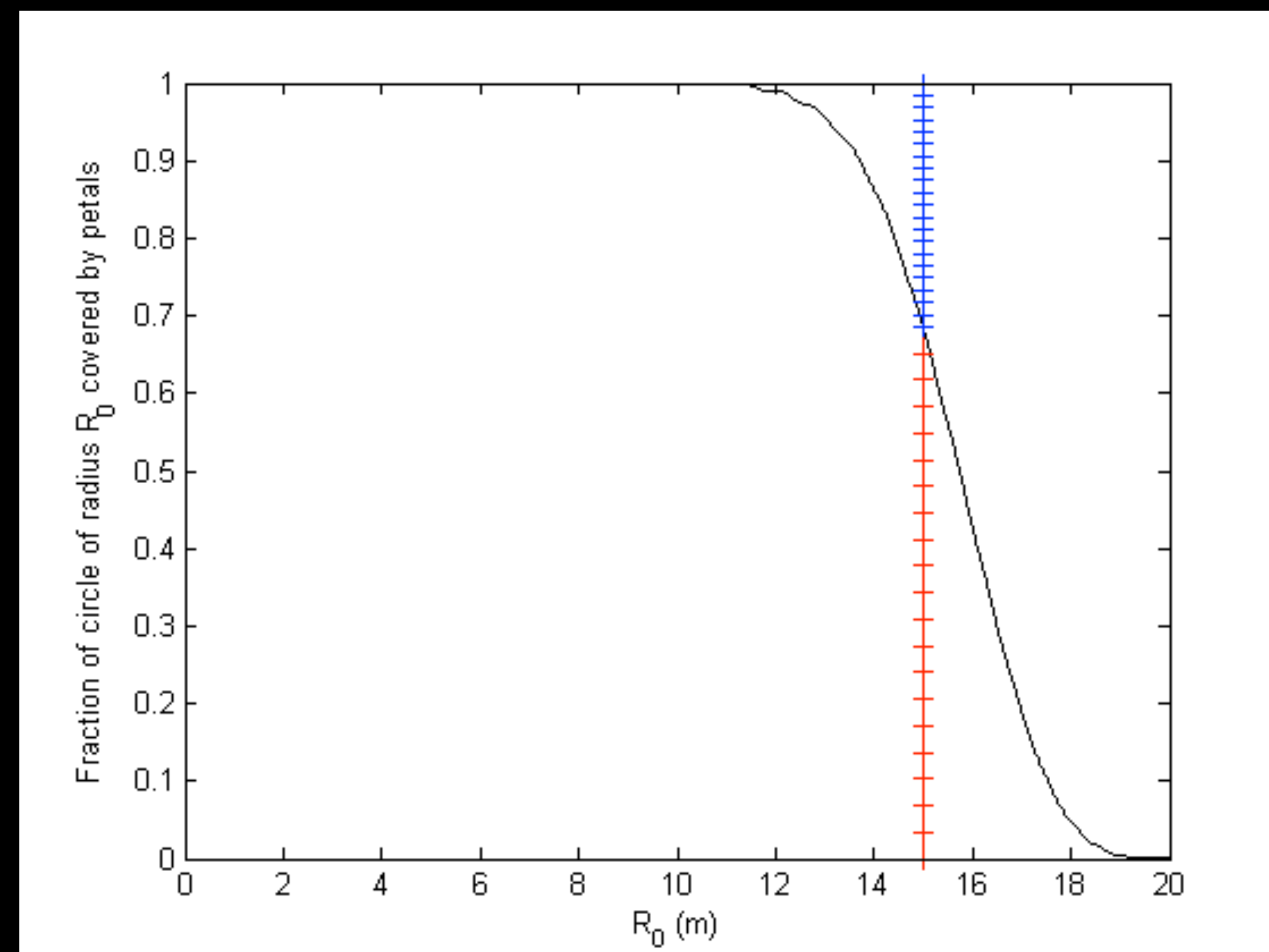
The increased degrees of freedom allow for smaller occulter design and flexibility to achieve constraints such as larger gaps or wider tips.

Convert apodization to binary occulter

Uses same approach as star-shaped pupil design.

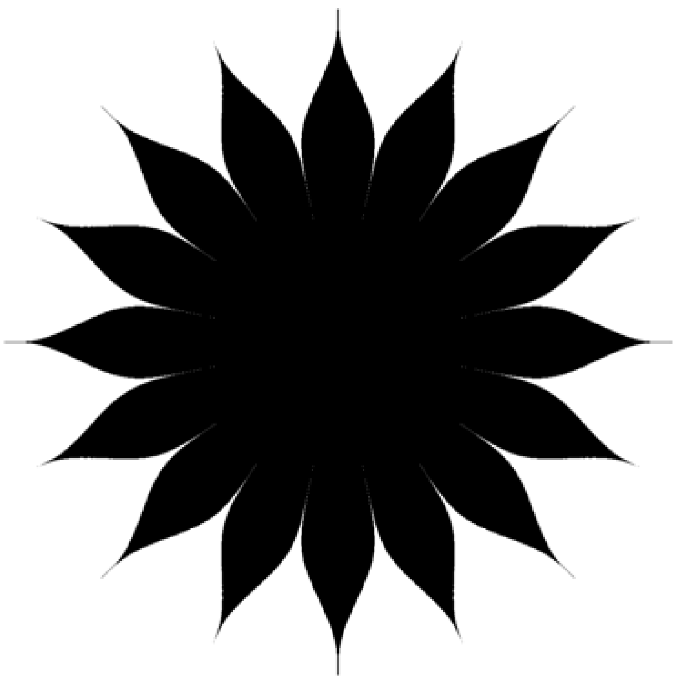
Marchal (1985), Simmons (2005), Cash (2006), Vanderbei et al. (2007)

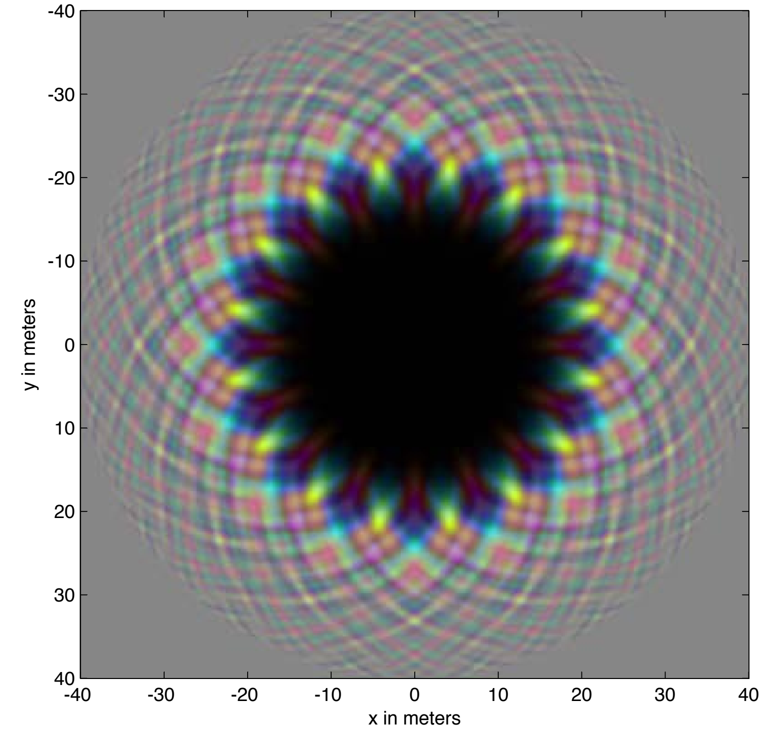


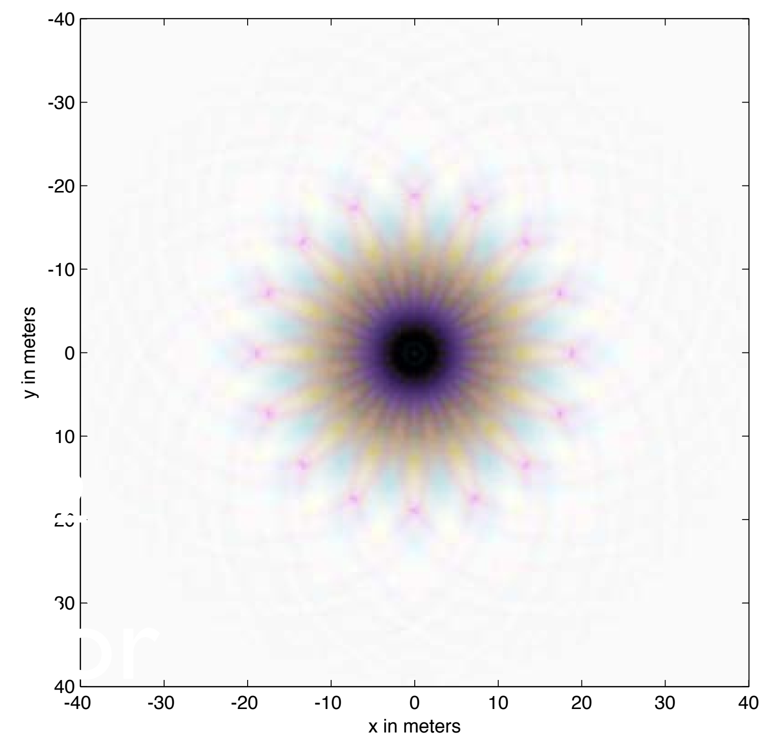


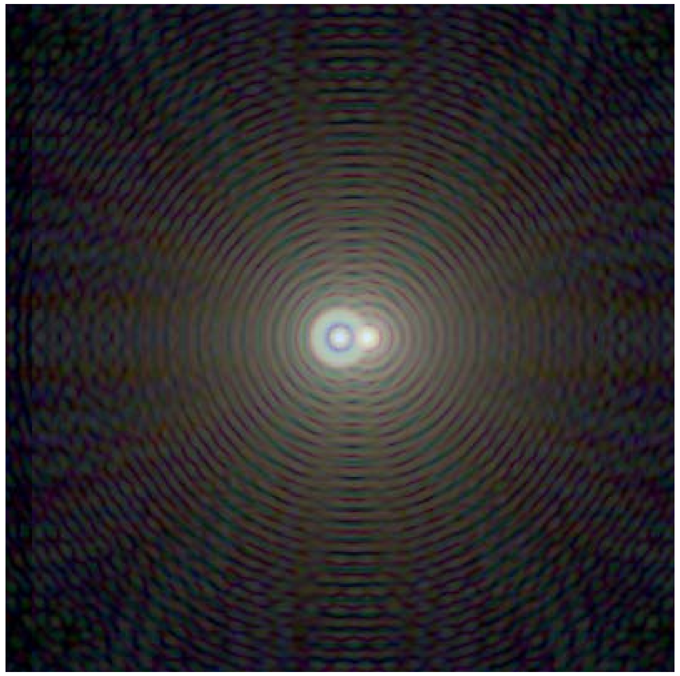
$$
\begin{aligned}
E_{o,\mathrm{petal}}(\rho,\phi) \;=\; & E_{o,\mathrm{apod}}(\rho) \\
& -E_0 e^{\frac{2\pi i z}{\lambda}} \sum_{j=1}^{\infty} \frac{2\pi(-1)^j}{i\lambda z} \left(\int_0^R e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} J_{jN}\left(\frac{2\pi r\rho}{\lambda z}\right) \frac{\sin\left(j\pi A(r)\right)}{j\pi} r\,dr \right) \\
& \times \left(2\cos\left(jN(\phi-\pi/2)\right)\right)
\end{aligned}
$$

Shaped Occulter

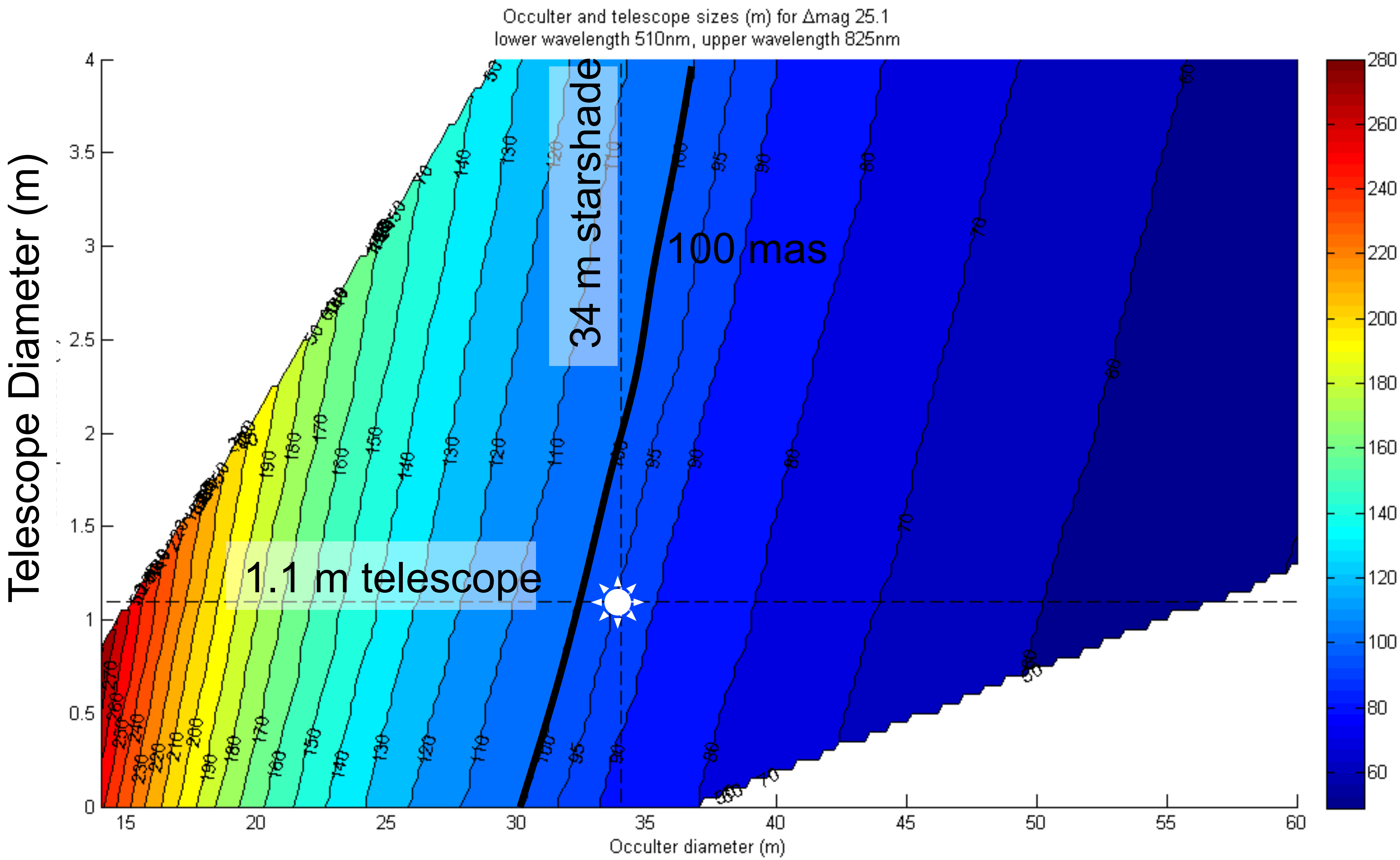








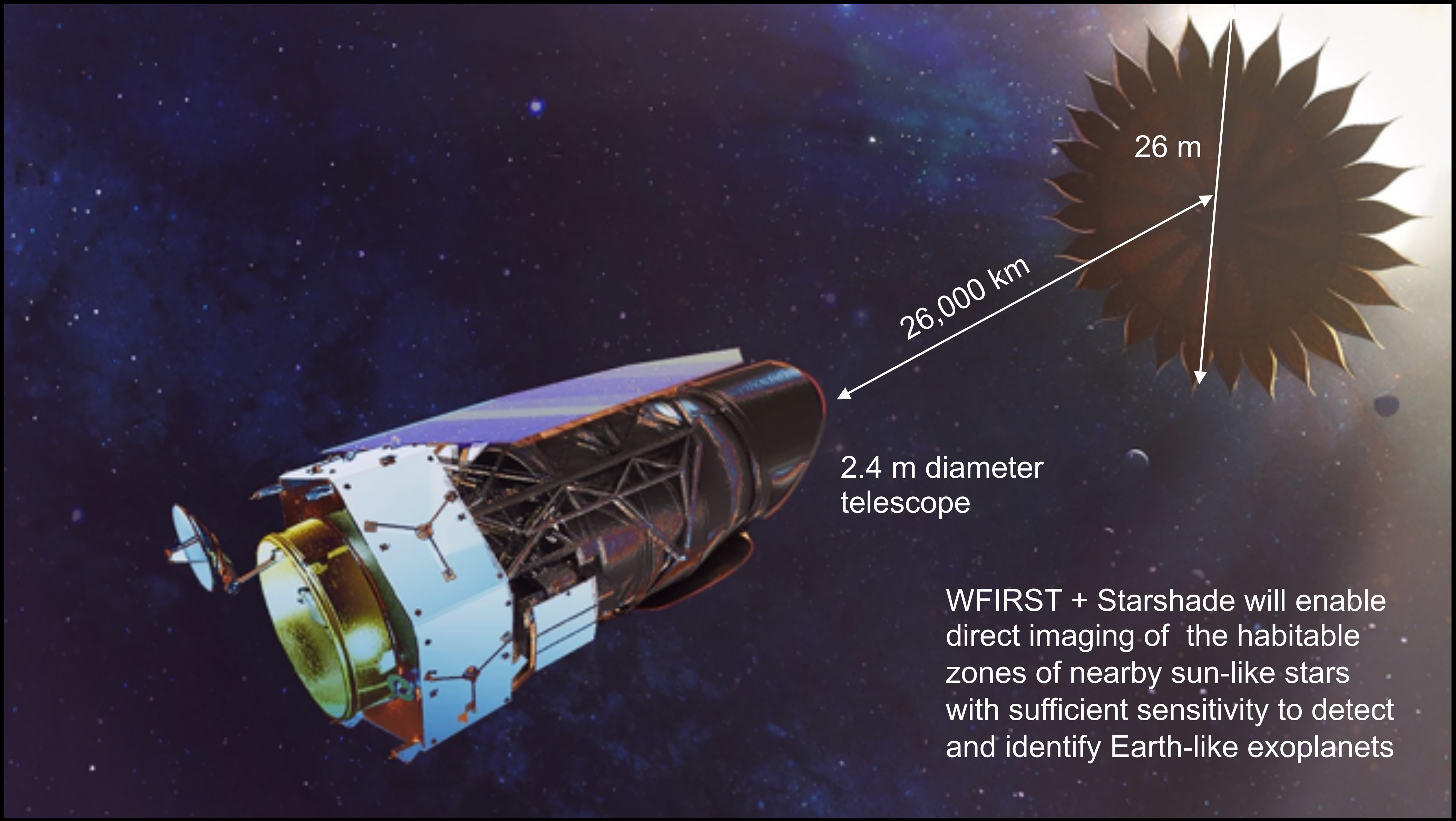
IWA contours for a range of telescope & starshade sizes



