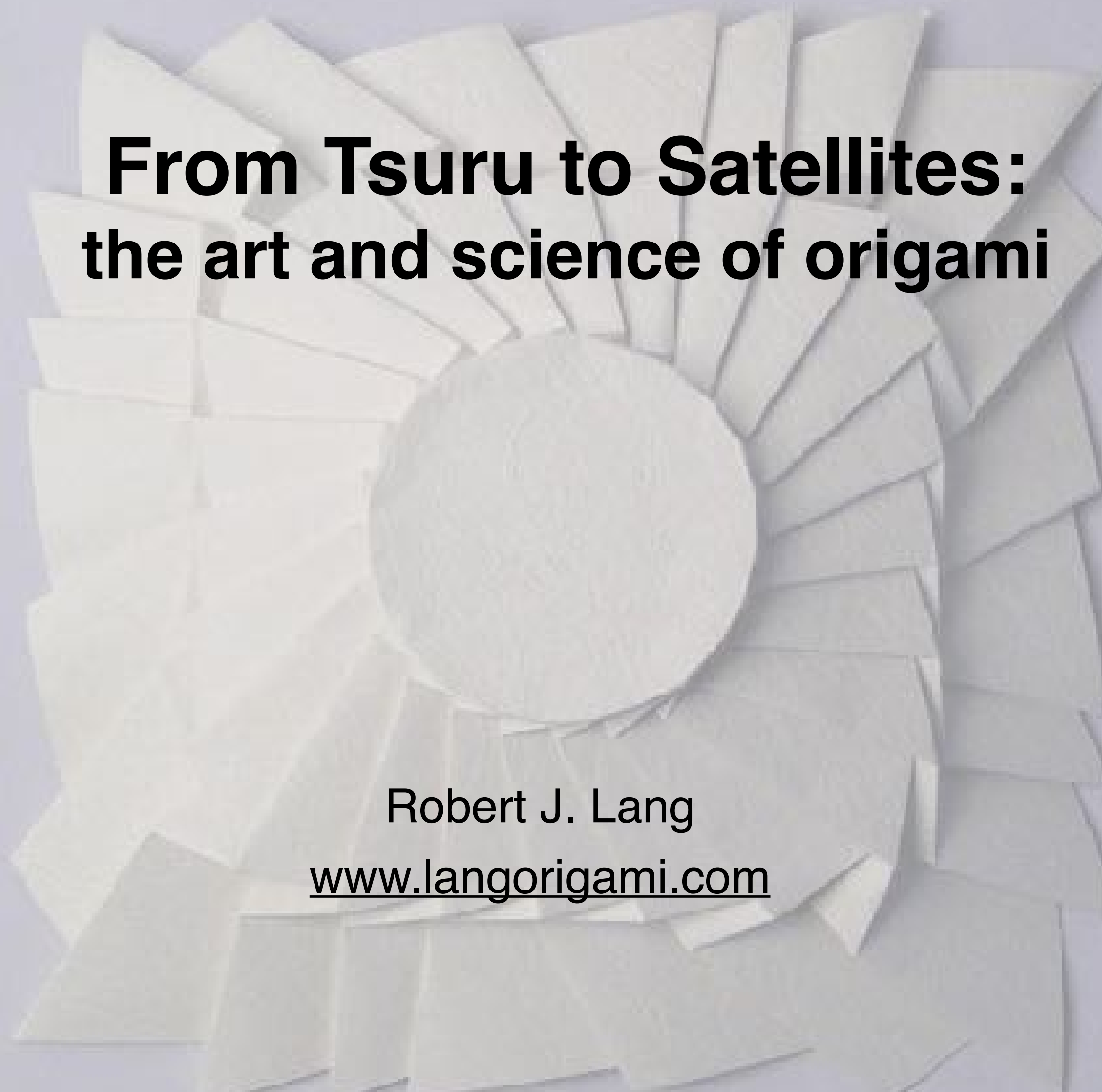


# From Tsuru to Satellites:
## the art and science of origami


Robert J. Lang

www.langorigami.com

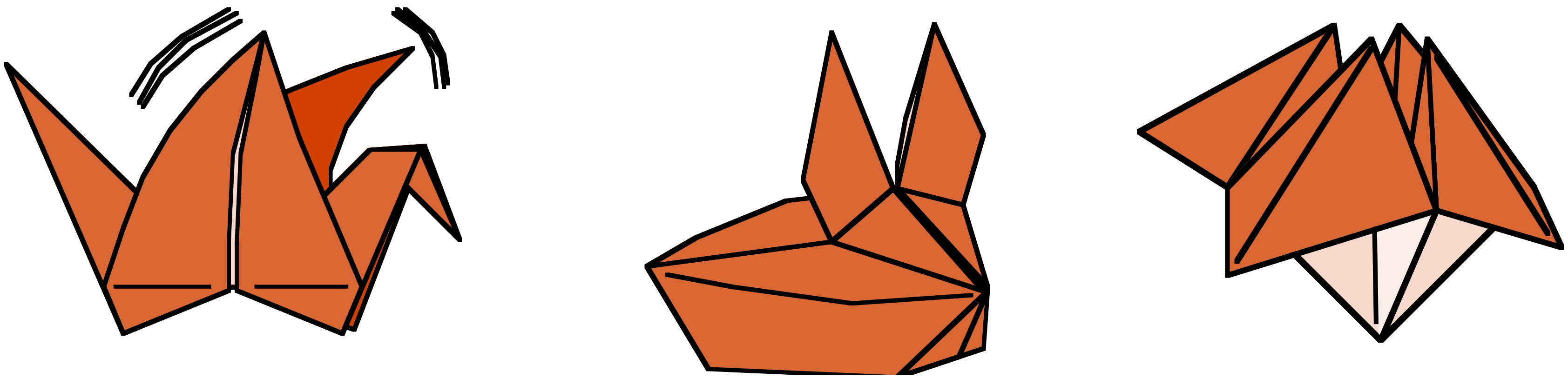


National Academies of Science

May 2024 | Washington DC

# What is Origami?

- Japanese paper-folding.

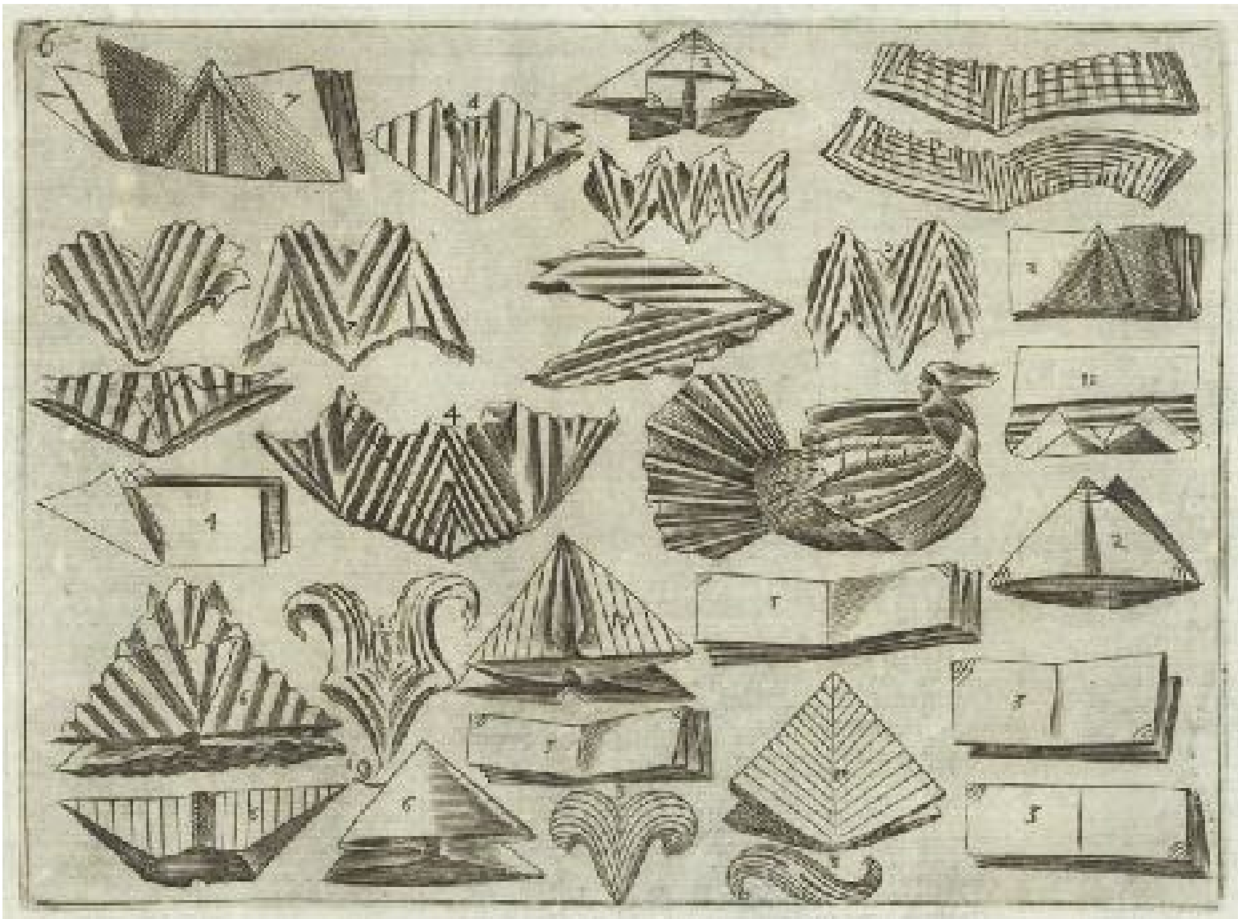


# History

- Paperfolding is ancient and a multi-cultural practice.