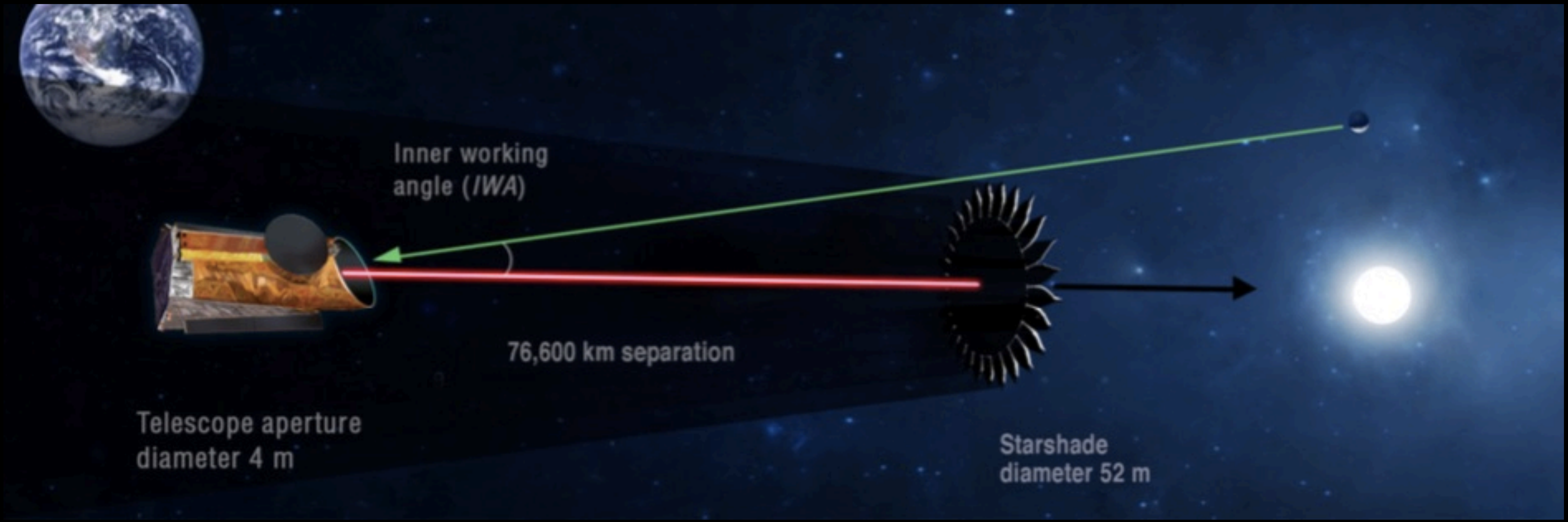
These curves are for Truss Diameter / Petal Length ratio = 20/7, bandpass 510-825 nm.

Eric Cady

Starshade Rendezvous



Raw Contrast:
1 x 10^{-10} (at IWA)
Spectral Bandwidth:
26% bands between 0.425-0.975 *μ*m
Inner Working Angles:
0.103” at 0.62-0.8 μm



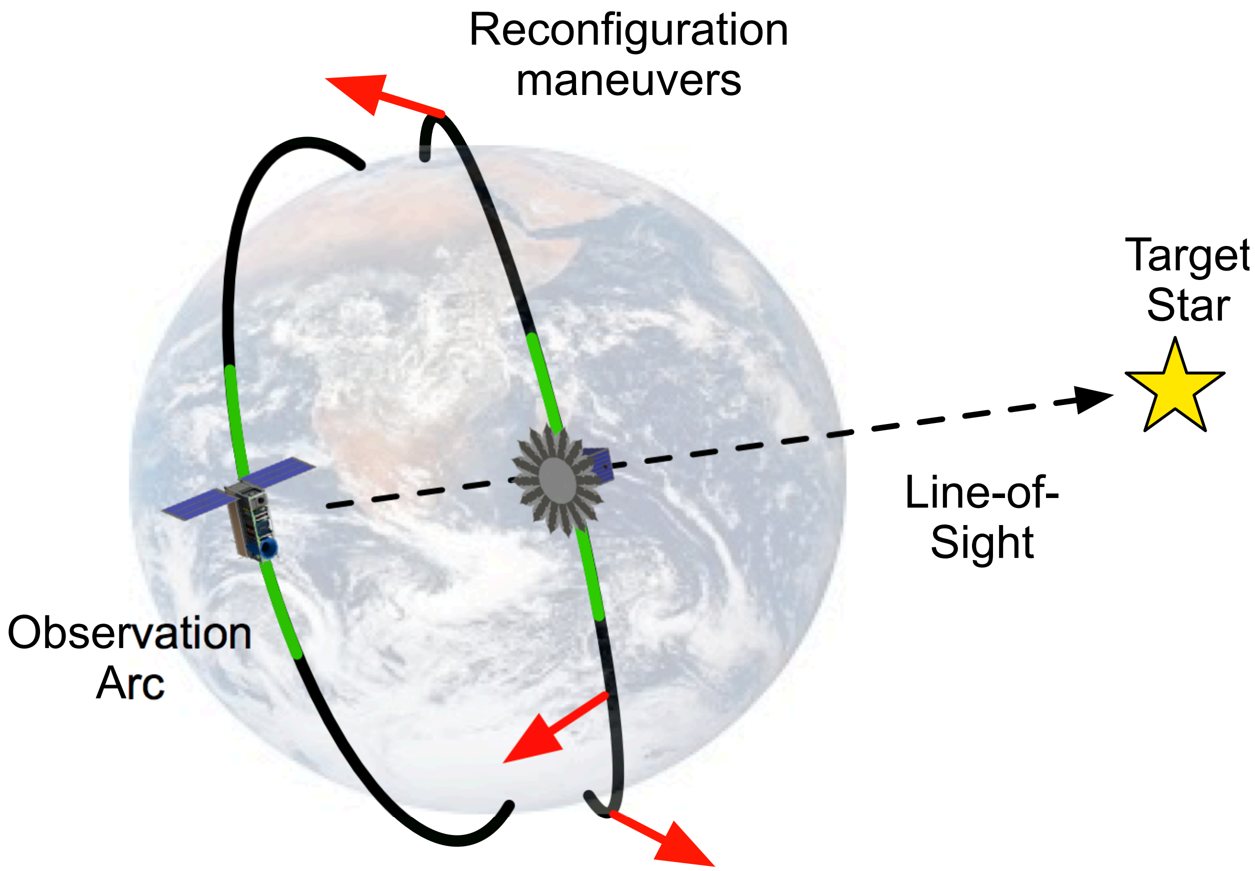
Raw Contrast:
1×10^{-10} (at IWA)
Inner Working Angles:
0.058” at 0.3-1 µm (ss)
0.062” at V band (cg)
Outer Working Angles:
6” (ss-imaging)
1” (ss-spectra)
0.83” (@ 0.5 µm, cg-imaging/spectra)

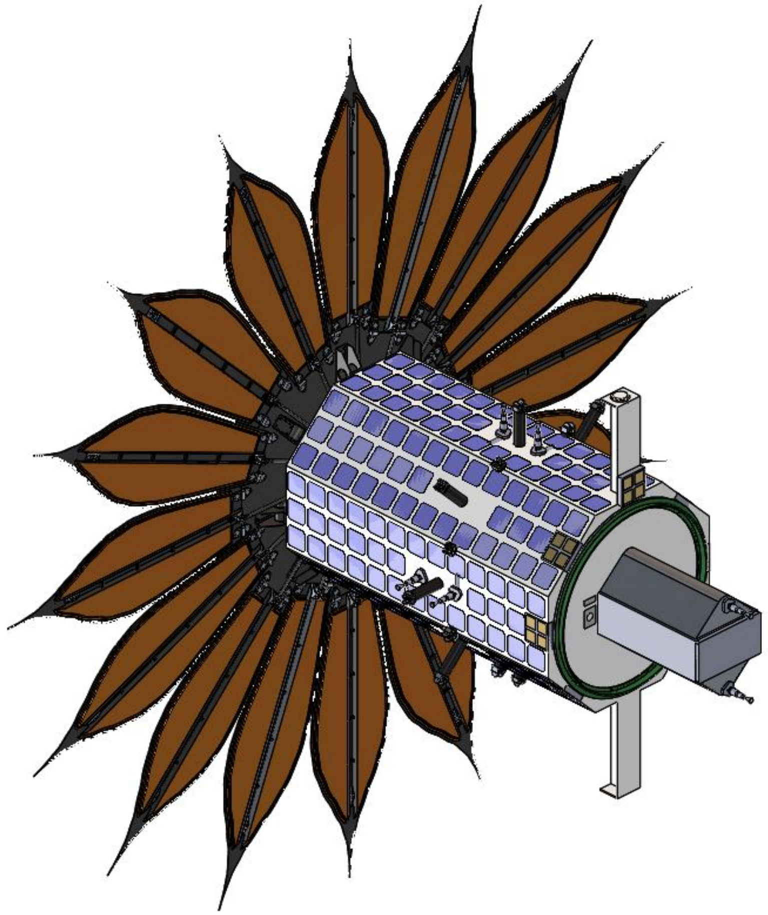
Spectroscopy:
R=7 from 200 to 450 nm (ss)
R=140 from 450 to 1000 nm (ss/cg)
R=40 from 1 to 1.8 µm (ss/cg)

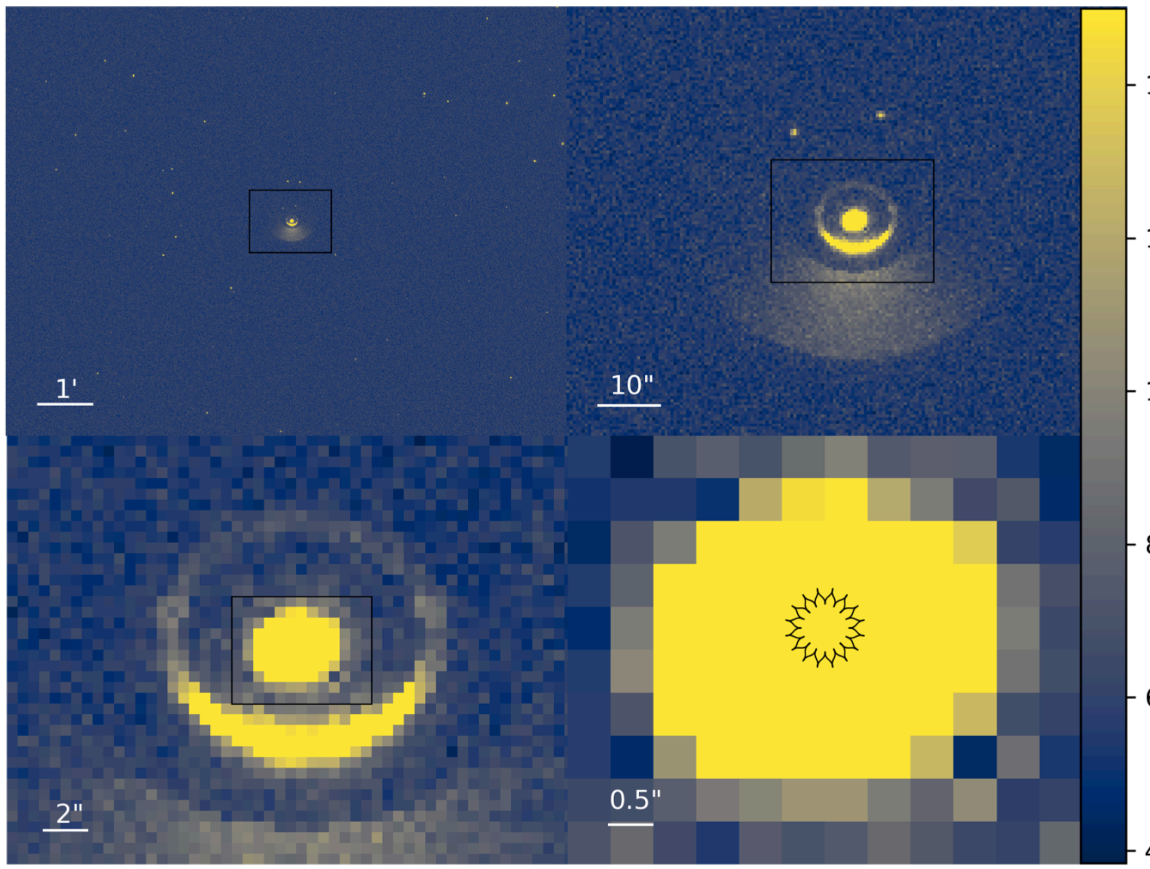
Wide field UV-optical camera,UV spectrograph, exoplanet camera and IFS

Miniature Distributed Occultor Telescope: mDOT
Simone d'Amico & Bruce Macintosh

- Low Earth Orbit (LEO) starshade smallsat
- 3-m diameter starshade
- 6U cubesat w 10 cm telescope
- Inner working angle 0.6", *B*-band contrast 10^{-7}
- Sensitive to dust disks to ~5 zodi

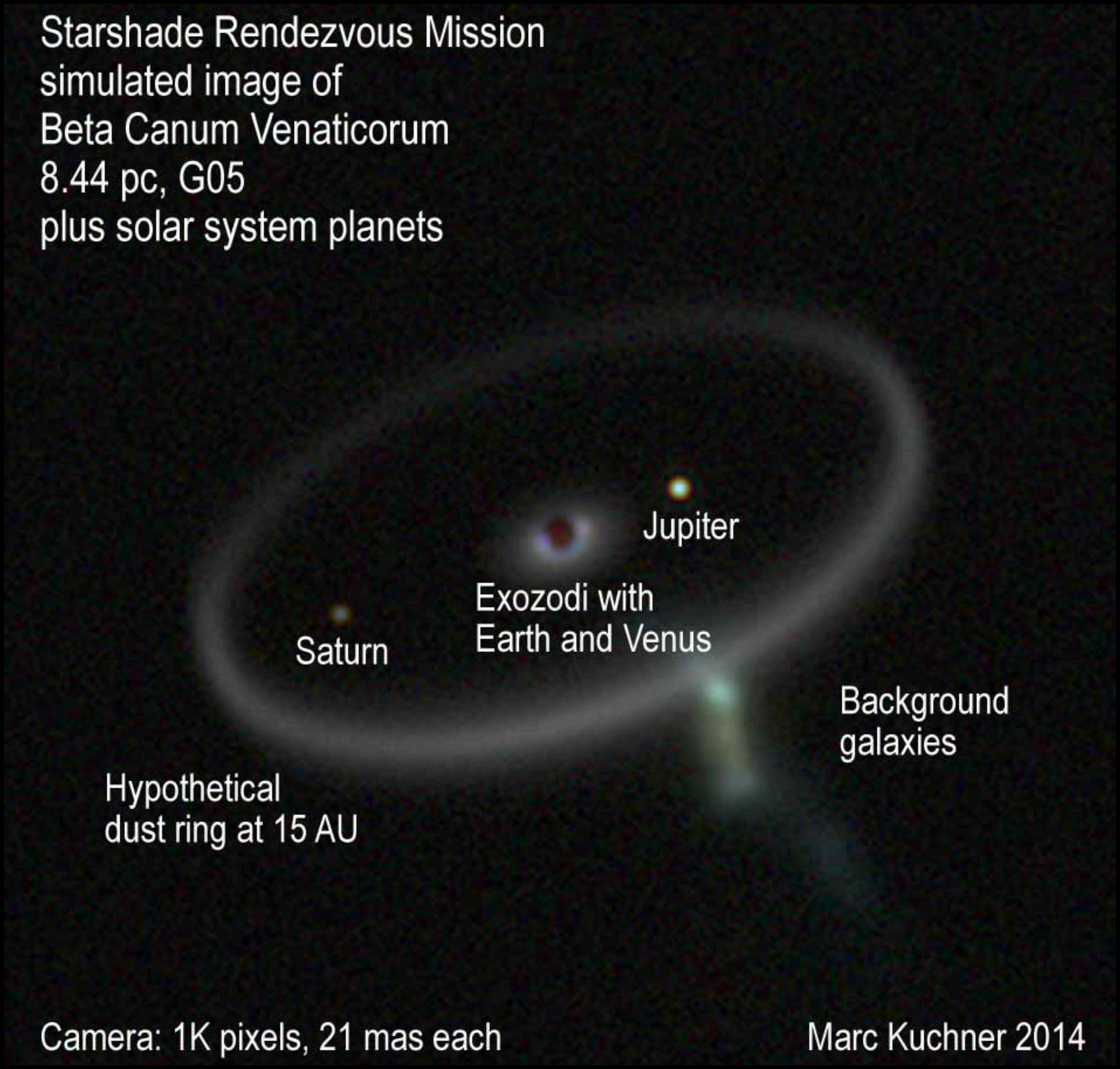




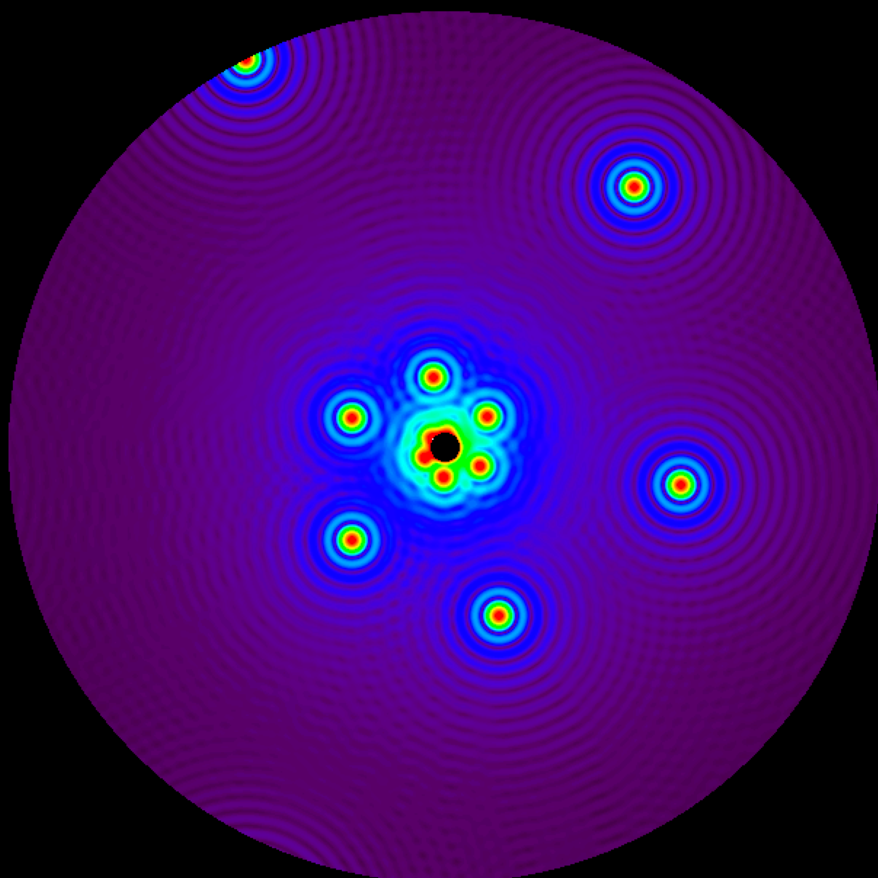


Alex Madurowicz, Stanford

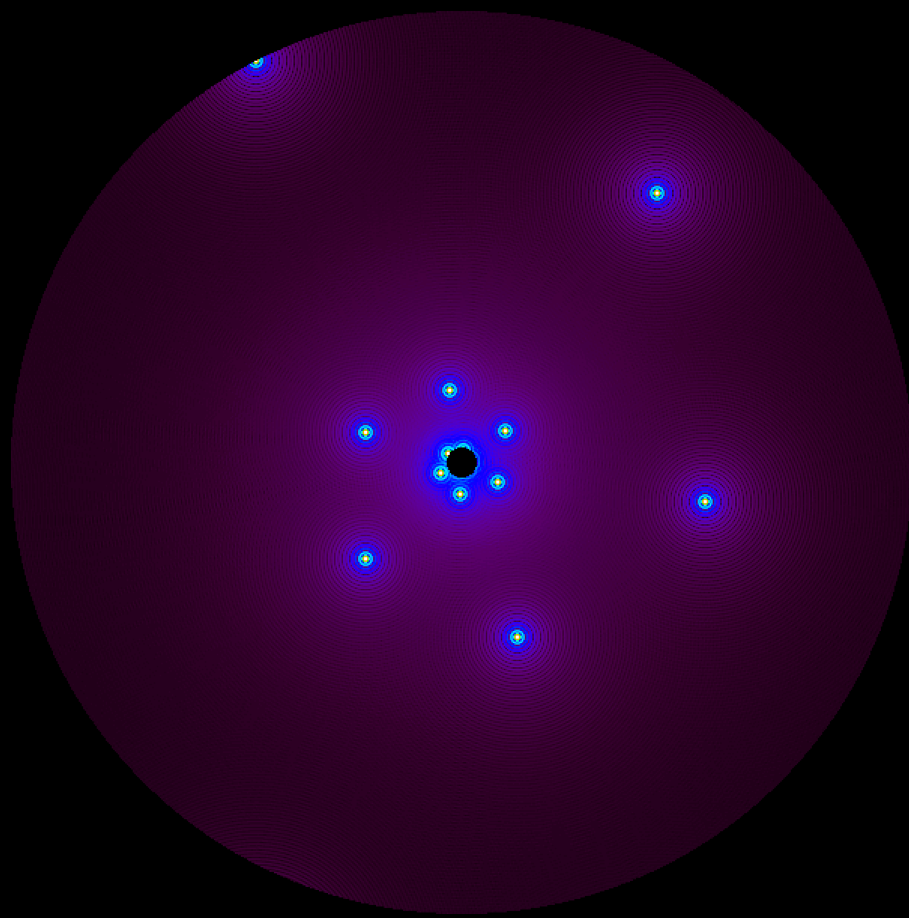
Simulated Solar System



A note on starshade IWA



20 PSFs log-uniform in semi-major axis seen with 1 **lambda**/D IWA



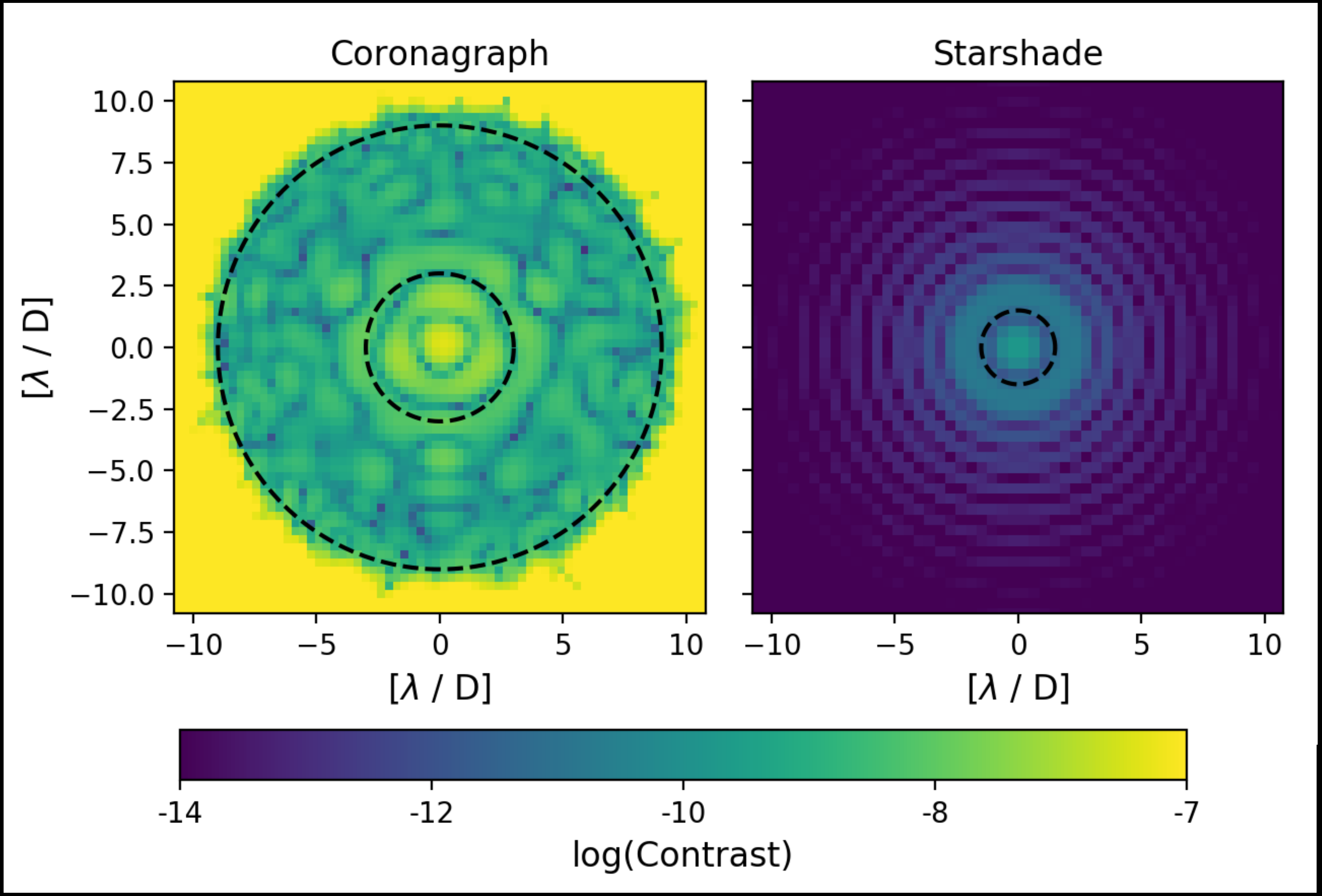
20 PSFs log-uniform in semi-major axis seen with 4 **lambda**/D IWA and 4x larger D

While starshade IWA nominally independent of lambda/D, image resolution still matters; IWA > lambda/D

Slide courtesy of Chris Stark

Comment on OWA

- Starshade has no OWA
- Contrast improves drastically outside starshade's IWA
- Defects ('speckles') confined to starshade's location in image



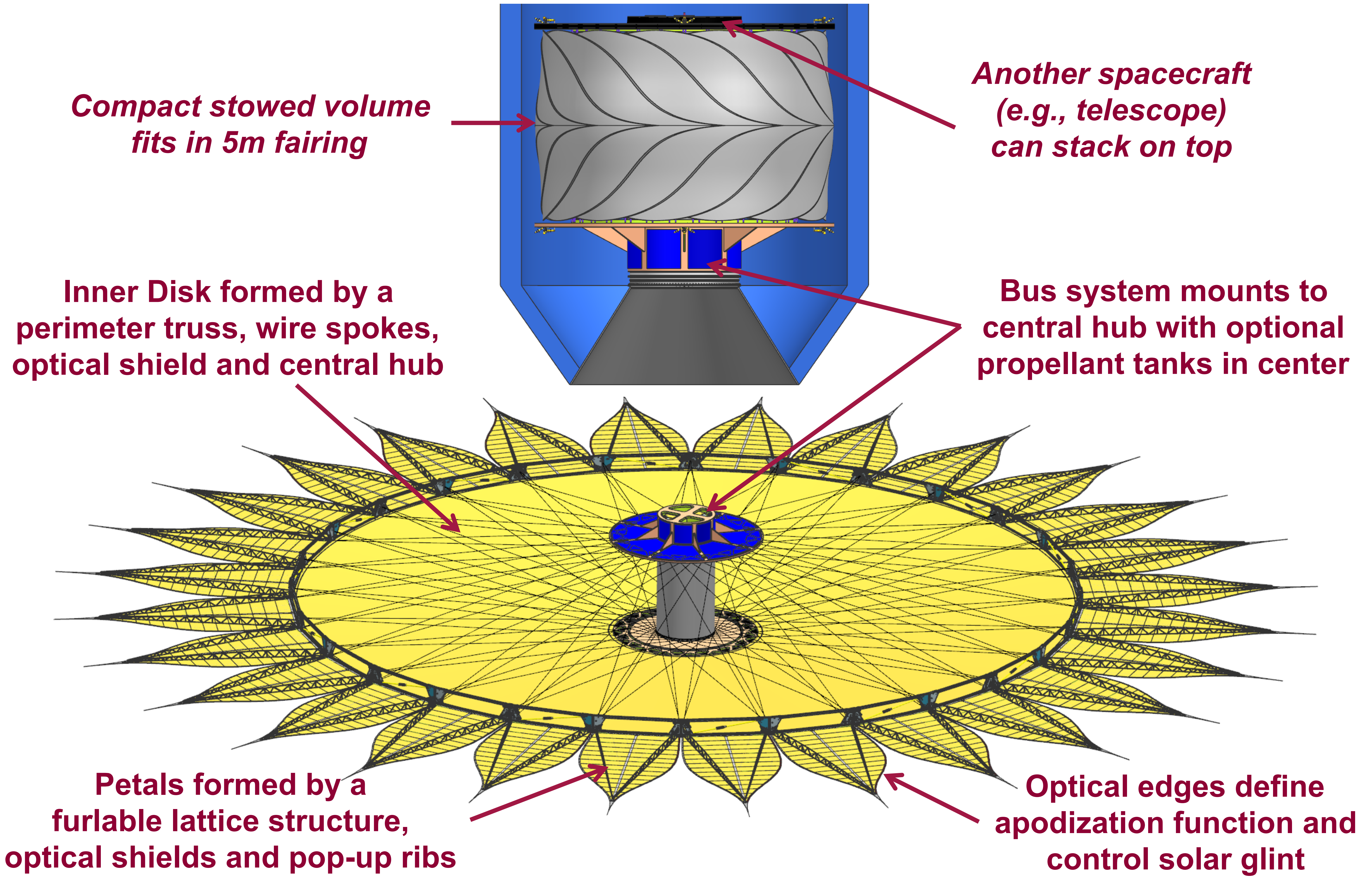
Simulation credit:
Leonid Pogorelyuk and Anthony Harness

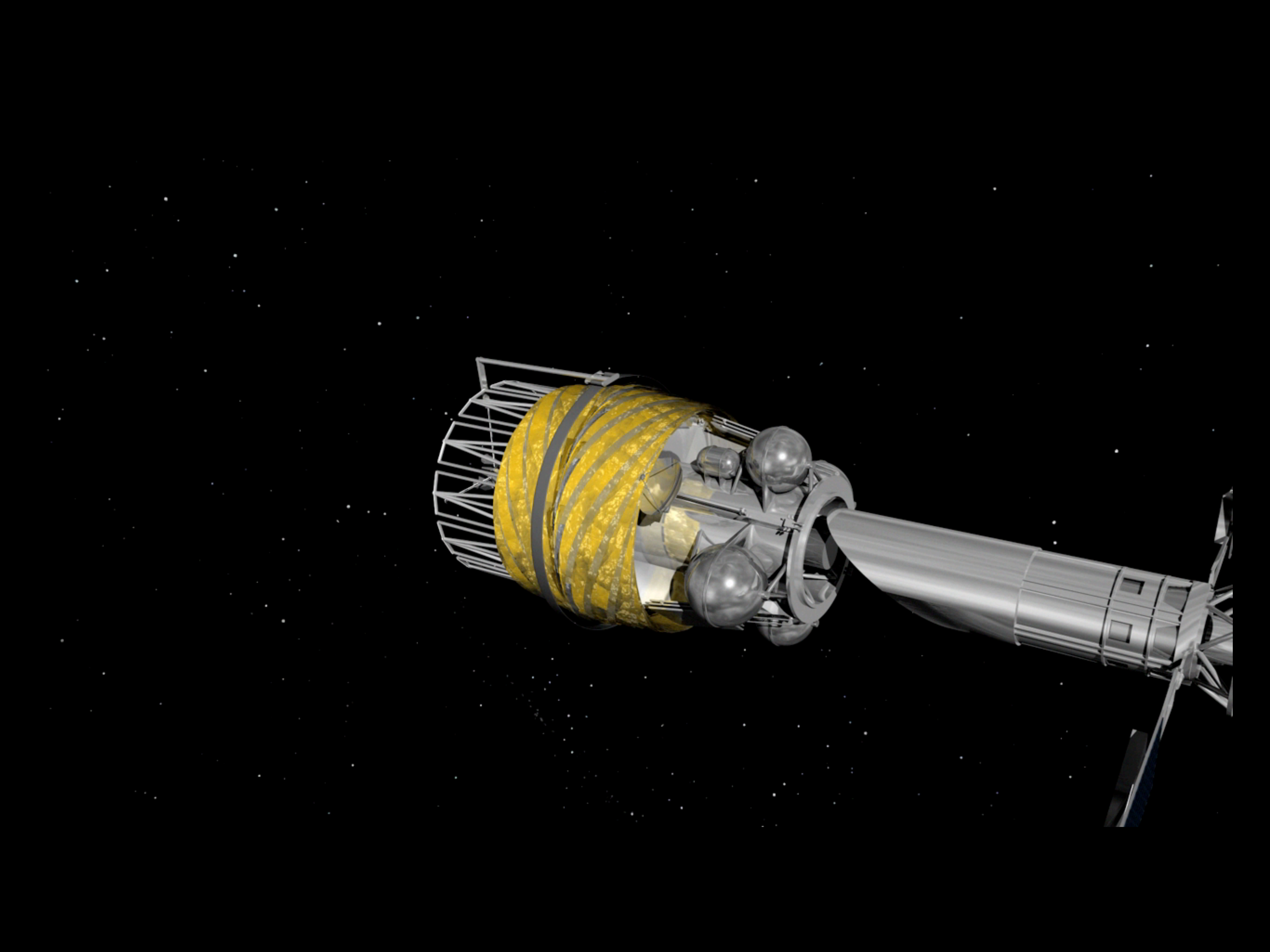
Making it Work

- Mechanical Design and deployment
- Error budgeting
- Manufacturing tolerances and stability
- Optical model verification

The next talk will cover the S5 technology development and verification program at NASA.