# Europe

**Li Tre trattati** 1639 by **Messer Mattia Giegher Bavaro**



# China

折纸图说 (zhe zhi tu shuo), 1914
by GUI Shaolie

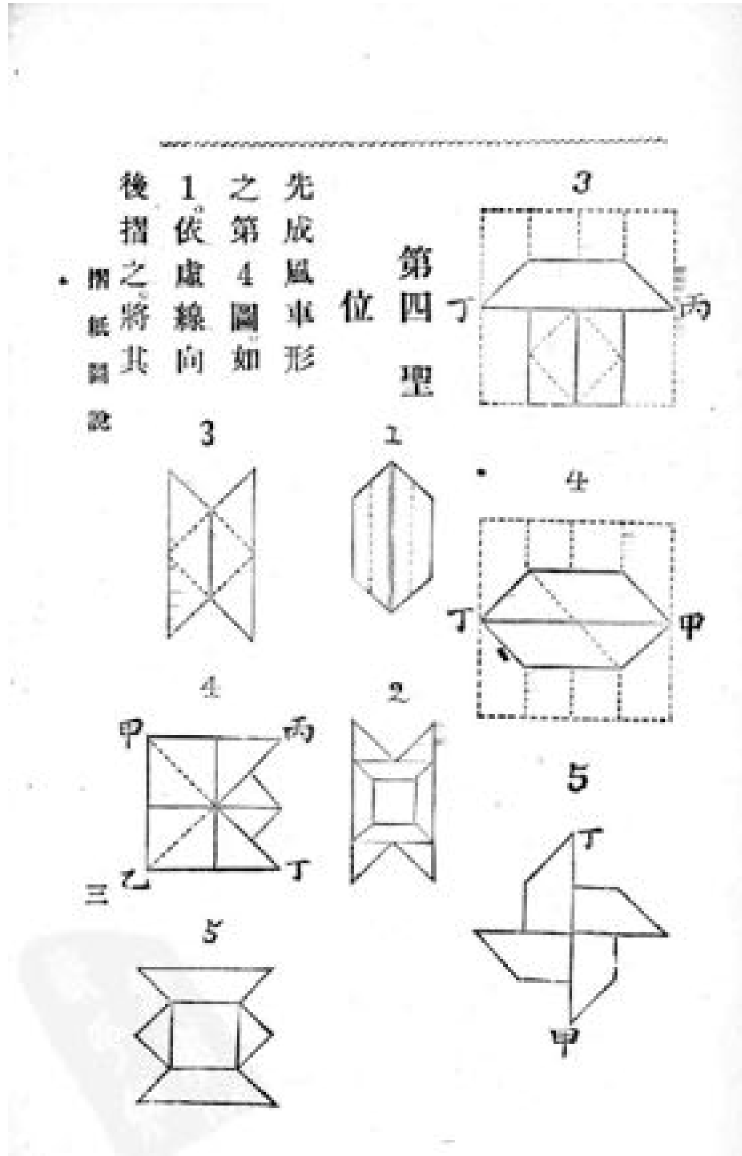
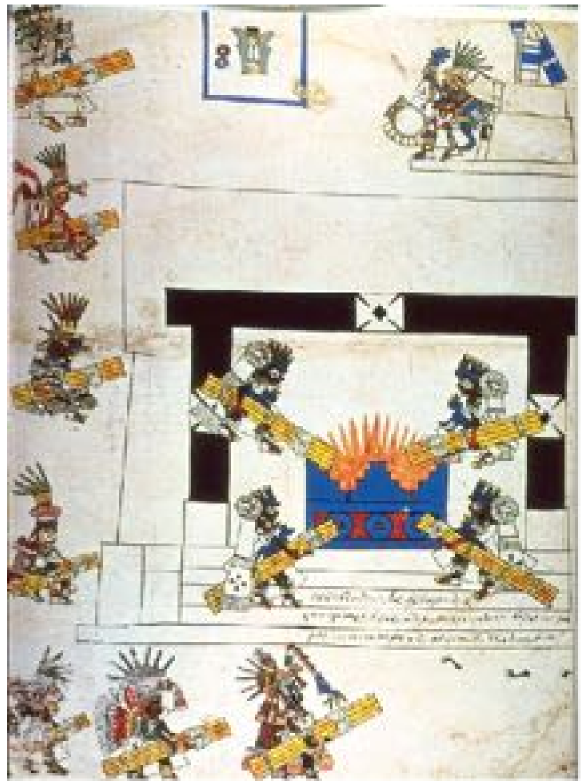



Image courtesy of Xiaoxian Huang

# Mesoamerica

- Aztec folded headdress from the Codex Borbonicus, ~1500s



Images courtesy of Mary Miller, Yale University

# Early Origami in Japan

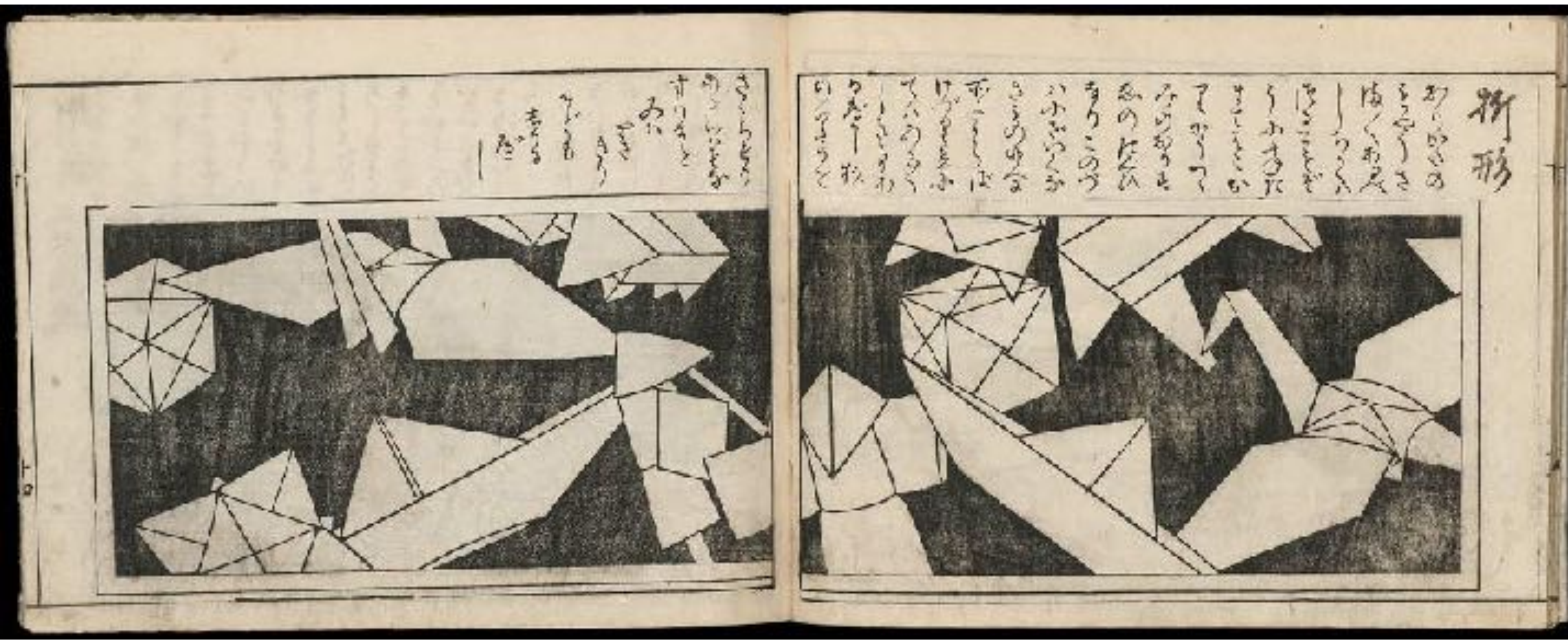
- Origami circa 1797.

# Oldest known crane

- A “kozuka” (samurai sword accessory) decorated with origami cranes
- Artist and style suggest ~1600 CE

# Earliest (but not first)

- Japanese book from 1734: Crane, boat, table, “yakko-san”
- By 1734, it is already well-developed



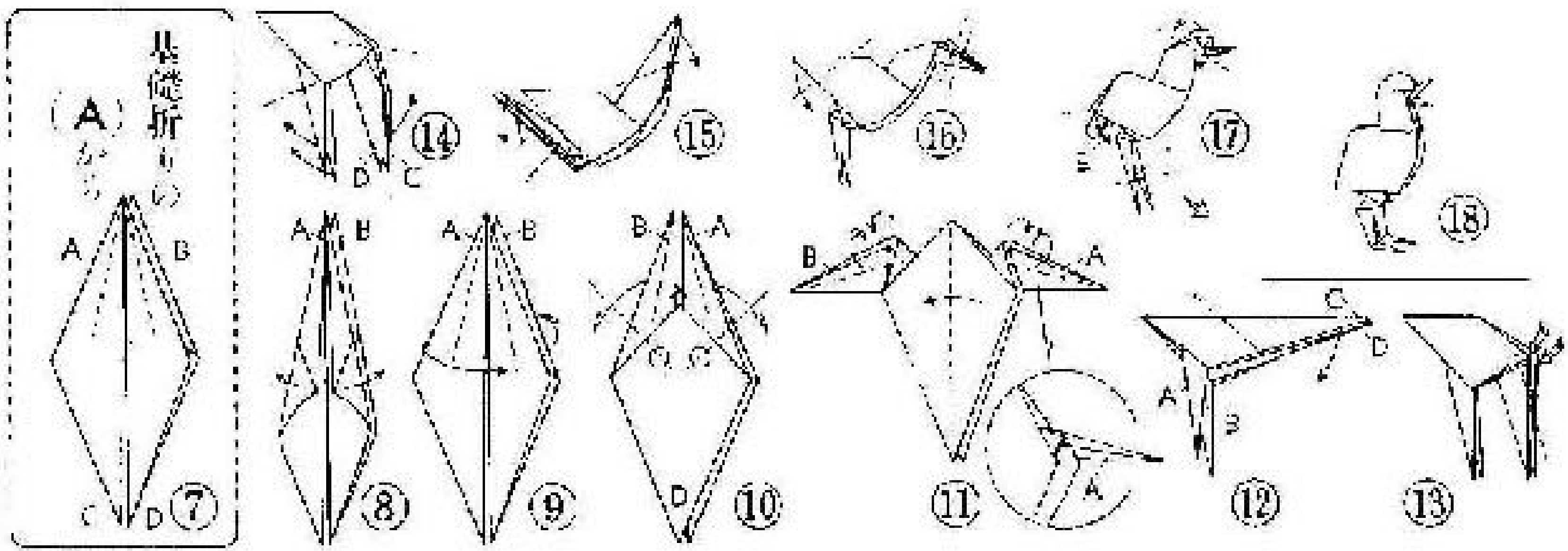
# Modern Origami

- Akira Yoshizawa (1911-2005)
- Inspired a worldwide renaissance of origami

# Modern Origami

ROBERT J. LANG ORIGAMI

- Yoshizawa: new creations plus a common language.