Generation 2 Perimeter Truss Design



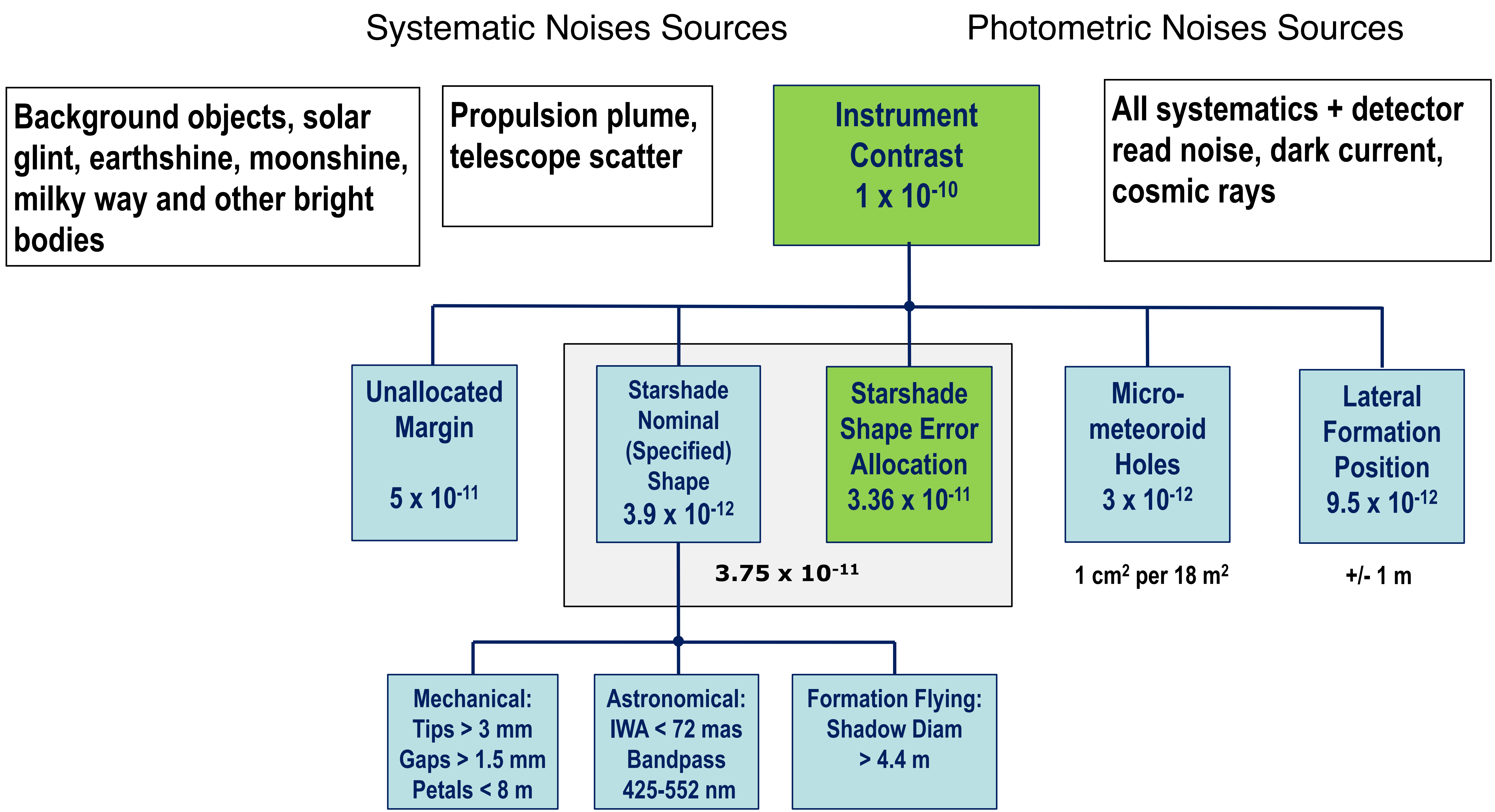


Gen 2 Deployment (no metrology)

Error Budget & Requirements

Employ a detailed error analysis examining all perturbations to set an error budget and requirements on manufacture and deployment.

Error Budget Tree



Shape Allocation breakdown

Char. Feature		CBE 3 sig	Cont.	Max Exp.	CBE Cont	Max Exp Cont
Petal Width (um)	Bias	20.00	0.25	2.50E+01	5.68E-13	8.88E-13
Edge Segment x and y position (um)	Random	20.00	0.25	2.50E+01	5.54E-13	8.66E-13
Edge Segment x and y position (um)	Bias	10.00	0.25	1.25E+01	4.97E-13	7.76E-13
Edge Segment clocking (urad)	Random	33.33	0.25	4.17E+01	4.27E-13	6.67E-13
Edge Segment shape (sinusoidals) (um)	Bias	13.00	0.50	1.95E+01	3.54E-13	7.96E-13
Petal Interface radial position (mm)	Random	0.17	0.25	0.21	1.85E-13	2.88E-13
Tip segment width (um)	Bias	13.00	0.50	1.95E+01	1.22E-13	2.75E-13
Petal higher order (sinusoids) (um)	Bias	1.00	1.00	2.00E+00	1.13E-13	4.52E-13
Edge Segment shape (sinusoidals) (um)	Random	13.00	0.50	1.95E+01	1.02E-13	2.29E-13
Tip segment shape (sinusoids) (um)	Bias	13.00	0.50	1.95E+01	7.62E-14	1.71E-13
Tip segment width (um)	Random	13.00	0.50	1.95E+01	6.76E-14	1.52E-13
Edge Segment Shape residual (f> 3 cycles/segment)	Bias	13.00	0.50	1.95E+01	5.41E-14	1.22E-13
Petal Interface radial position (mm)	Bias	0.04	0.25	0.04	4.82E-14	7.53E-14
Petal Interface clocking angle (urad)	Random	100.00	0.25	0.00	4.39E-14	6.85E-14
Tip segment shape (sinusoids) (um)	Random	13.00	0.50	1.95E+01	4.23E-14	9.51E-14
Edge Segment clocking (urad)	Bias	5.00	0.25	6.25E+00	2.97E-14	4.63E-14
Petal Interface elliptical mode (mm)	Bias	0.10	0.50	0.15	2.34E-14	5.26E-14
Petal Interface tangential position (mm)	Random	0.03	0.25	0.03	6.30E-15	9.84E-15
Tip segment x and y position (um)	Random	20.00	0.25	2.50E+01	2.02E-15	3.16E-15
Tip segment x and y position (um)	Bias	10.00	0.25	1.25E+01	9.12E-16	1.42E-15
Petal Interface higher order polygon modes (mm)	Bias	0.10	0.50	0.15	8.66E-16	1.95E-15
Petal 1-cycle in-plane shape error (width preserving) (mm)	Random	0.03	0.50	3.75E-02	1.77E-16	3.97E-16
Quadratic bending (cantilever beam bending) (mm)	Random	0.05	0.50	7.50E-02	4.31E-18	9.71E-18
Tip segment clocking (urad)	Random	33.33	0.25	4.17E+01	3.85E-18	6.02E-18
Quadratic bending (cantilever beam bending) (mm)	Bias	0.05	0.50	7.50E-02	3.58E-22	8.06E-22
Tip segment clocking (urad)	Bias	5.00	0.25	6.25E+00	2.06E-23	3.22E-23
SUM					3.32E-12	6.04E-12

Experiment vs. Requirement

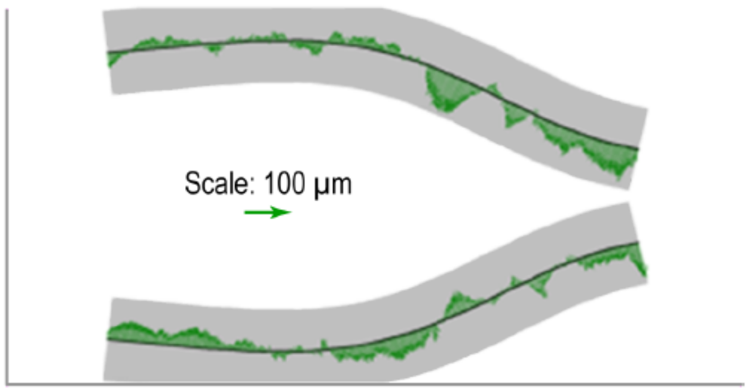
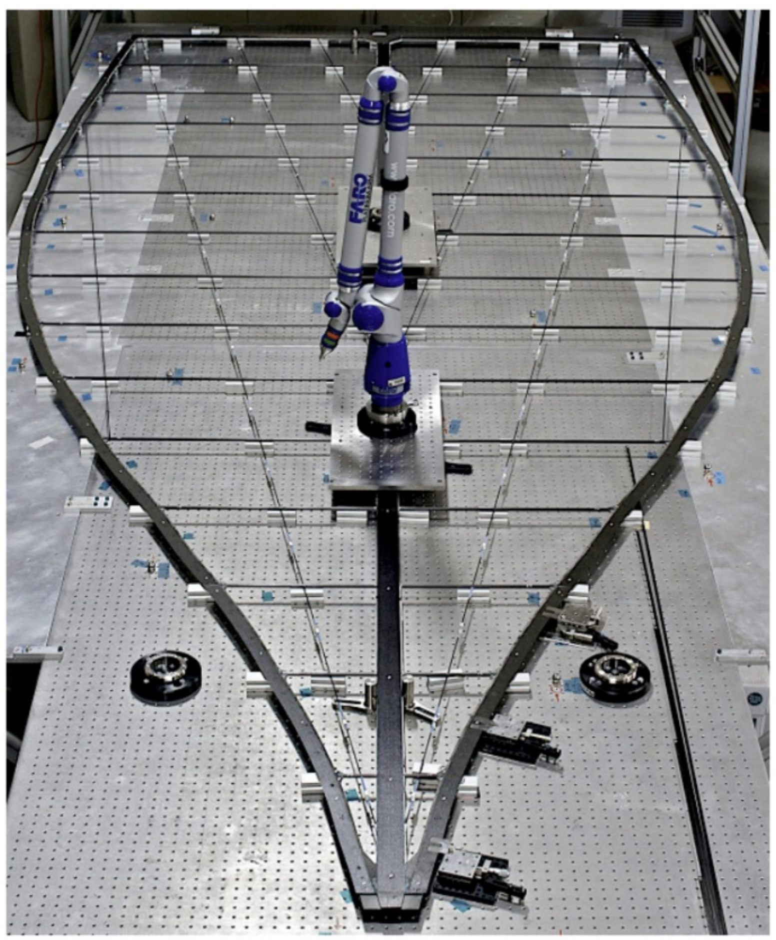


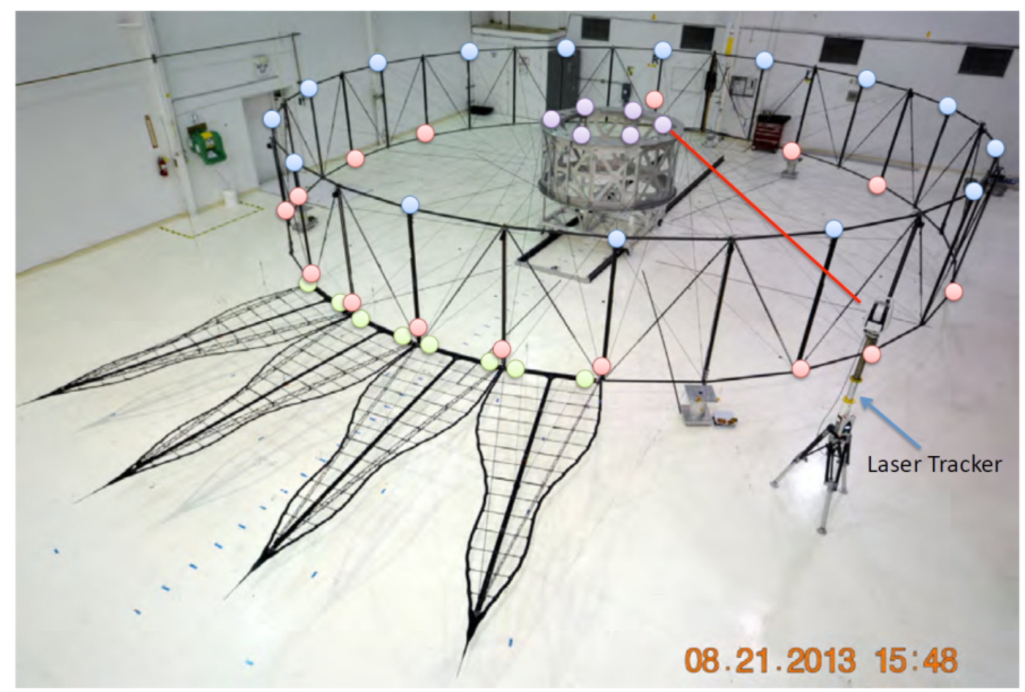
Figure 9.4-2. Measured petal shape error (green arrows) vs. 100 μm tolerance for 1 × 10^{-10} imaging (gray band) shows full compliance with the allocated tolerance.

Table 6.4-4. Comparison of TDEM results with Exo-S requirements.

Key Technology	Demonstration	Achieved Tolerance	Required Tolerance
Petal Segment Shape (Random)	TDEM-09	±45 μm	±68 μm
Petal Segment Position (Random)	TDEM-09	±45 μm	±45 μm
Radial Petal Position (Bias)	TDEM-10	±100 μm	±150 μm



Kasdin TDEM-10 Final Report

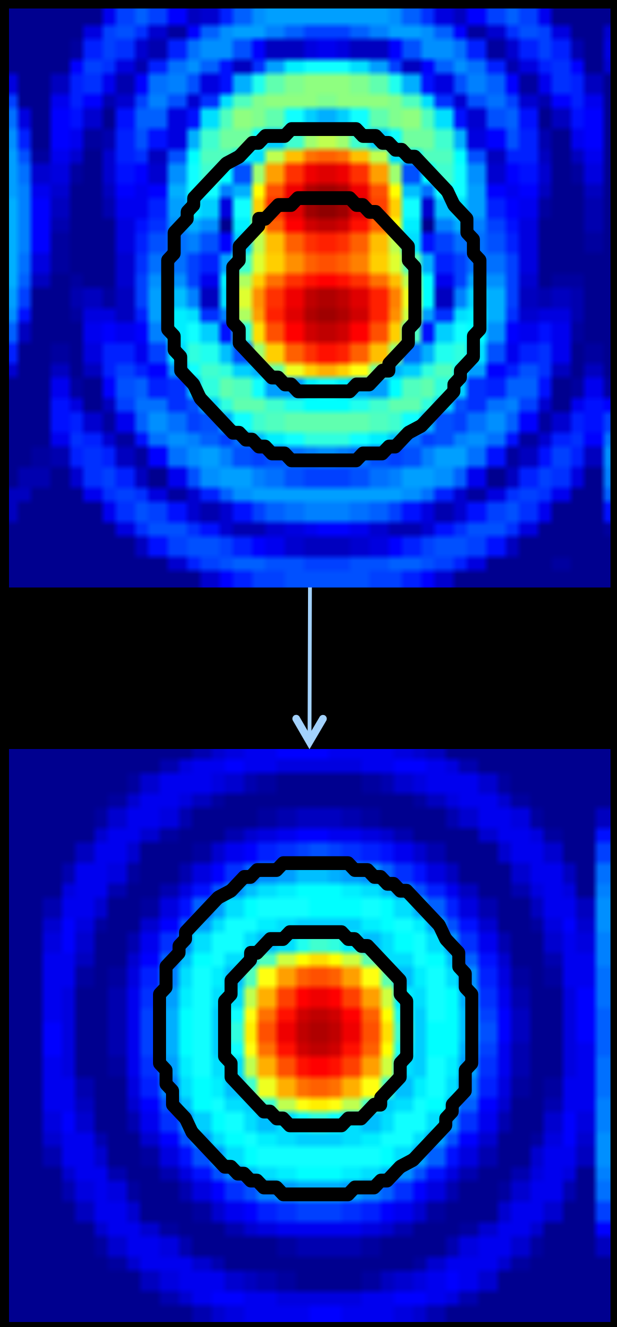


Kasdin TDEM-11 Final Report

Mean contrast at worst-case wavelength of 2.15 x 10^{-10}

Spinning the Starshade

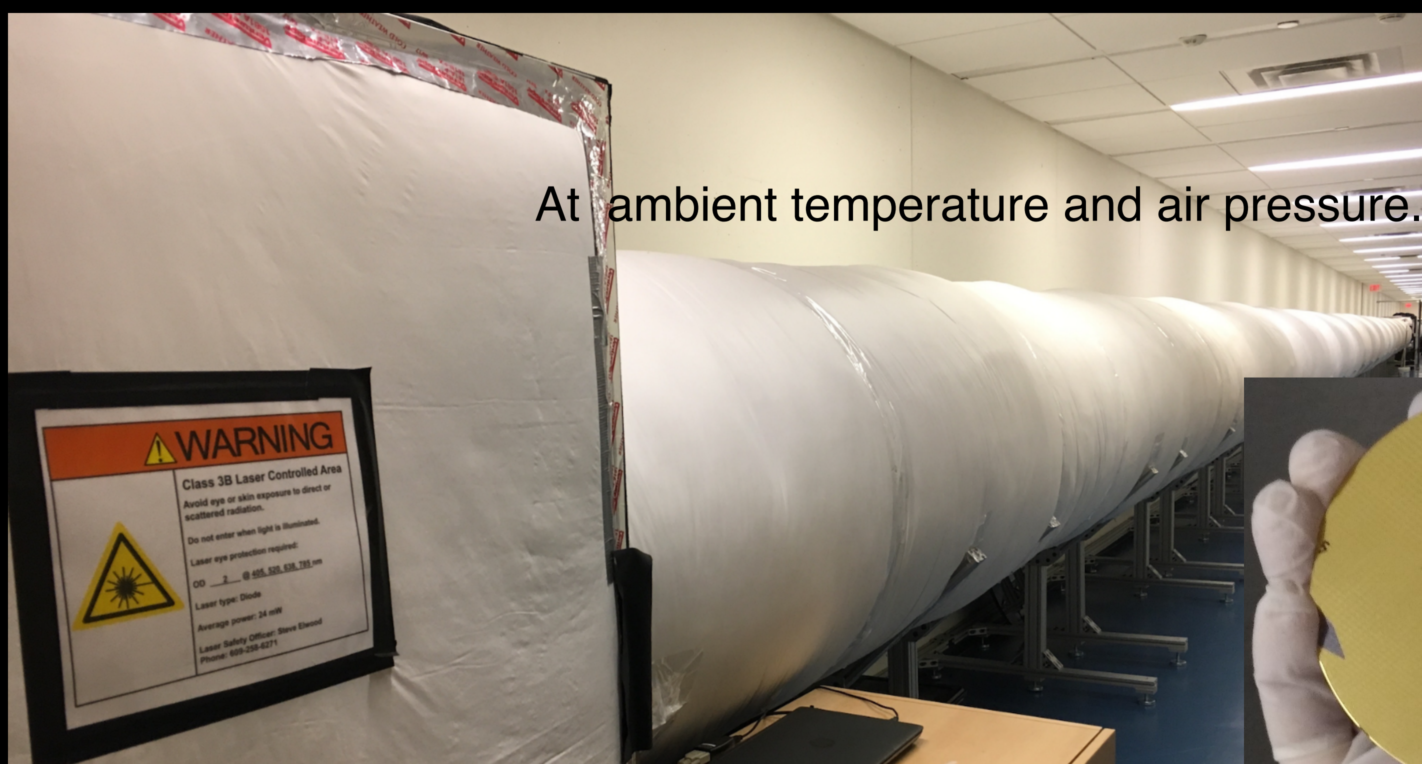
- Local errors, e.g. a displaced petal, scatter into speckles in the image plane.
- Speckles look like planets.
  - Speckle requirement is 1e-11 contrast.
- Spinning the starshade smears the speckles into annuli.
  - Background requirement is 1e-10.
- *This leads to a 3x relaxation of shape requirements.*
- Same requirements apply to planet detection and characterization since limited by zodi and exozodi rather than instrument.
- Spin rate: up to 12 rev/hr, limited by retargeting fuel (assumed 1 kg per 90 deg turn)