A. Yoshizawa, *Origami Dokuhon I*

# Origami Today

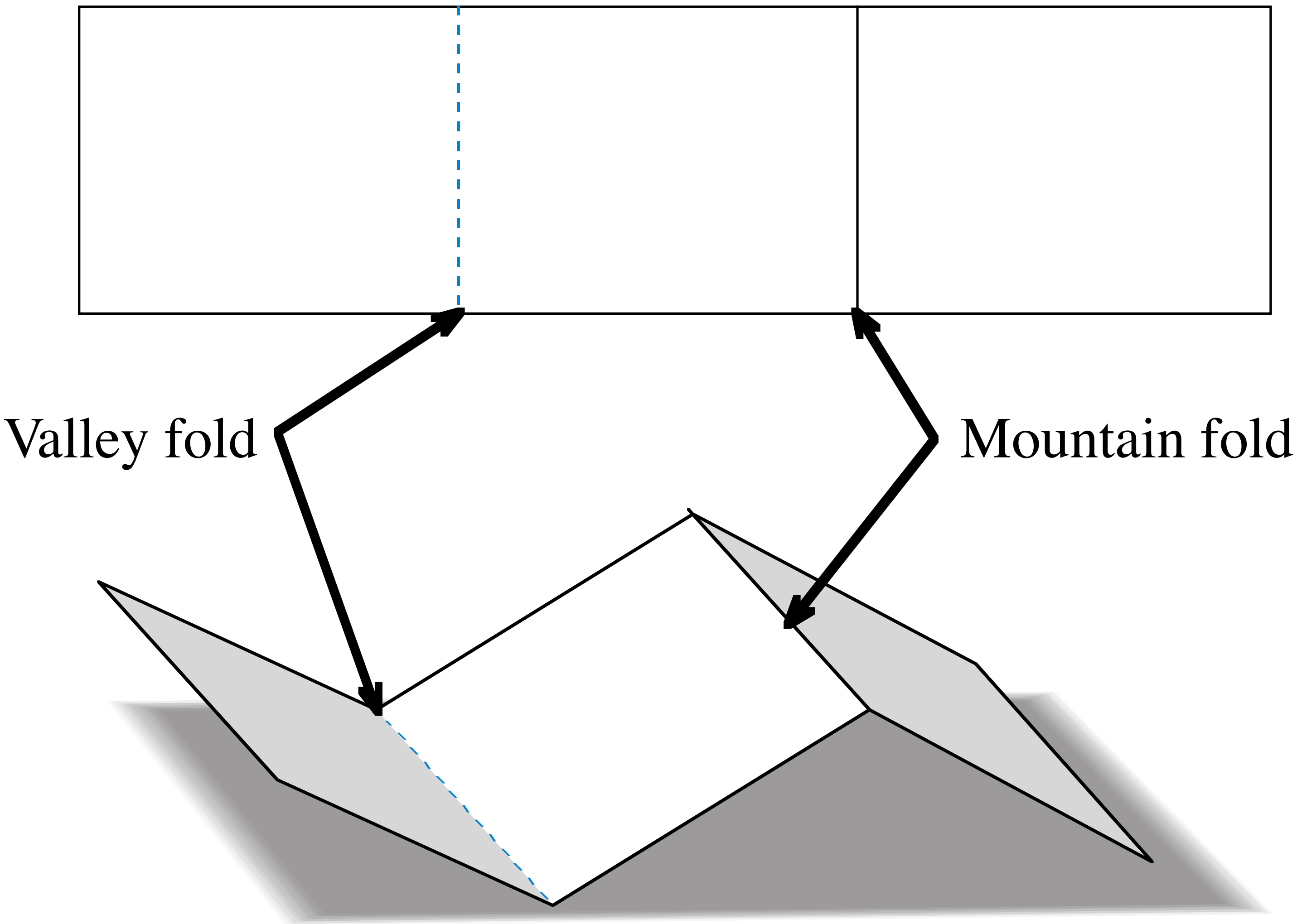
- “Black Forest Cuckoo Clock,”
- One sheet, no cuts

# Ibex

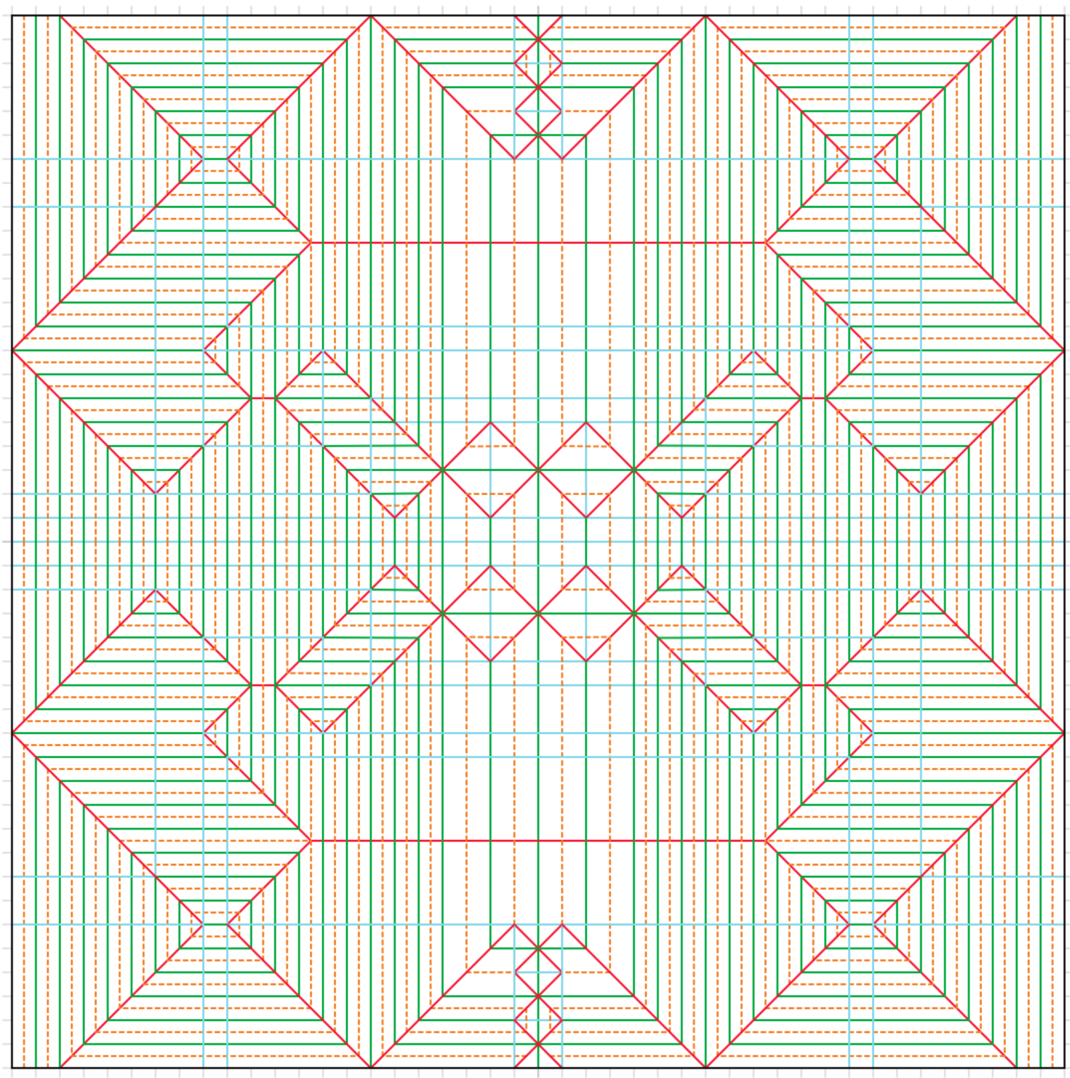
# What Changed?

- Math!

# Basic Folds of Origami



# Crease Patterns

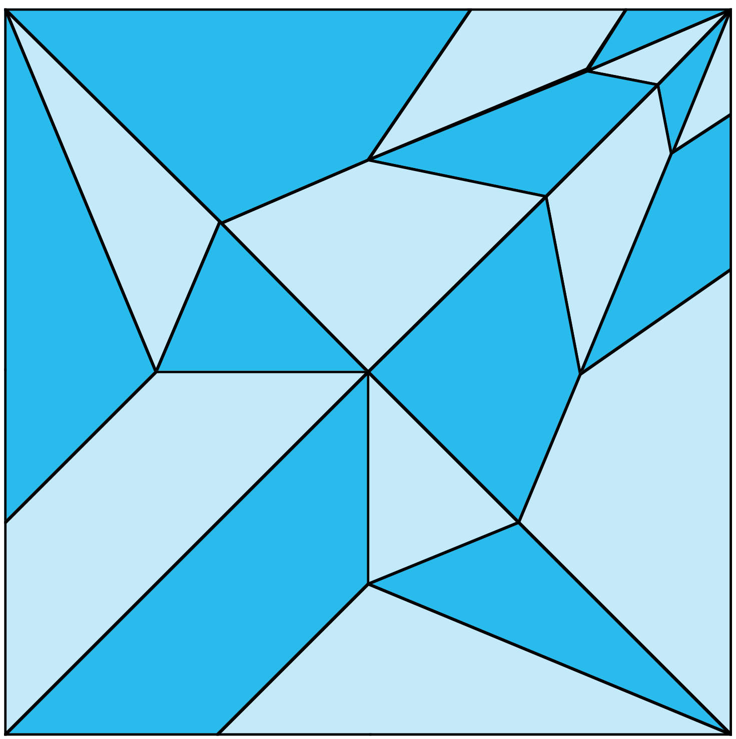


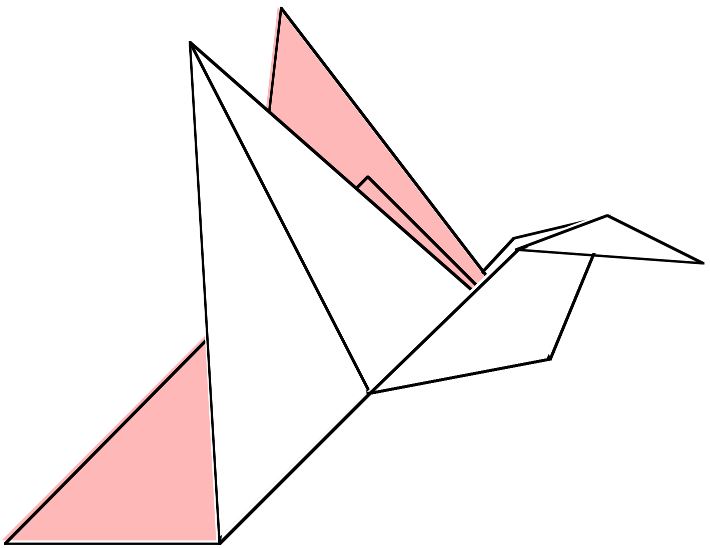
# Crease Patterns

- The design of an origami figure is encoded in the crease pattern
- What constraints are there on such patterns?

# Properties of Crease Patterns

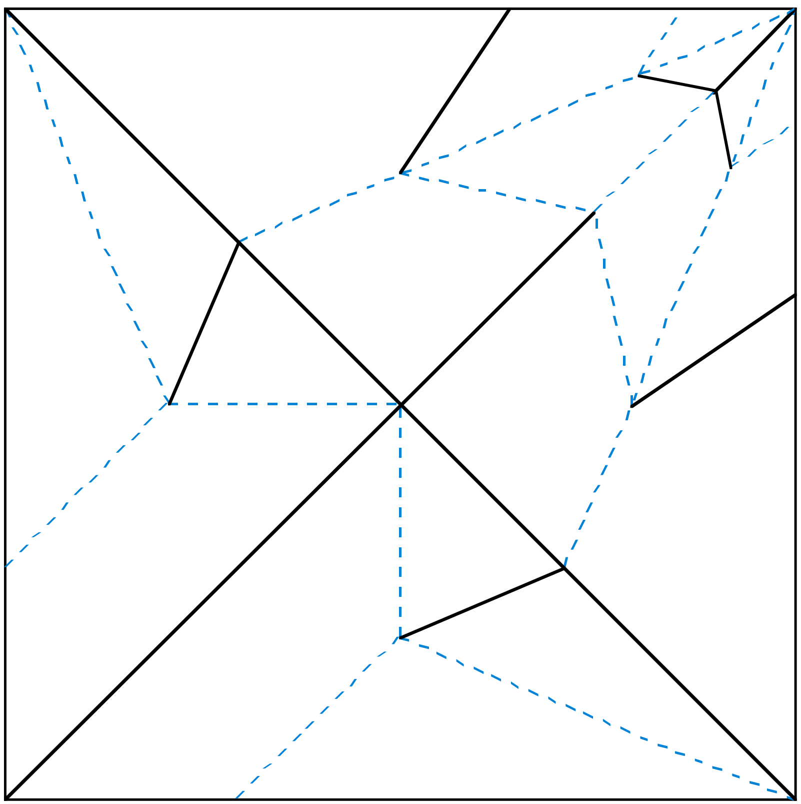
- 2-colorability

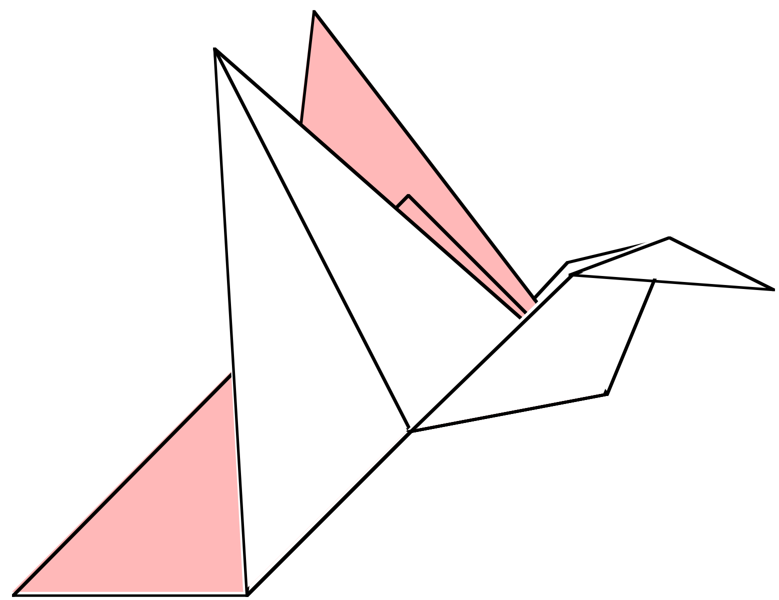




# Mountain-Valley Counting

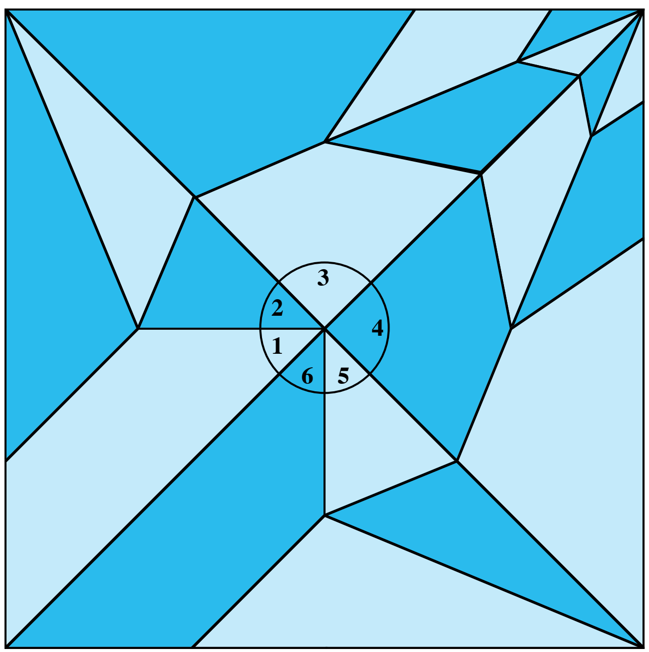
- Interior vertices: M – V = ±2

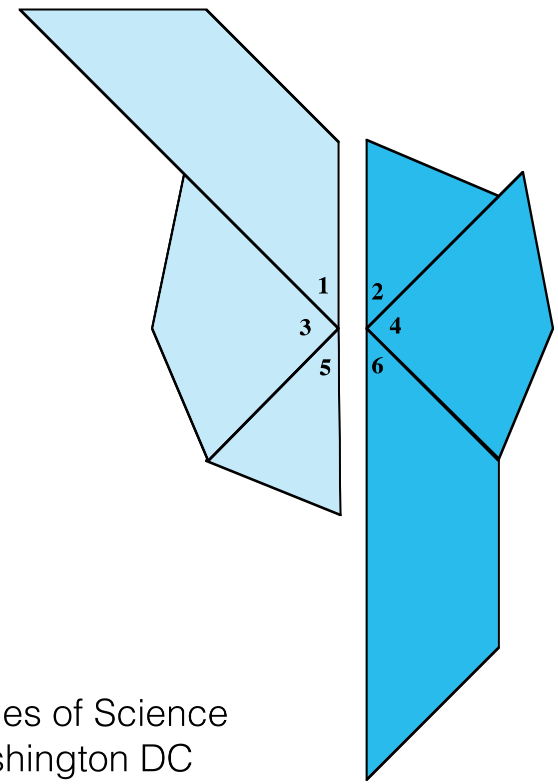




# Angles Around a Vertex

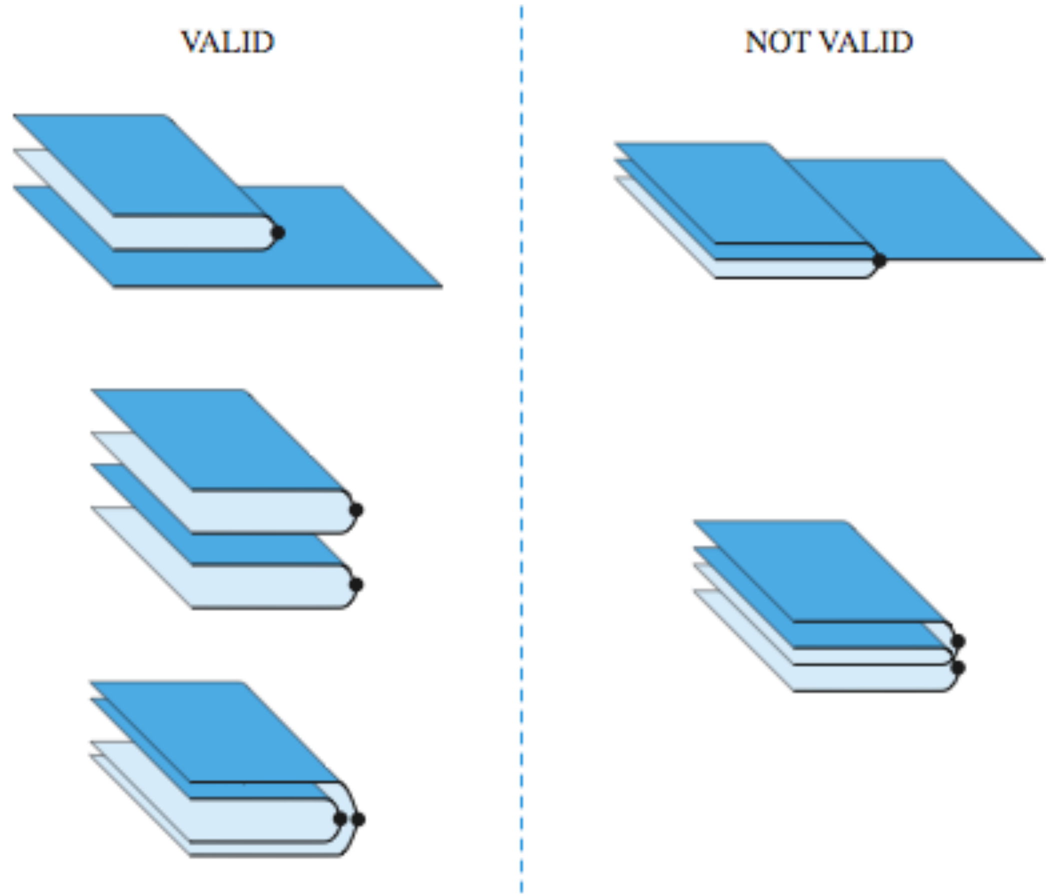
- Alternate angles around a vertex sum to a straight line



# Layer Ordering

- No self-intersection at overlaps.



# Flat-Foldability

- A crease pattern is “flat foldable” iff it satisfies:
  - 2-colorability (easy, all vertices even degree)
  - Maekawa Condition (M-V parity) at every interior vertex
  - Kawasaki Condition (Angles) at every interior vertex
  - Justin Conditions (Ordering) for all facets and creases