Shaklan, SPIE, 2011

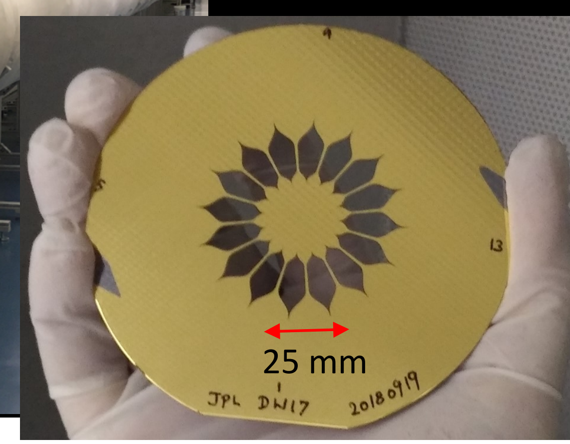
Experimental Optical Verification

Verify the scalar optical modeling used for design and performance predictions is correct via subscale tests

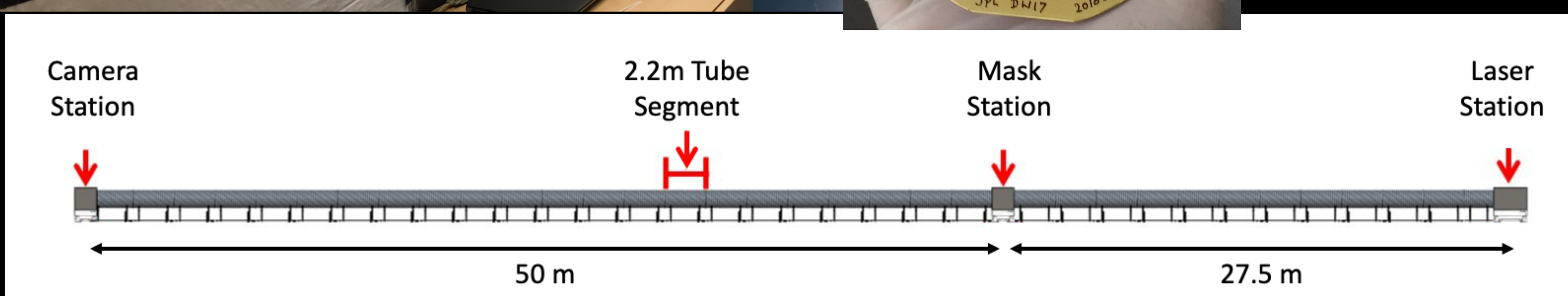


At ambient temperature and air pressure.

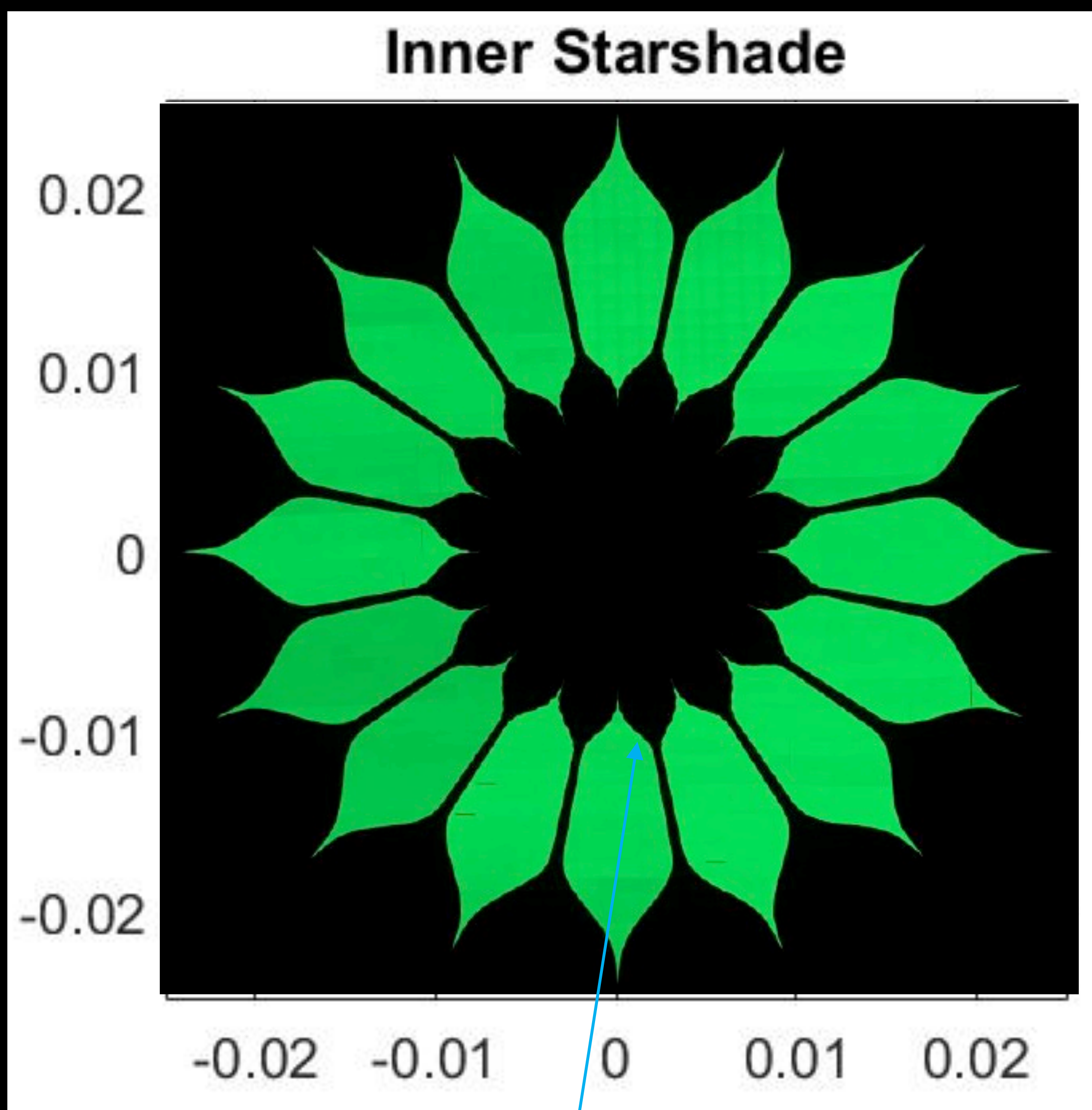
Princeton Starshade Testbed

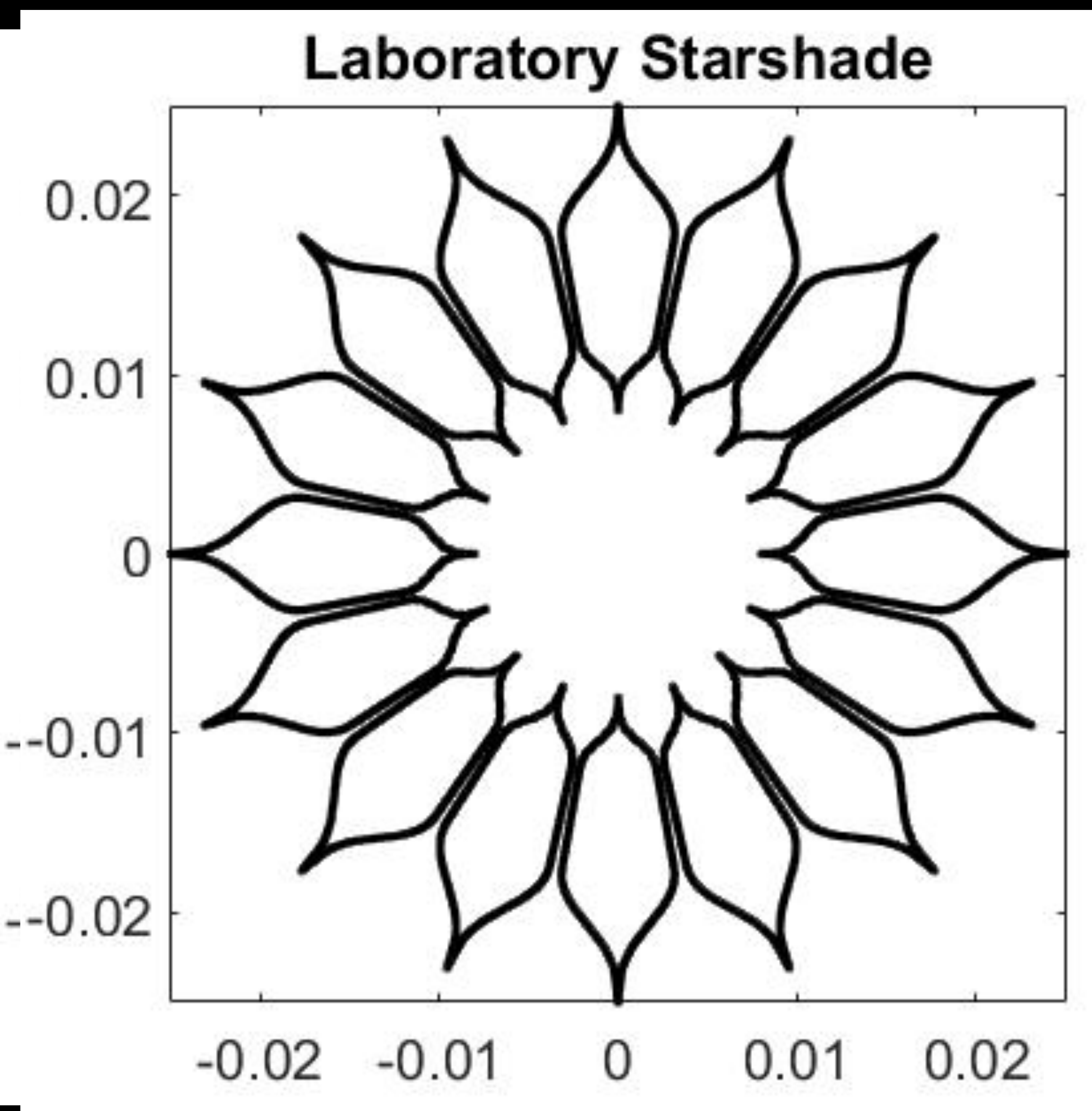


Conducted by Anthony Harness



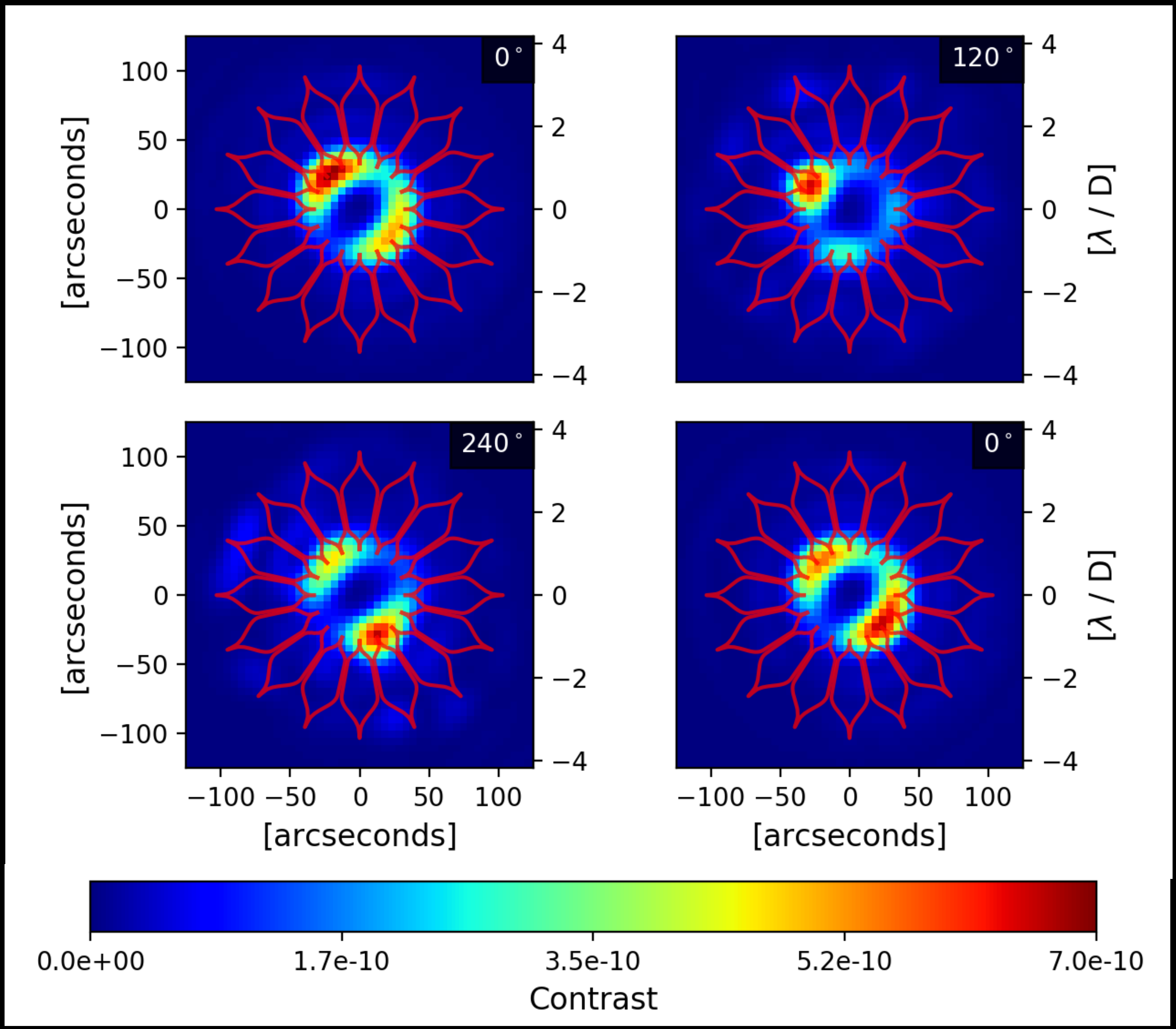
Laboratory Starshade Design at Flight Fresnel Number





Inner Tips: 16.2 um wide, 500 um long
Outer Tips: 27 um wide.

Sample Lab Results



Single wavelength: 641 nm

Bright lobes are due to interaction with the mask edge as light propagates through narrow valleys

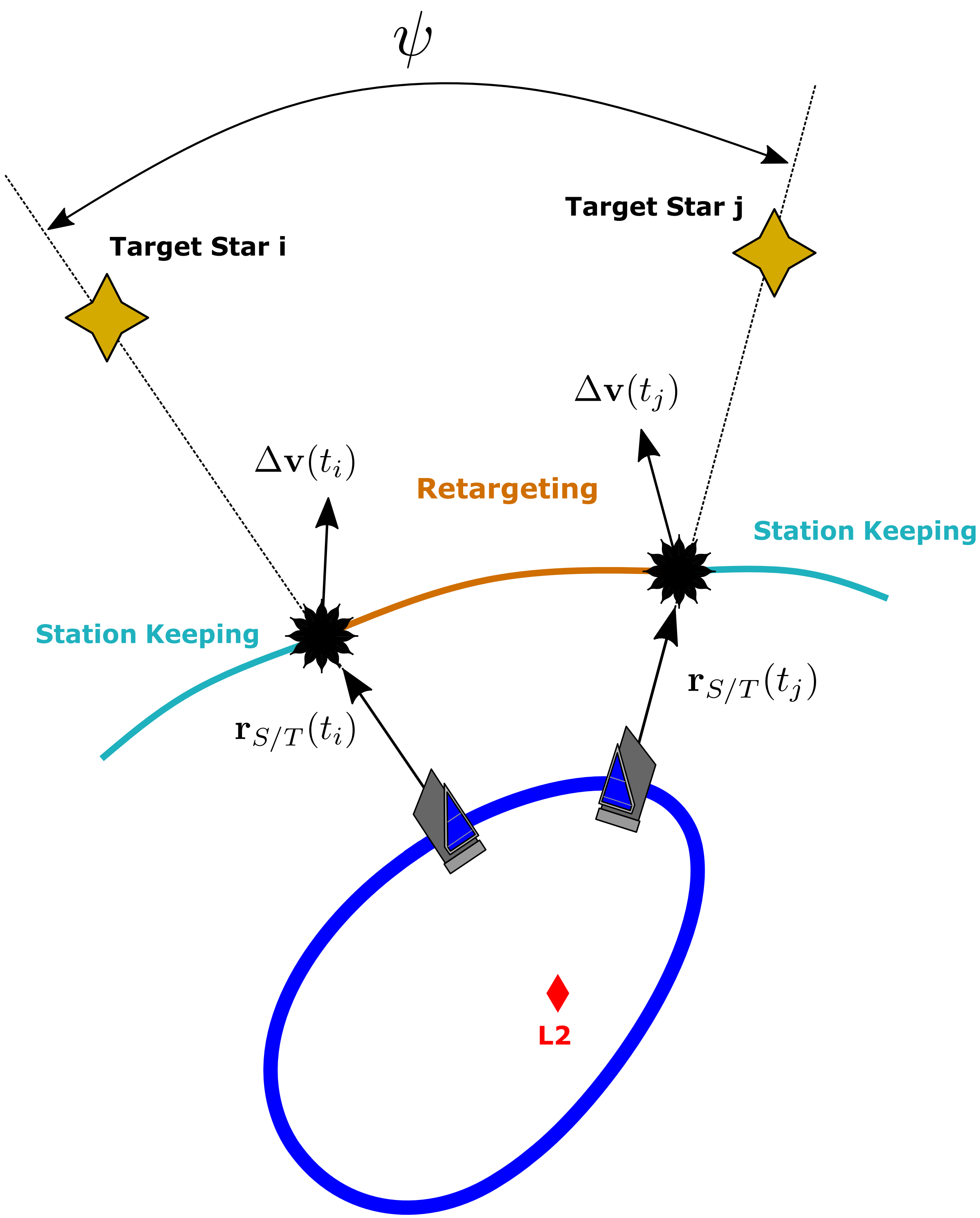
"Thick Screen Effects"