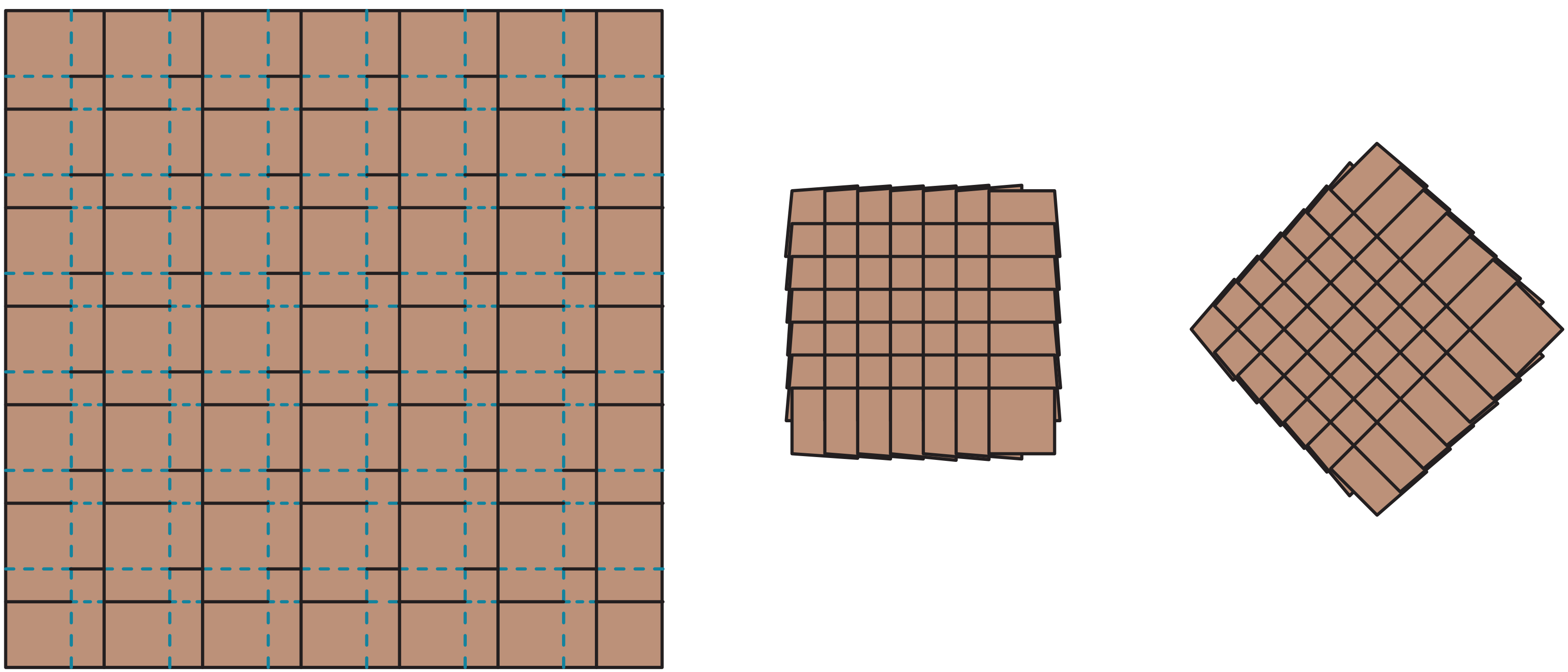
That’s all we need to describe flat origami.

# But is it useful, or just fun?

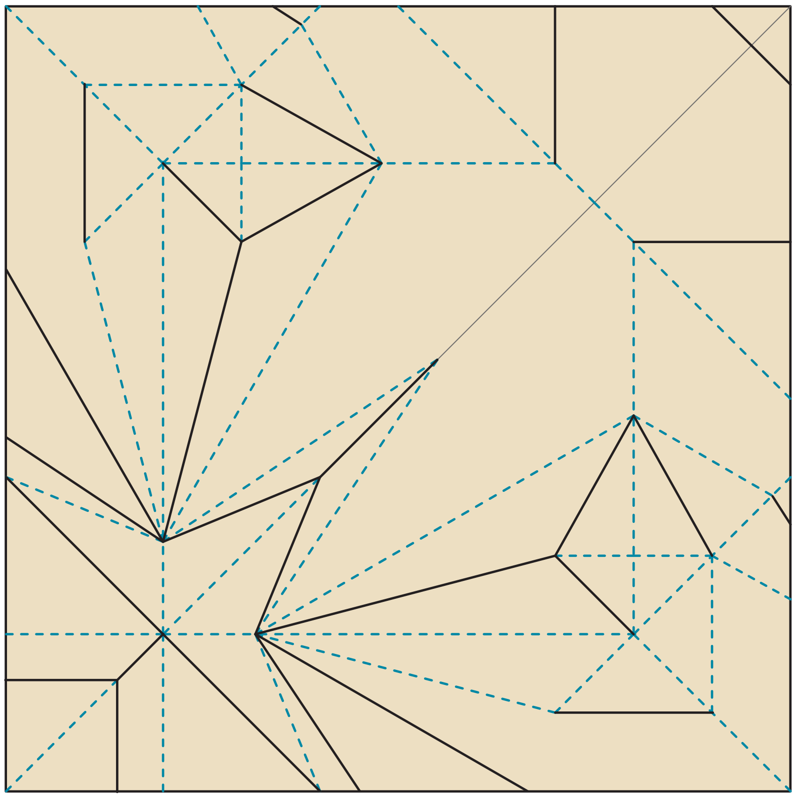
- Flat-foldability rules (math)…
- lead to crease pattern matching rules (application)…
- and thus, the generation of beauty (art)…
- and even practical functional objects ($$$)!

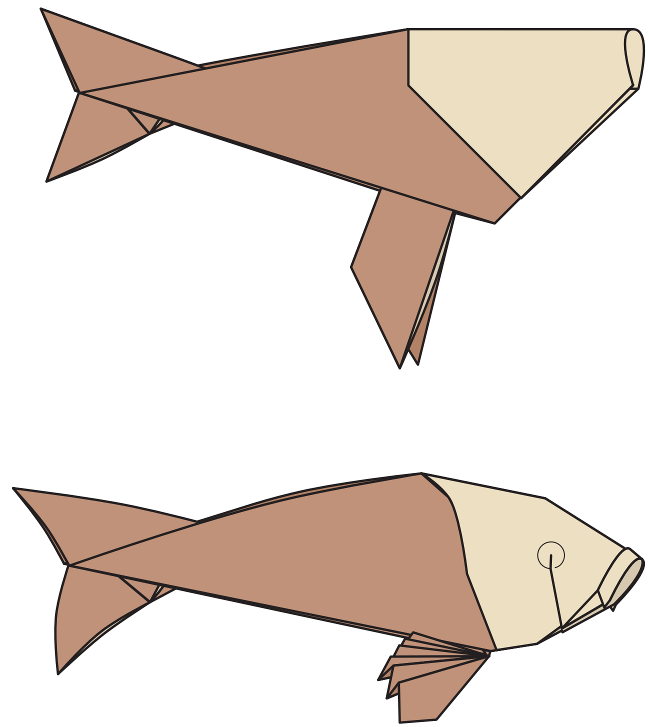
# Textures

- Patterns of pleats
- Integrate with existing forms



# The recipient form





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Western Pond Turtle

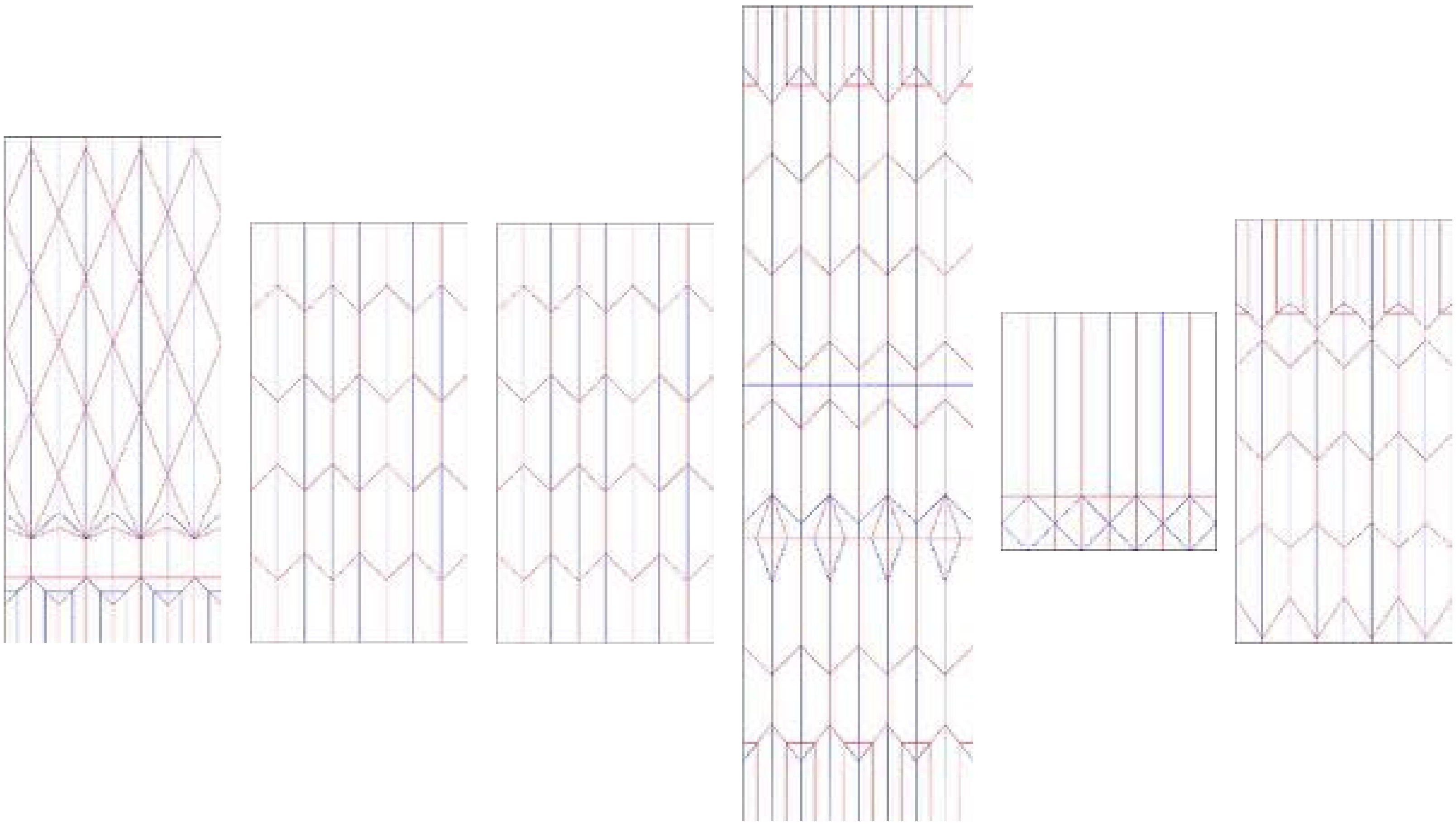
# Rattlesnake

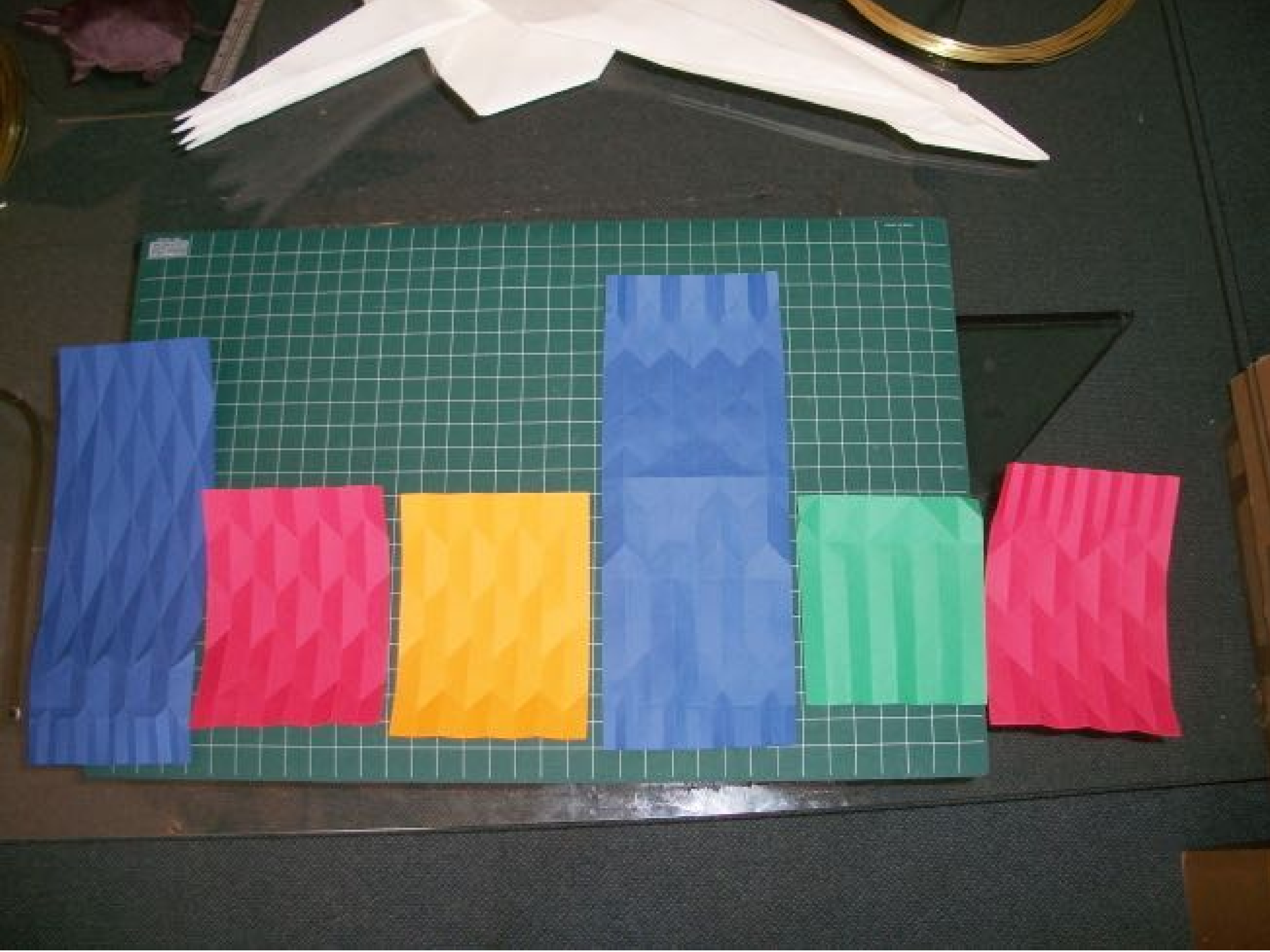
# Rattlesnake II

# On demand

- A geometric design
  - five letters for a corporate logo
  - one week to design and fold

# Six crease patterns...





# March 14, 2012

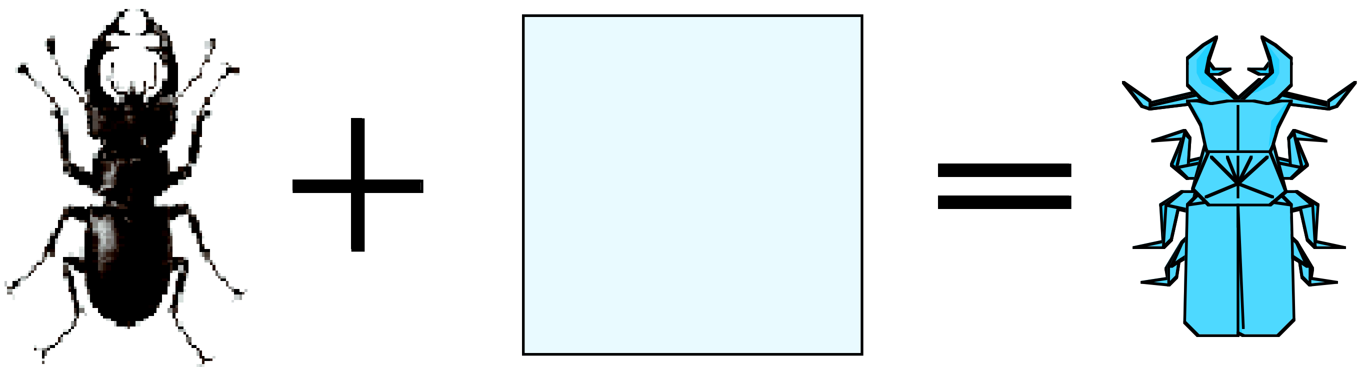
ROBERT J. LANG ORIGAMI

- The Google Doodle honored Akira Yoshizawa

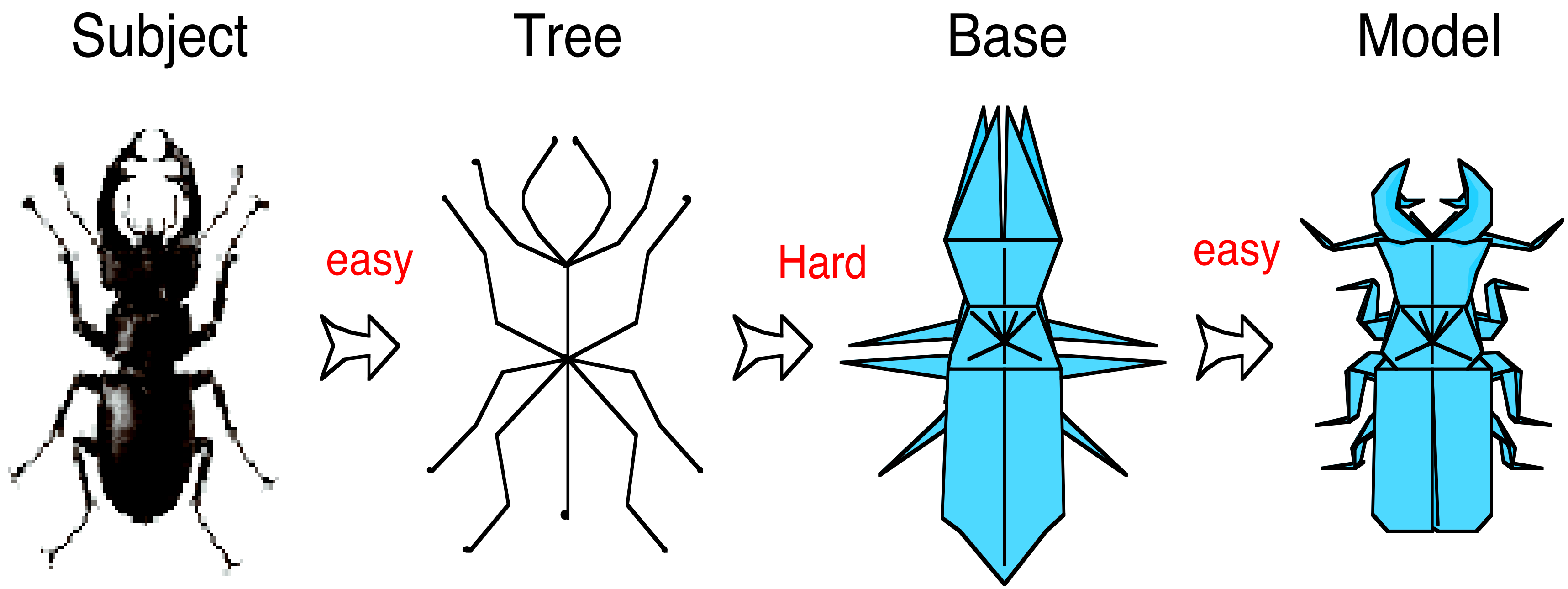

read the whole story at:
http://tinyurl.com/yoshizawa-doodle

# Origami design

- The fundamental equation:
- given a desired subject, how do you fold a square to produce a representation of the subject?