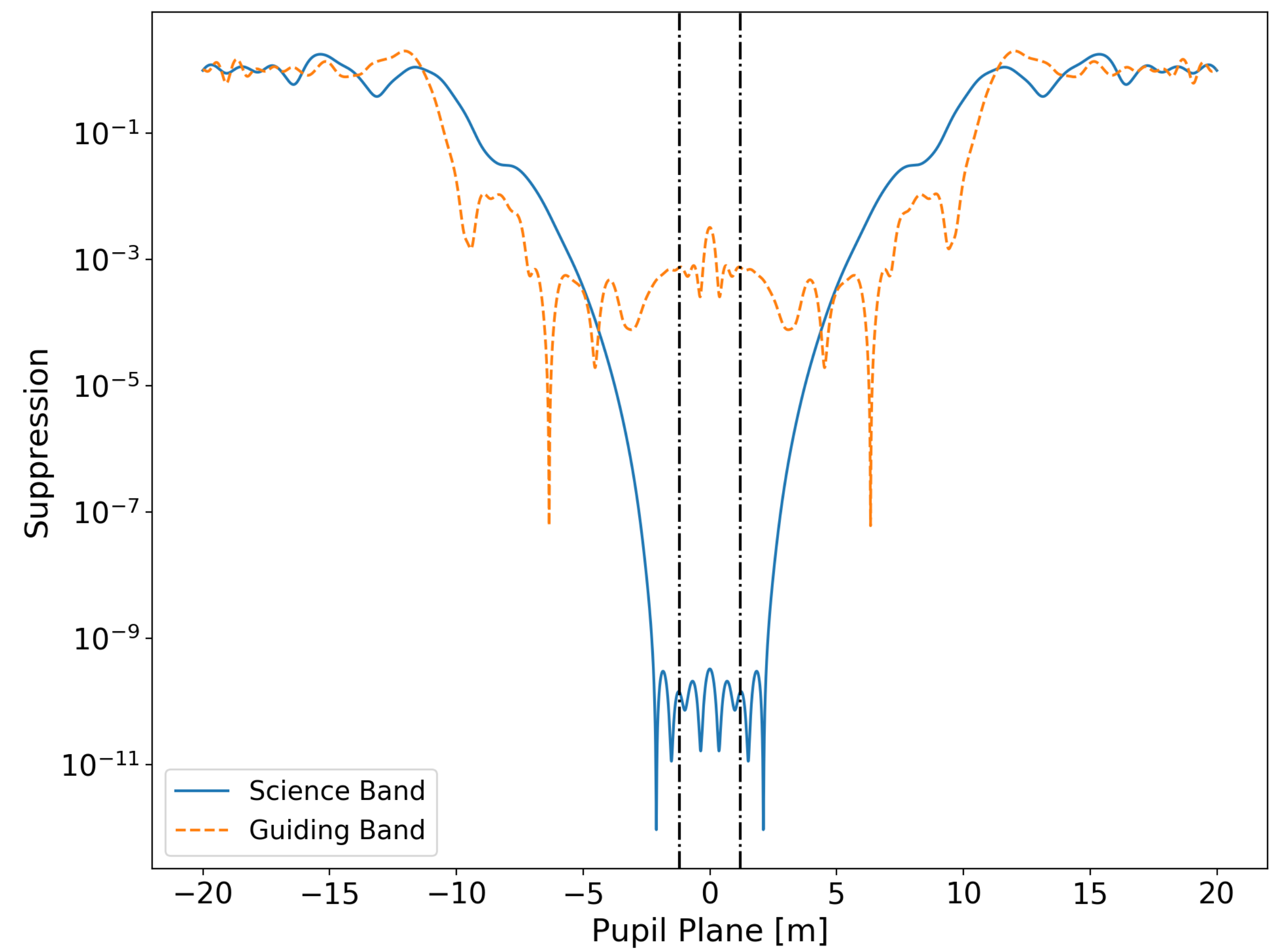
Demonstrated ability to achieve 1e-10 contrast with lab starshade.

Operational Considerations

- Formation flying
- Viewing Constraints
- Solar diffraction and glint
- Slew time and DRMs

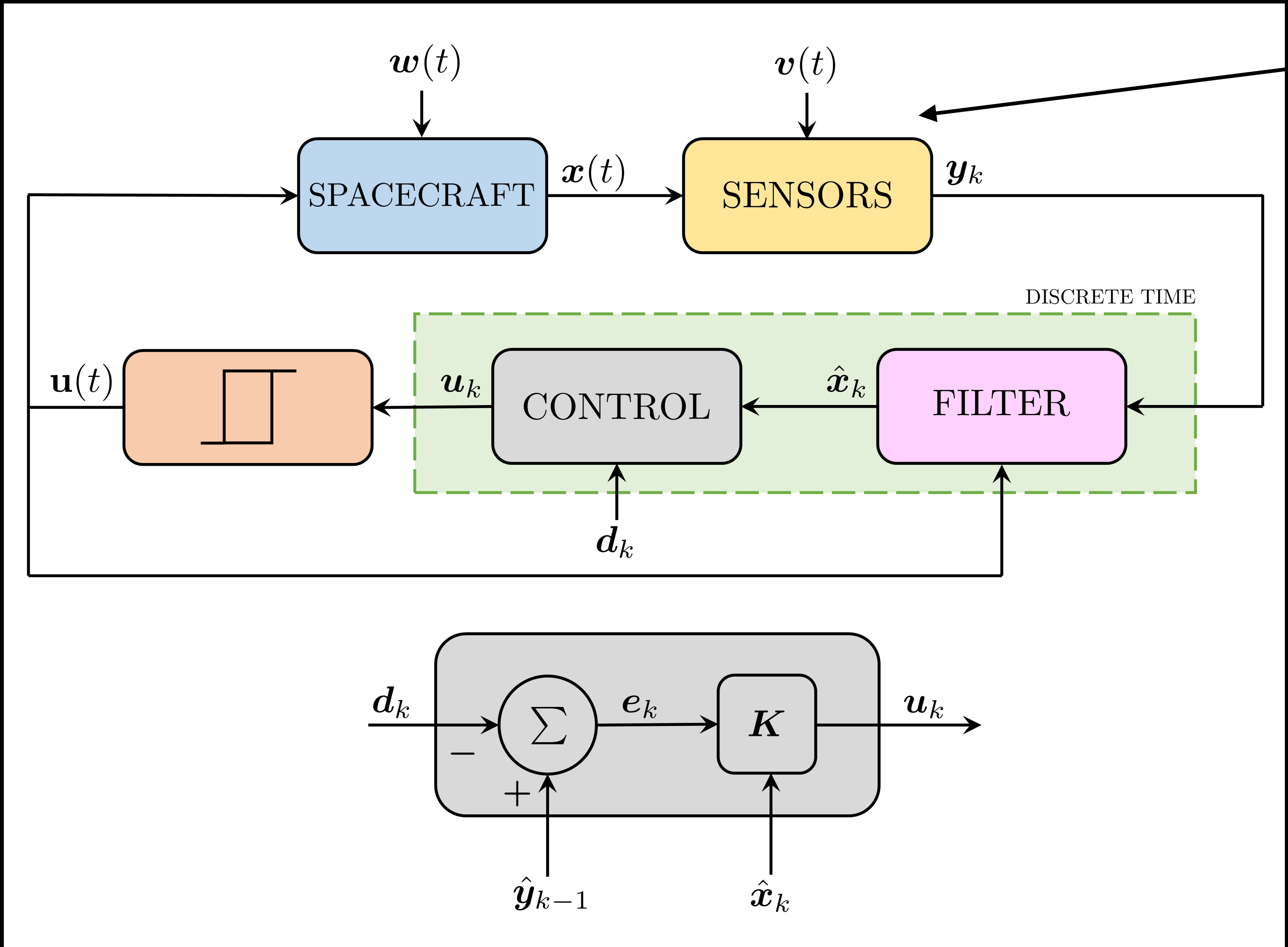
Retargeting and Stationkeeping





Soto, et al.

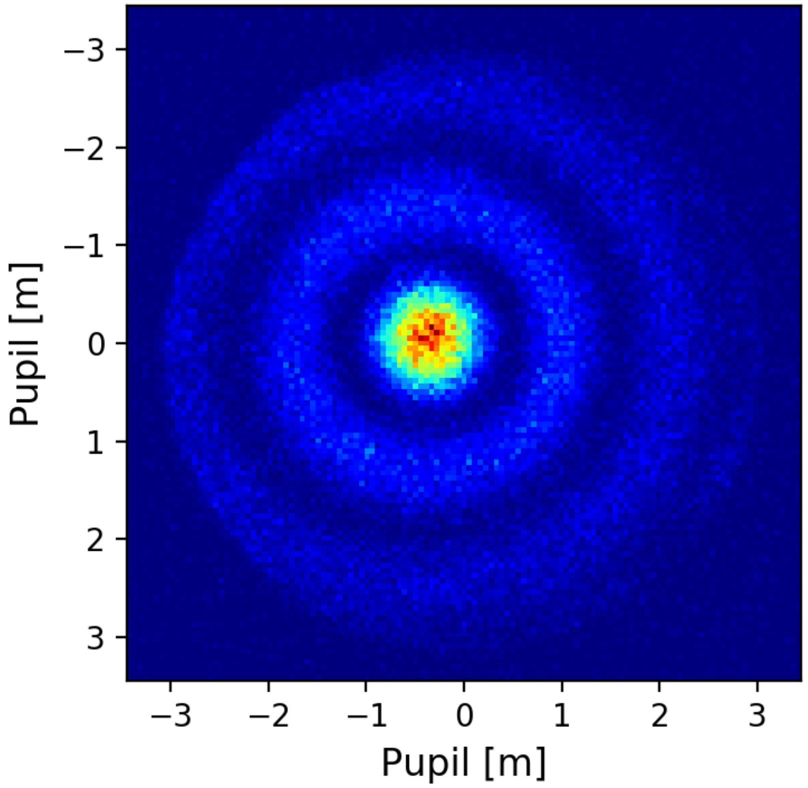
Control Loop

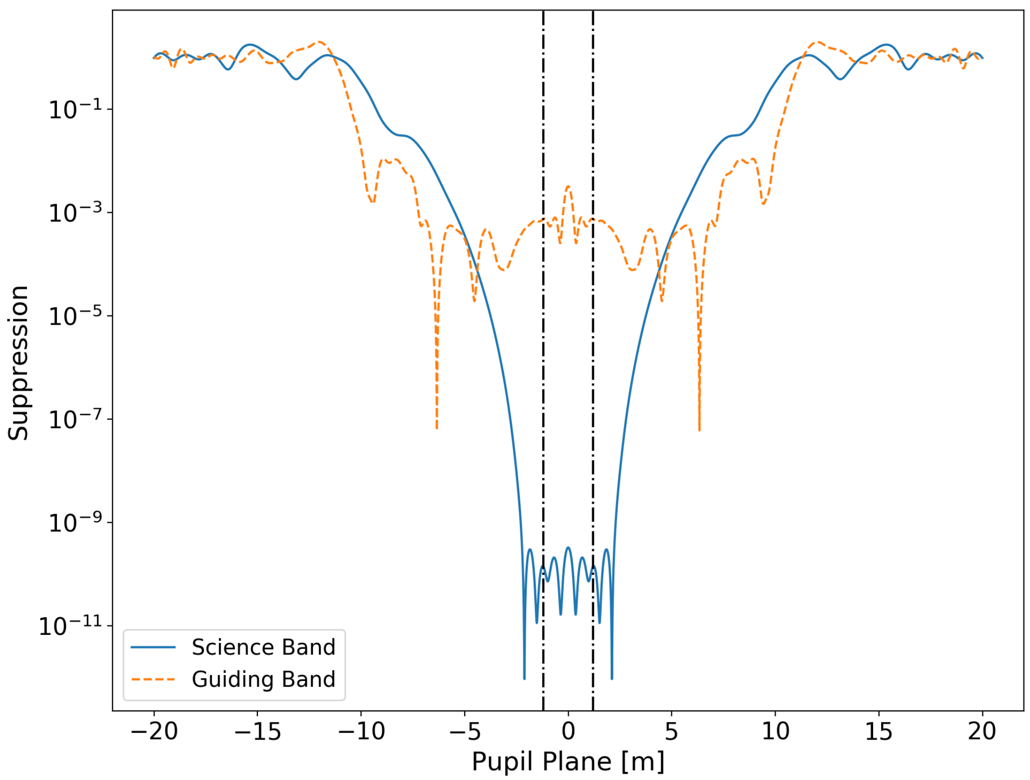


measure position by fitting pupil image

Linear Quadratic Regulator with Integral Control and Unscented Kalman Filtering

Position Sensing





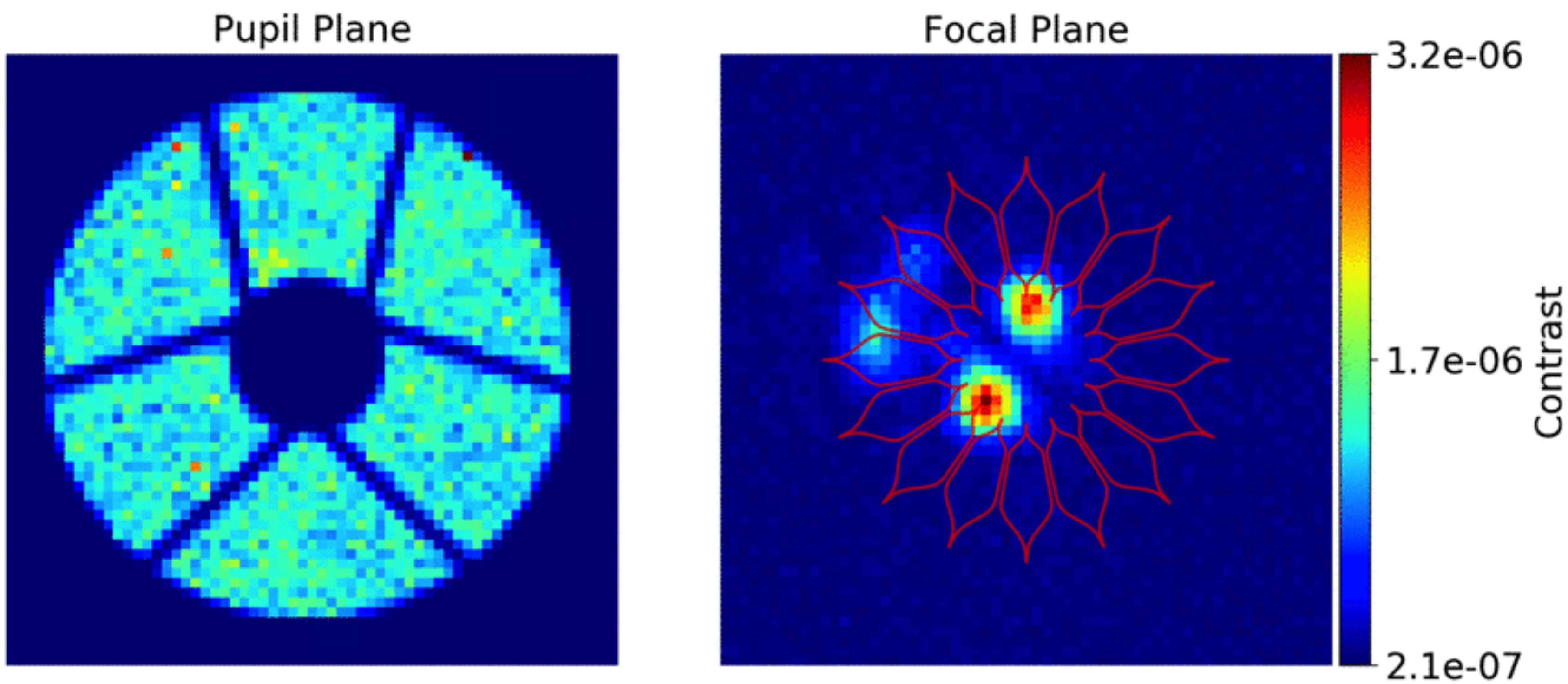
- Starshade's diffraction pattern approximated by a Bessel function:

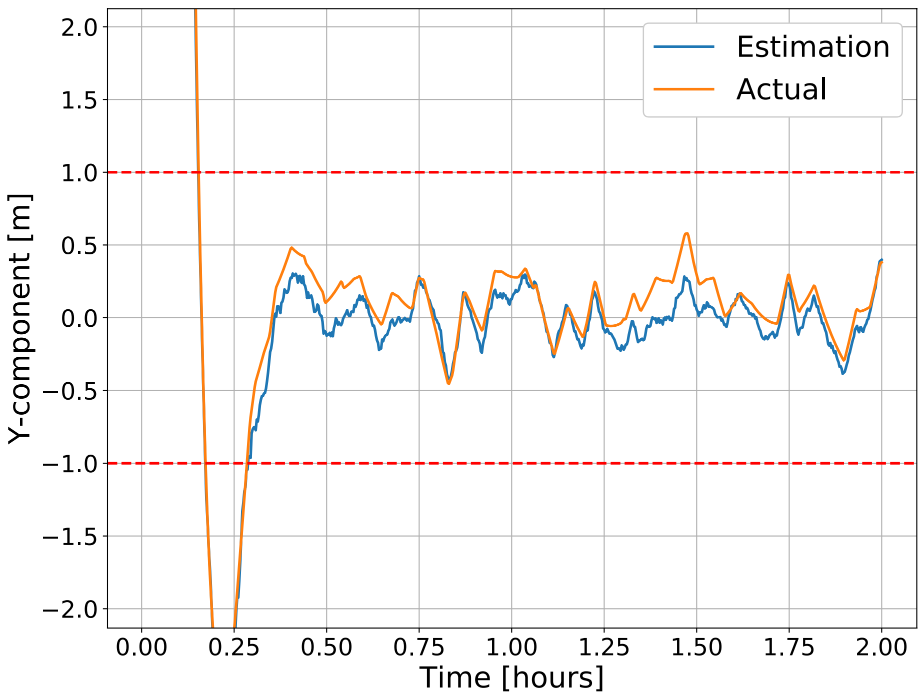
$$I(x,y) \approx J_0^2\left(\frac{2\pi R\sqrt{(x-x_s)^2+(y-y_s)^2}}{\lambda z}\right) \tag{4}$$