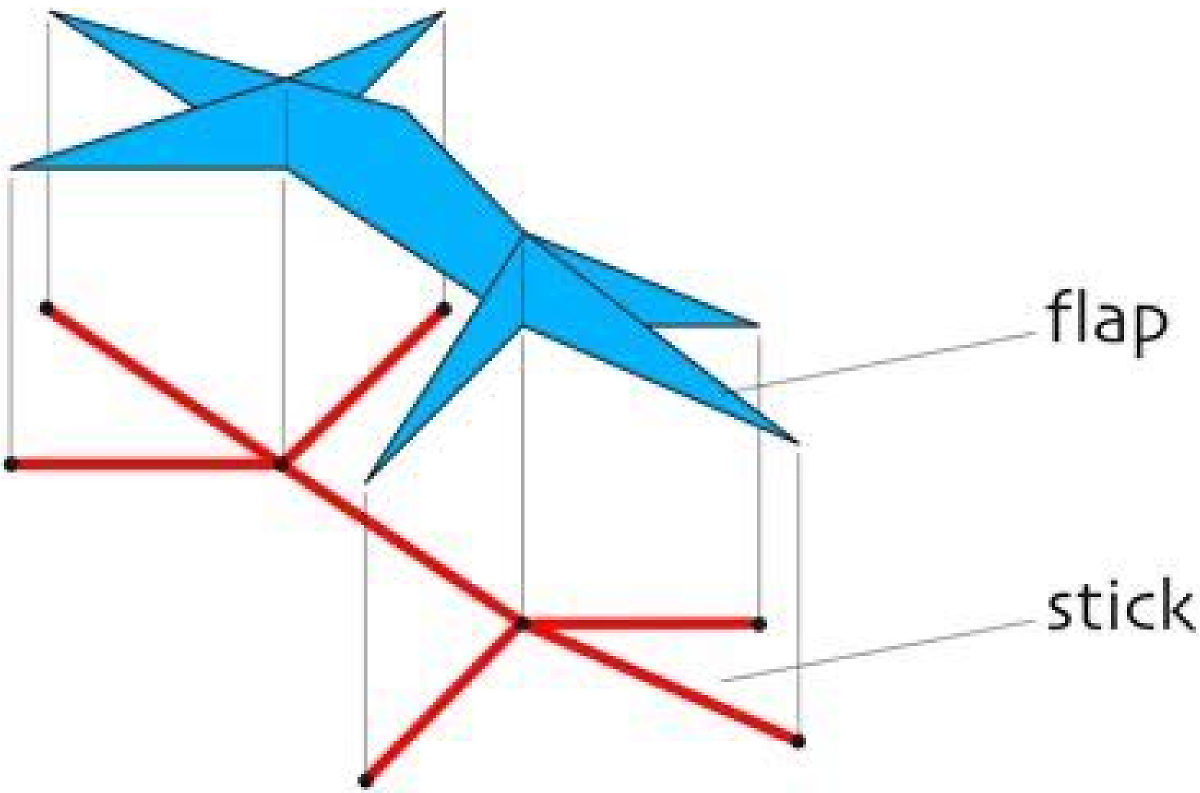


# A four-step process

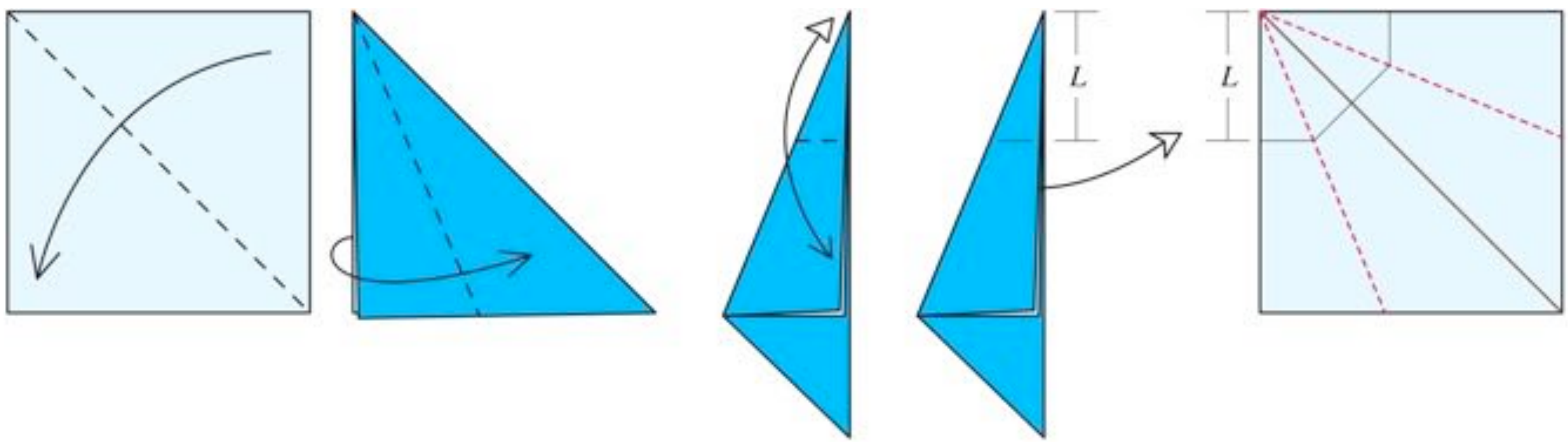


# The hard step

- How do you make a specified bunch of flaps?

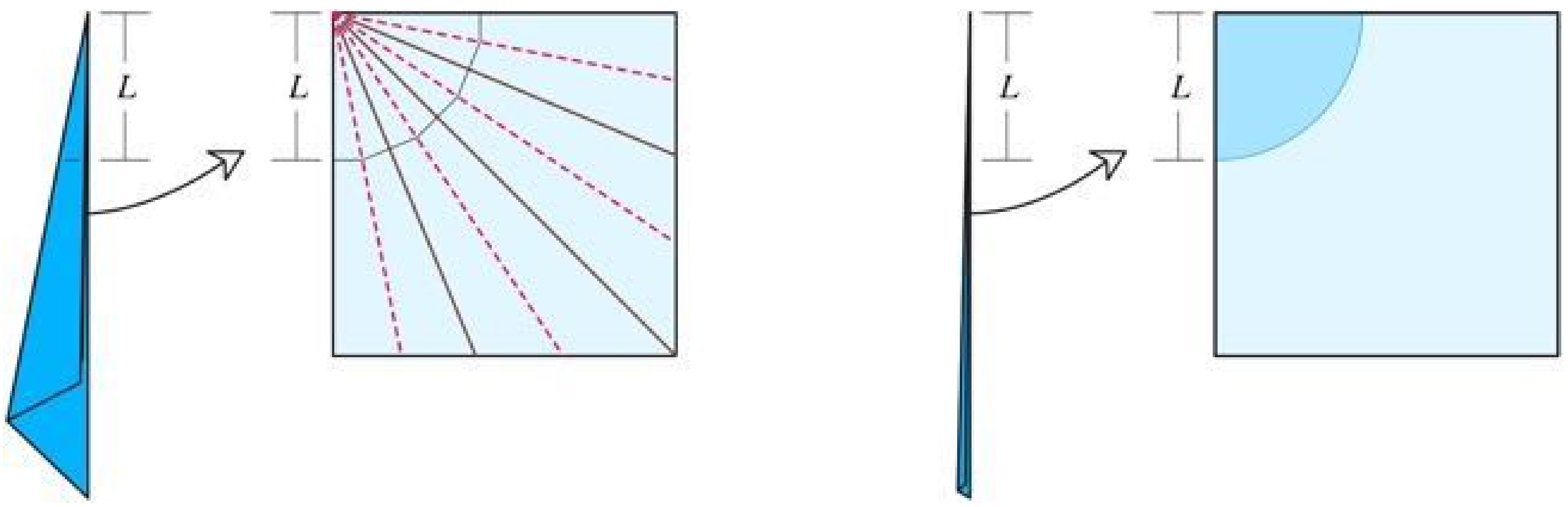


# How to make a flap



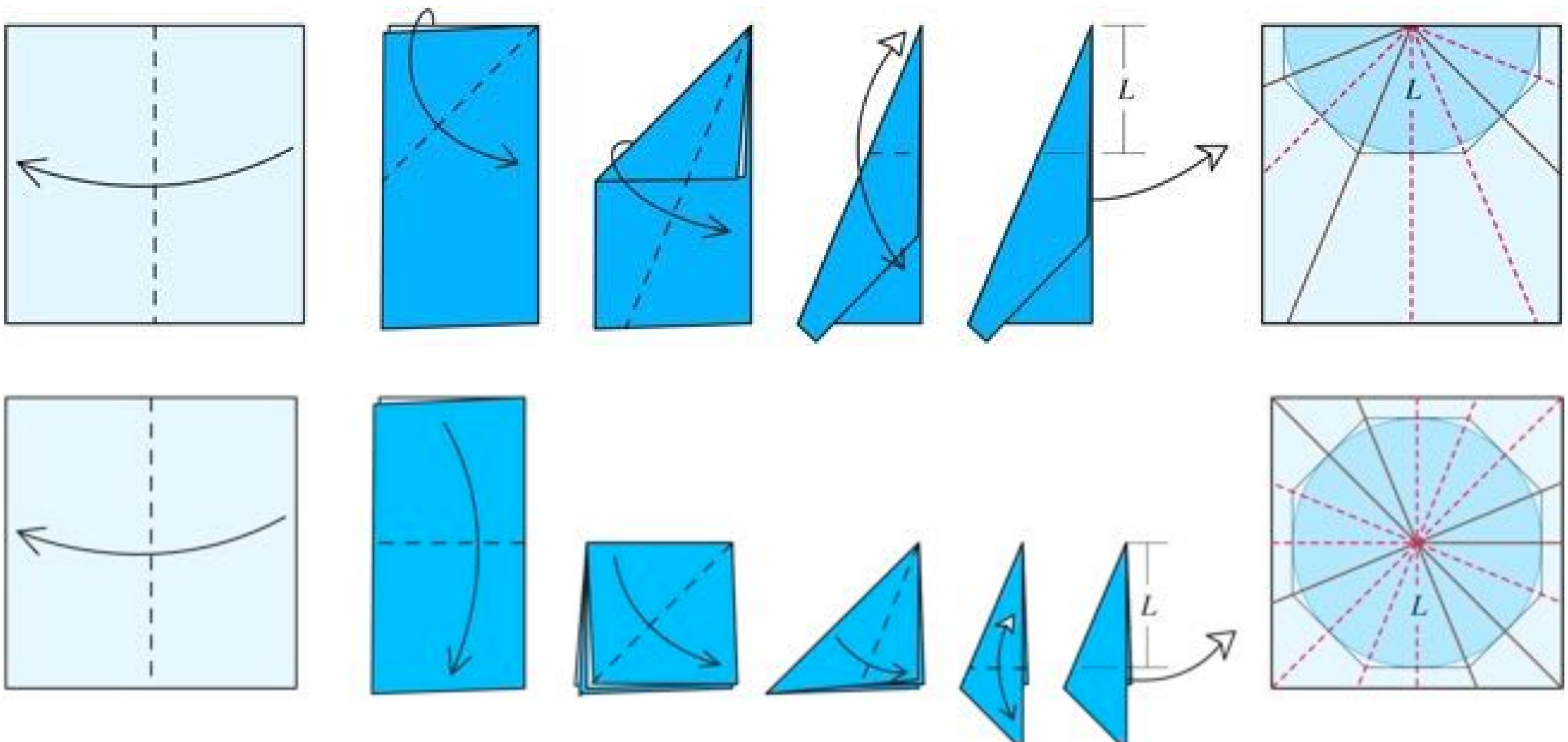
# Limiting process

- Skinnier flap leads to…
- A (quarter) circle!



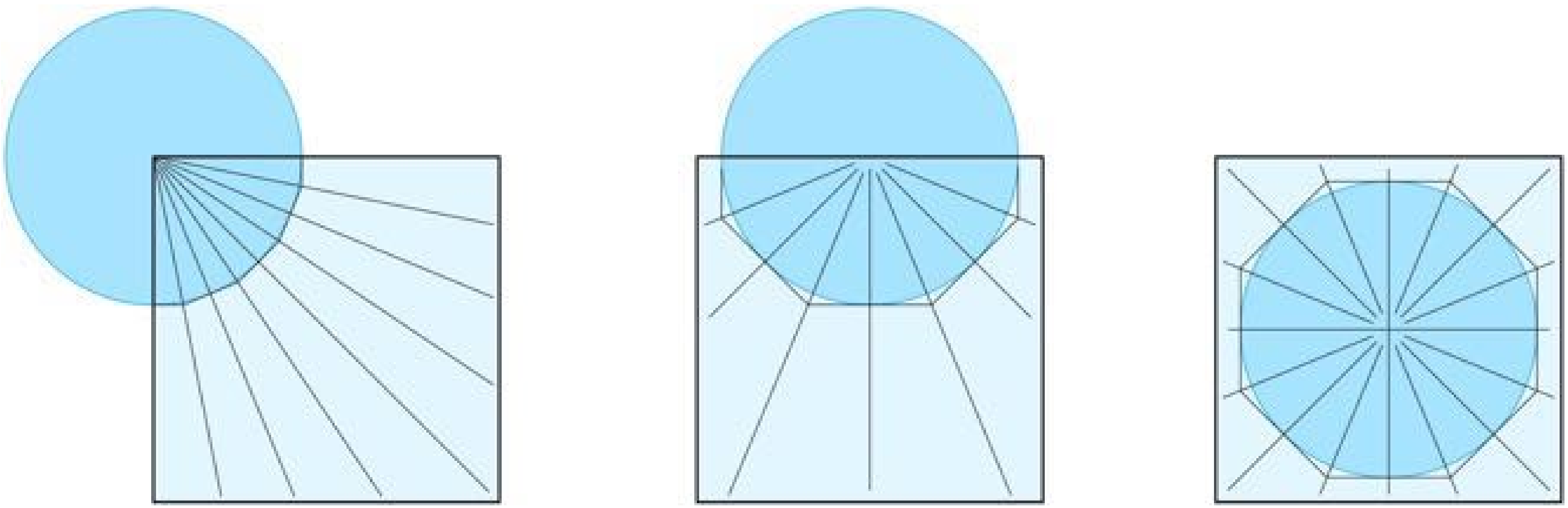
# Other types of flap

- Flaps can come from edges…
- …and from the interior of the paper.