- x_s and y_s solved via non-linear least squares

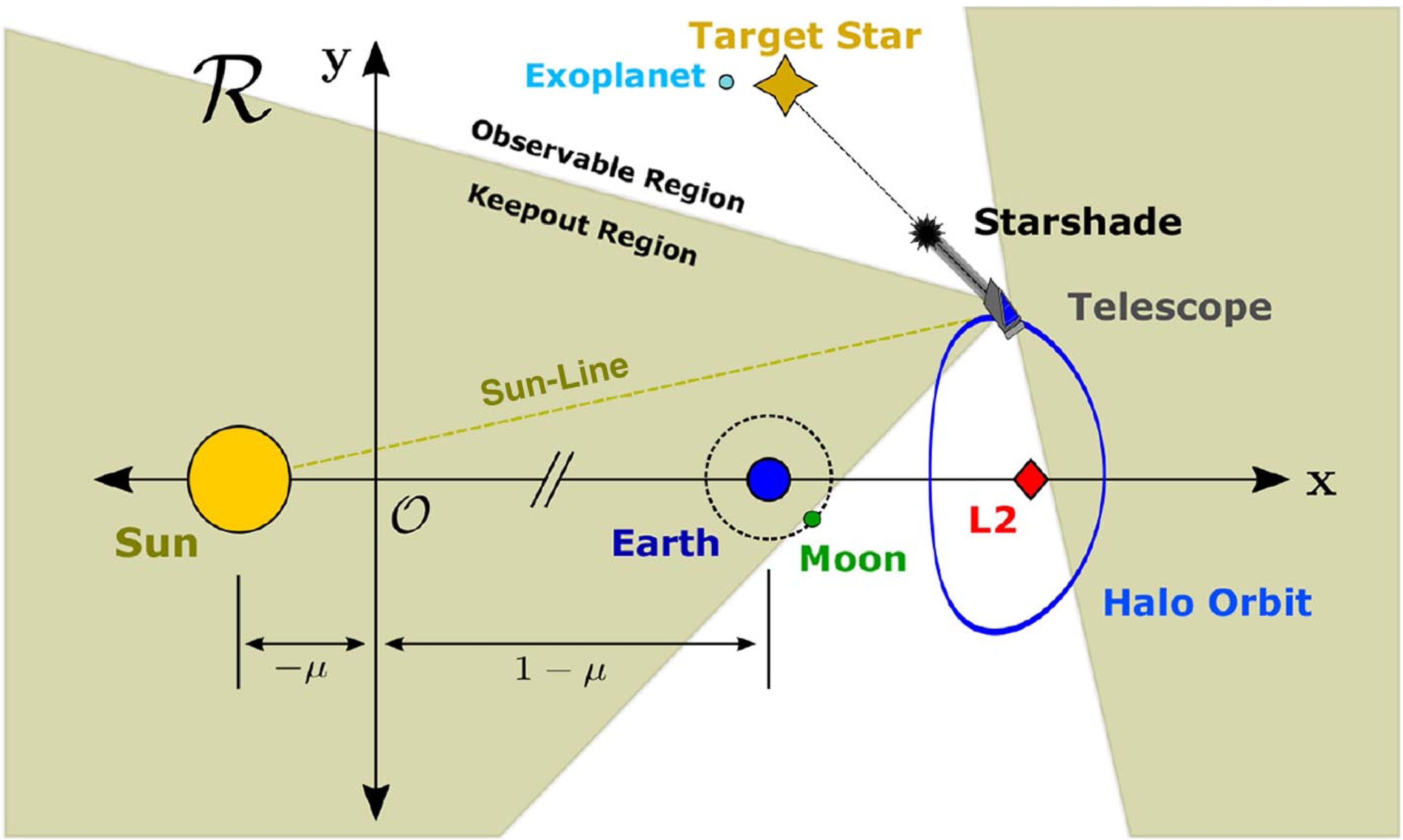
Hardware-in-the-loop Stationkeeping Test

Simulated Formation keeping with actual position measurements from Princeton testbed





Viewing Constraints

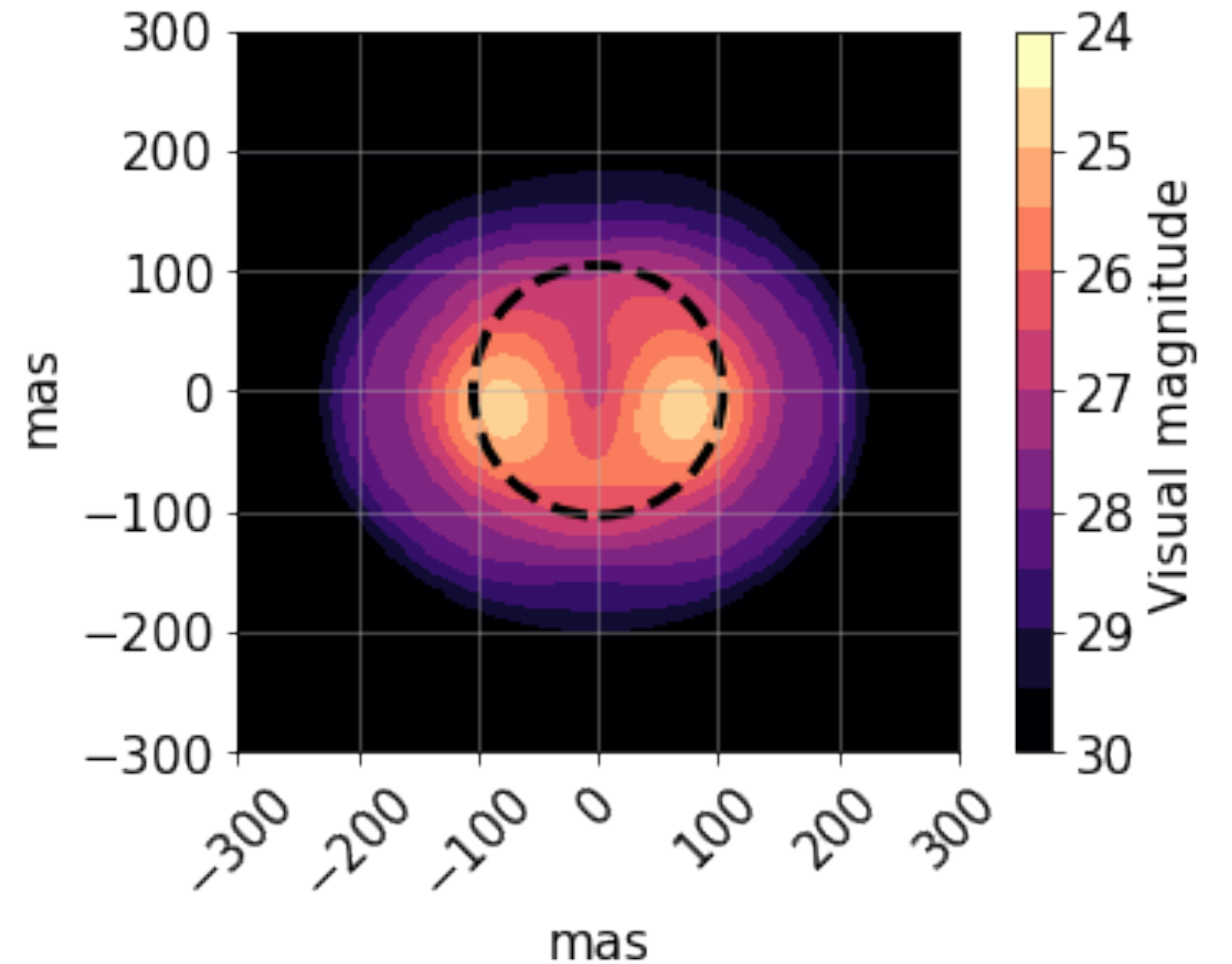


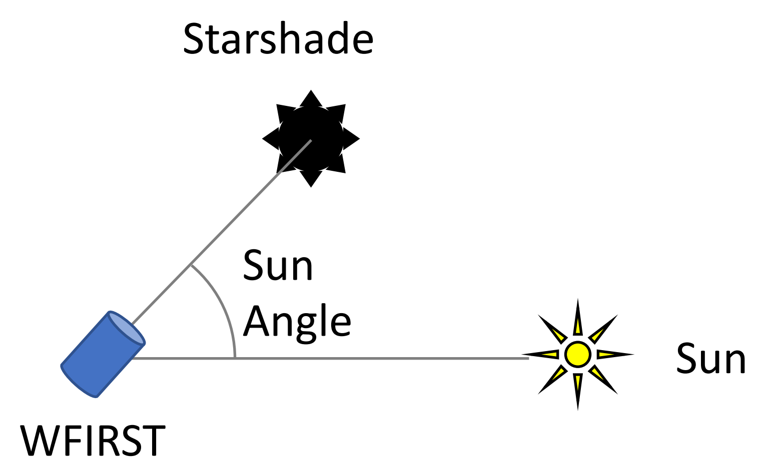
40 to 83 deg Field of Regard

Soto, et al.

Solar Edge Glint & Diffraction

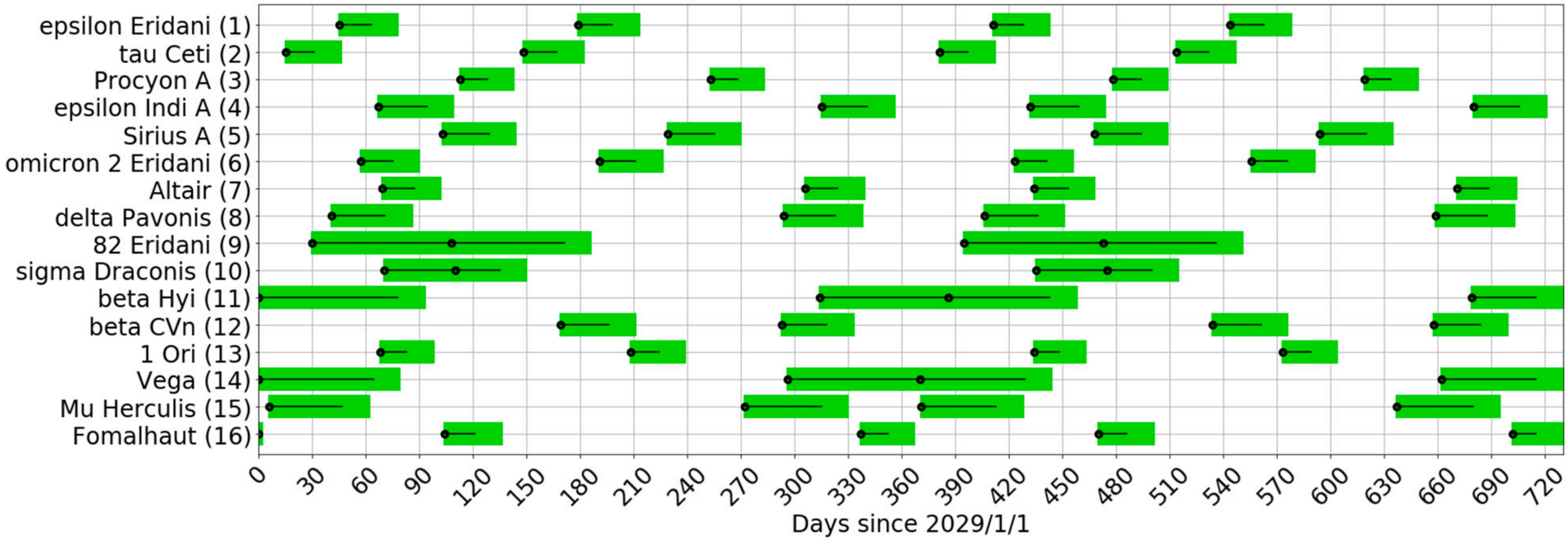
- Model of solar glint from 26 m starshade at 26,000 km observed with WFIRST at 63° sun angle.
- Model is based on measured data of scatter from an etched amorphous metal edge, the design baselined for flight.
- The inner working angle (104 mas) is shown for reference.





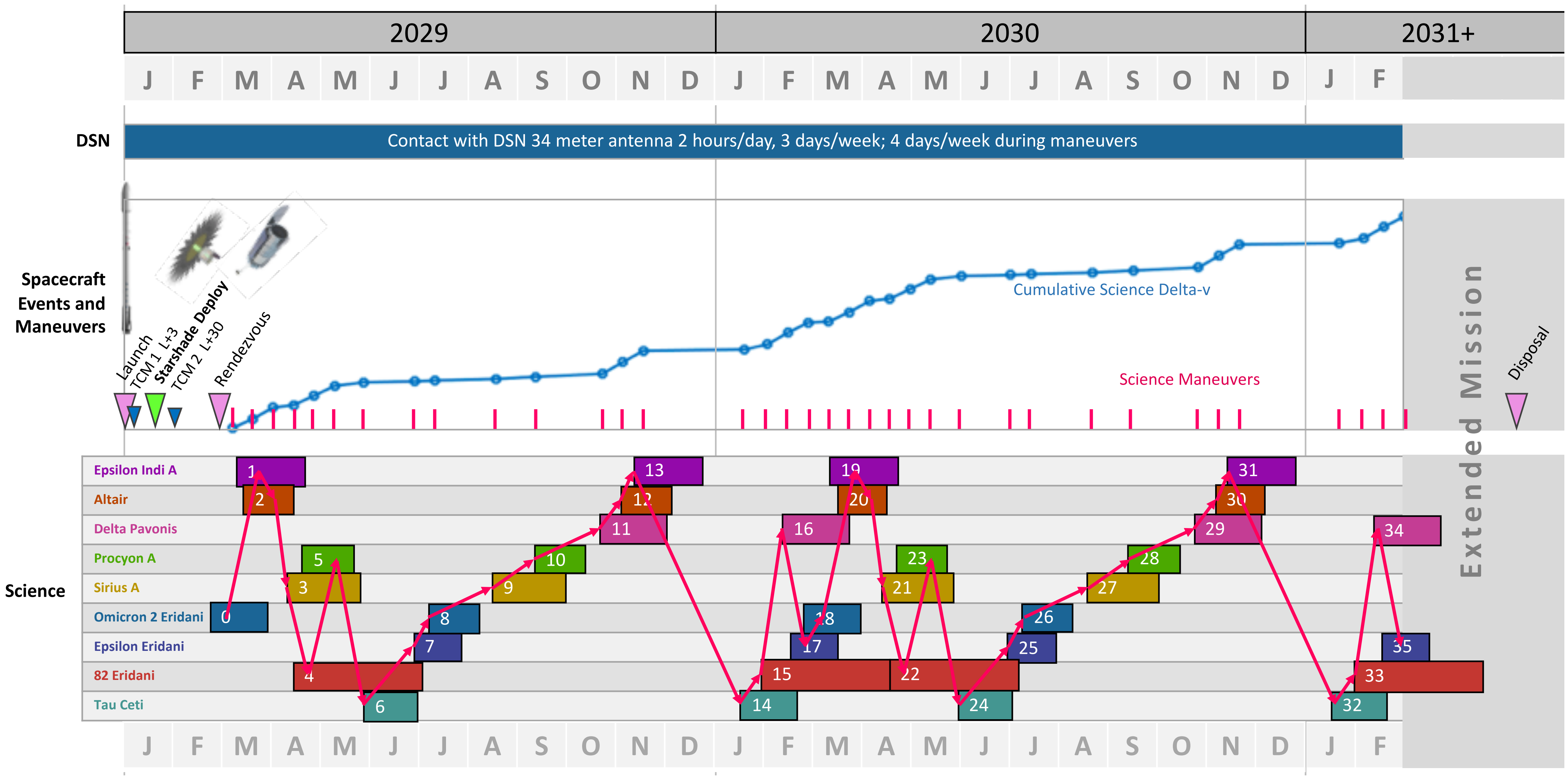
Sample Target Availability

Starshade Rendezvous



- Selected high completeness (>0.5) targets with no optical companions.
- Targets are distance range between 3 – 8 pc.
- Viewing windows determined by solar exclusion angles.
- Two ~30-day windows per target per year is typical.

Example DRM – Rendezvous



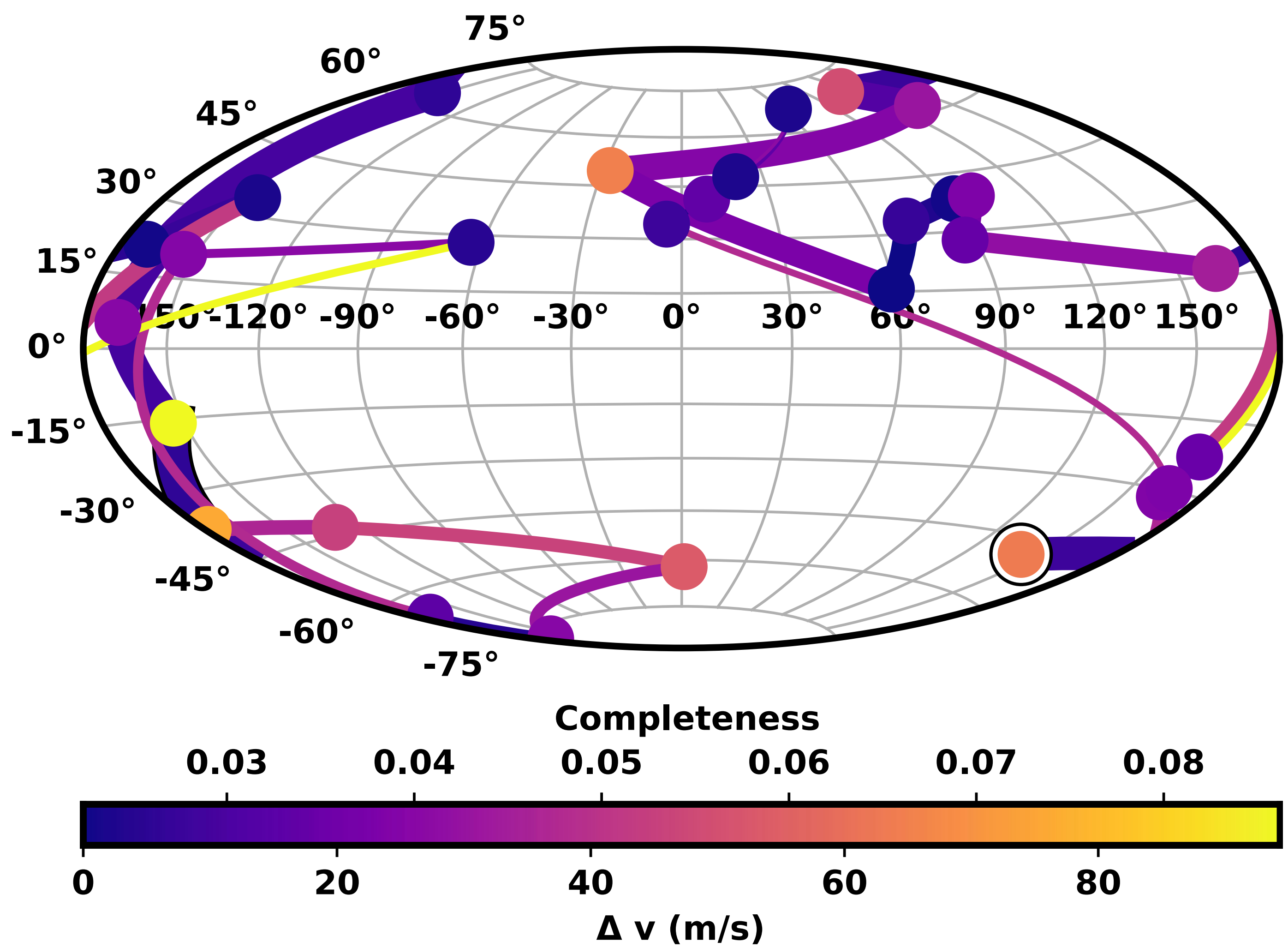
Mission Timeline

Red line segments are slews (2 days to 2 weeks)

Red dots: single day's observation

Horizontal bars: target star observability windows based on Sun angular constraints

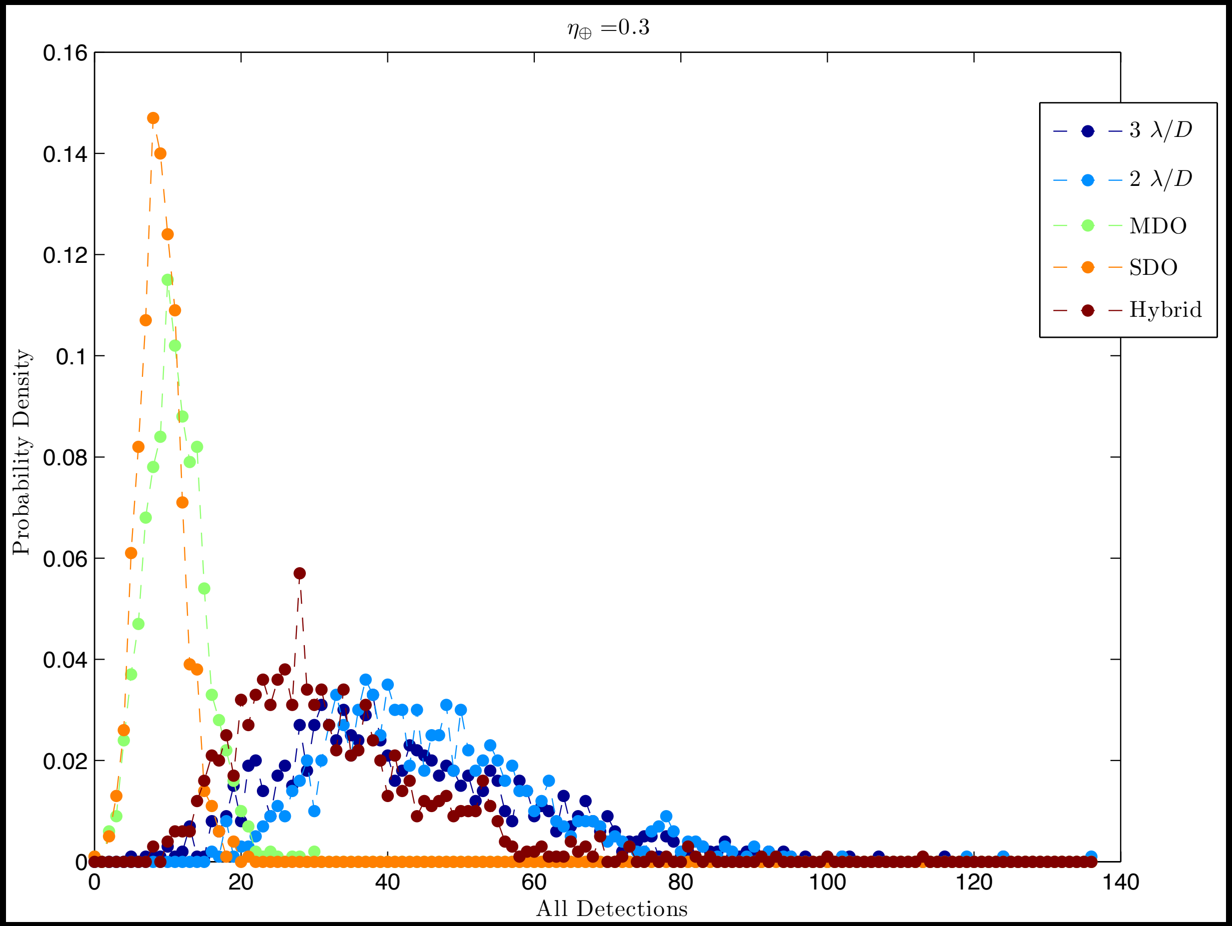
Optimized Mission Planning – EXOSIMS

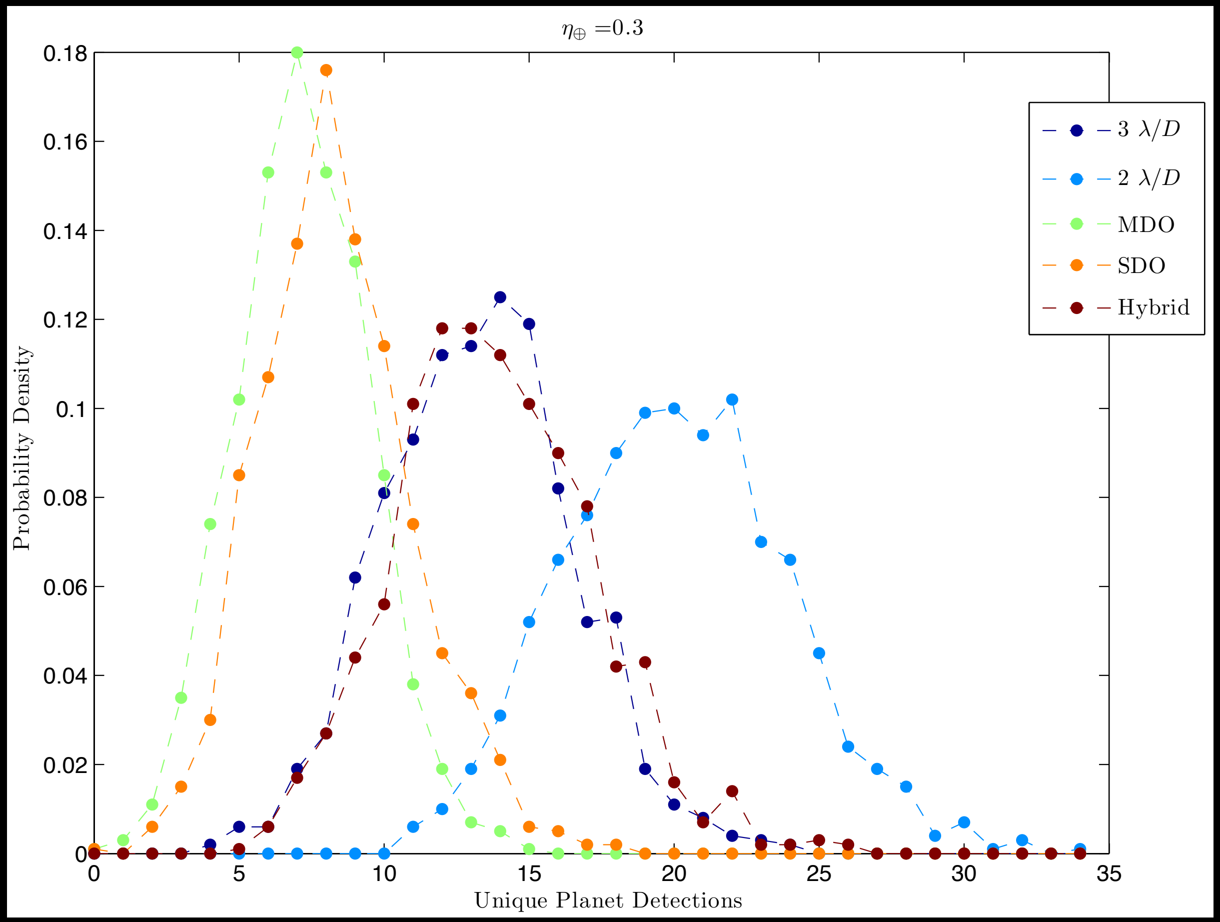


Monte Carlo simulation accounting for optimal integration times, fuel use, retargeting time, and keep out zones to balance completeness, spectroscopy, revisits, and number of targets.

Soto, et al., 2019

4 m telescope - Example Yield Results

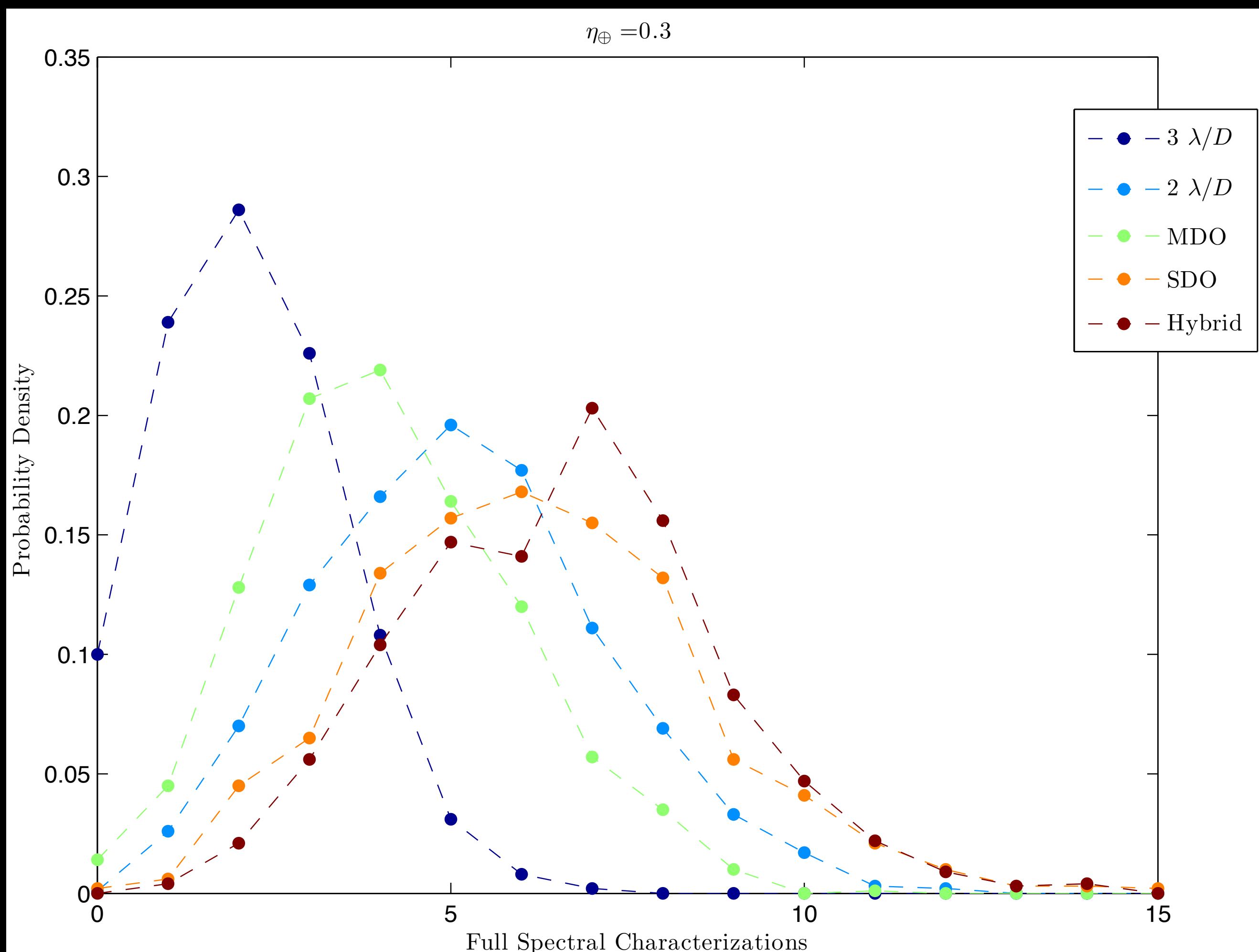


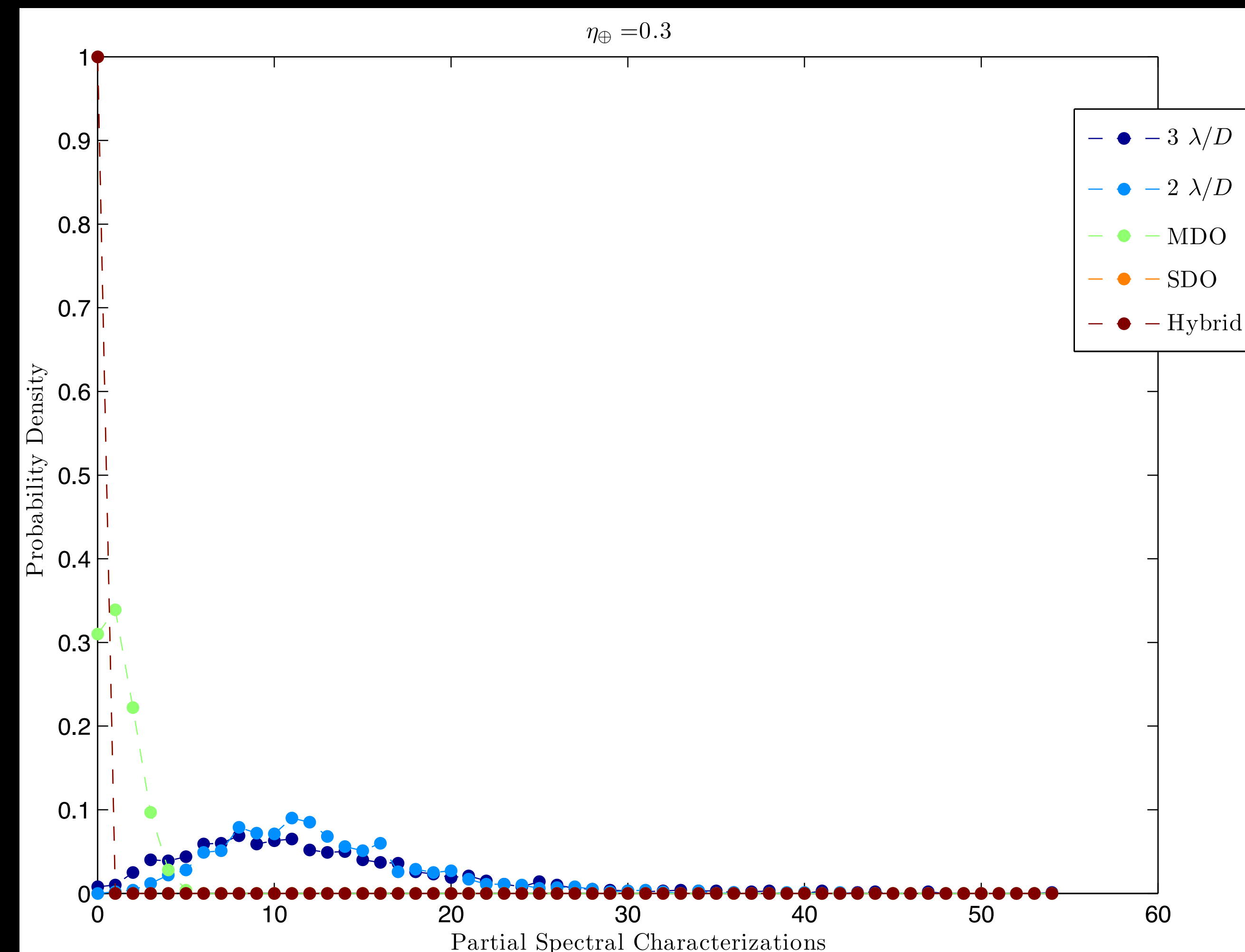


Sample comparison of probability distributions of detecting an Earth using a coronagraph with IWA of 2 and 3 lambda/D, a multi- and single-distance starshade and a hybrid mission with both coronagraph and star shade (such as HabEx) with a 4 m telescope.

Savransky 2010

4 m telescope - Full and Partial Spectra





- 2 l/D coronagraph is necessary to get any spectra
- 3 l/d has non-negligible probability of zero planets.
- Number of full spectra for coronagraph limited by red end (1000 nm)
- SDO & MDO close in performance
- Hybrid best performance but assumes 3 l/D coronagraph is possible

Thank You