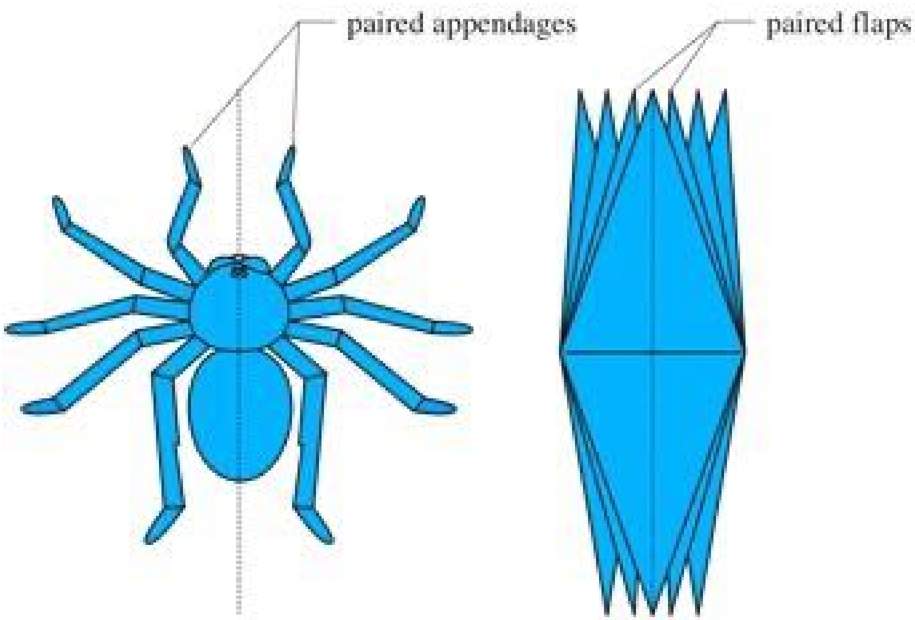
# Unify

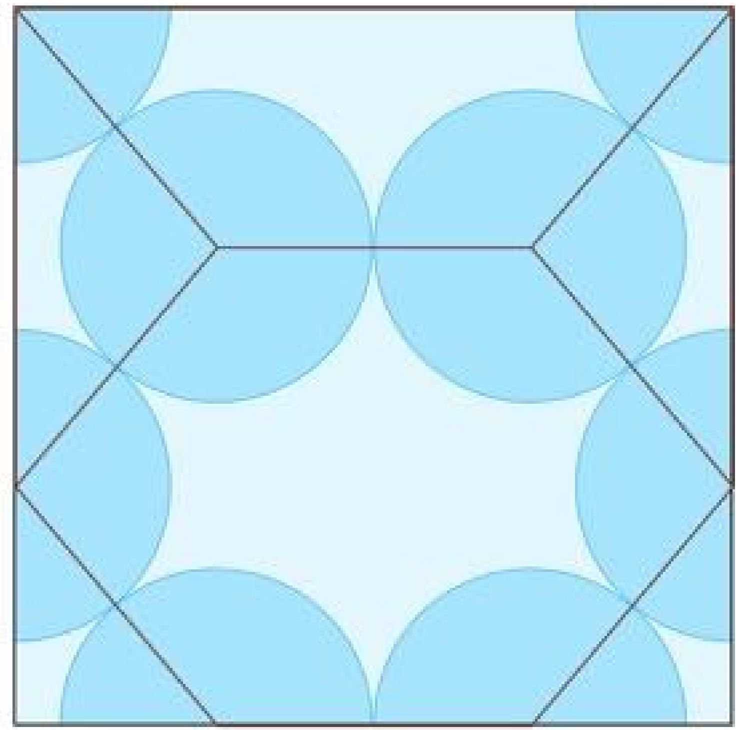
- They're all circles



# Circle Packing

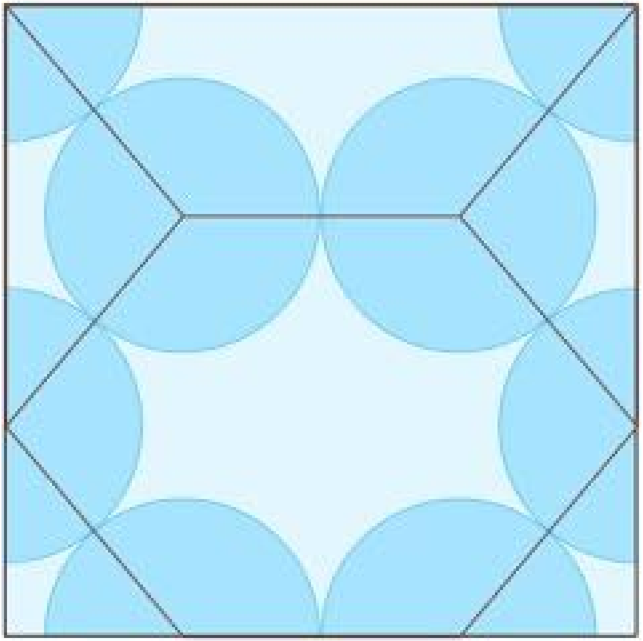
- Many flaps: use many circles.



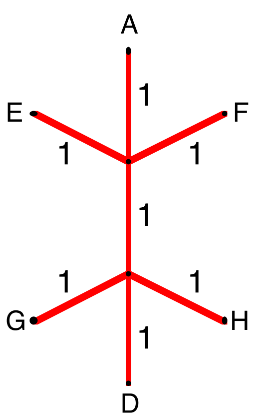
# Creases

- The lines between the centers of touching circles are always creases.
- But there needs to be more. Fill in the polygons, but how?

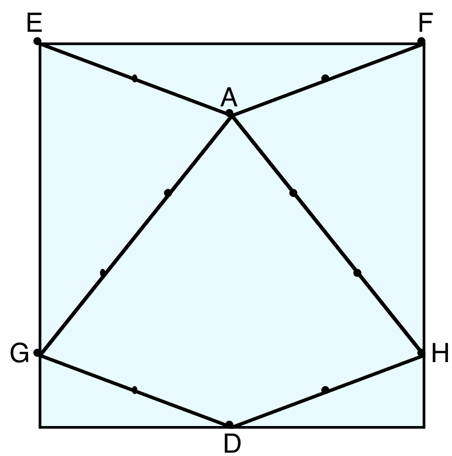


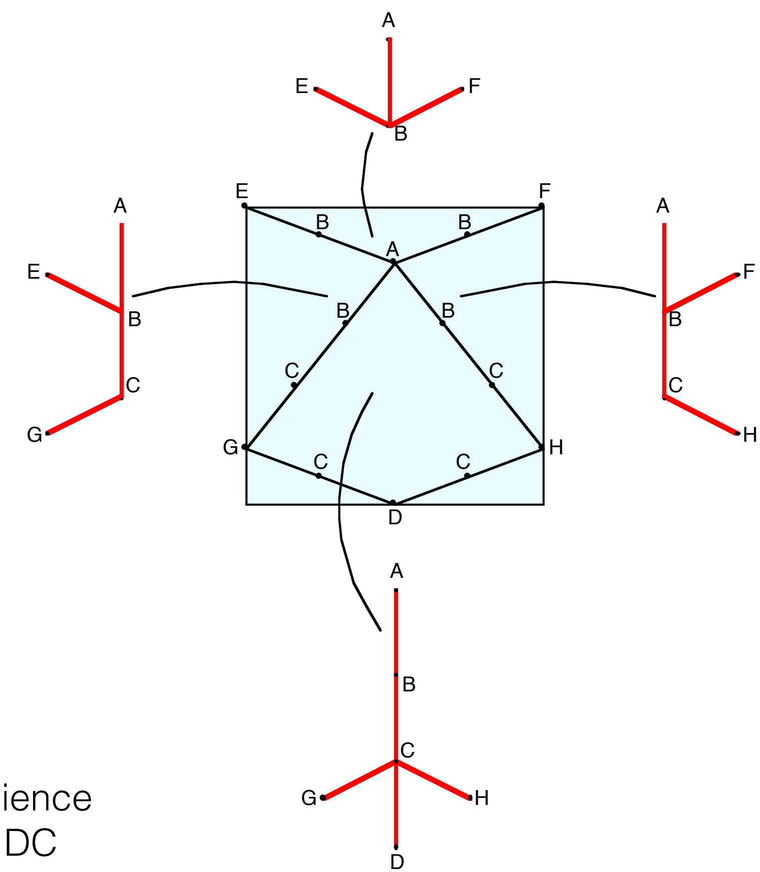
# Divide and conquer

- The creases divide the square into distinct polygons that correspond to pieces of the stick figure.



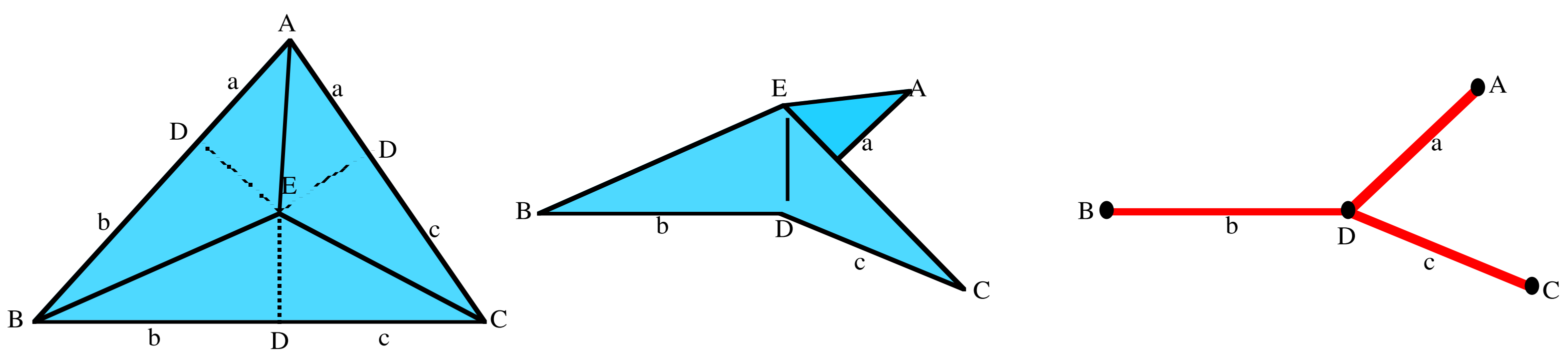

m=0.267



# Molecules

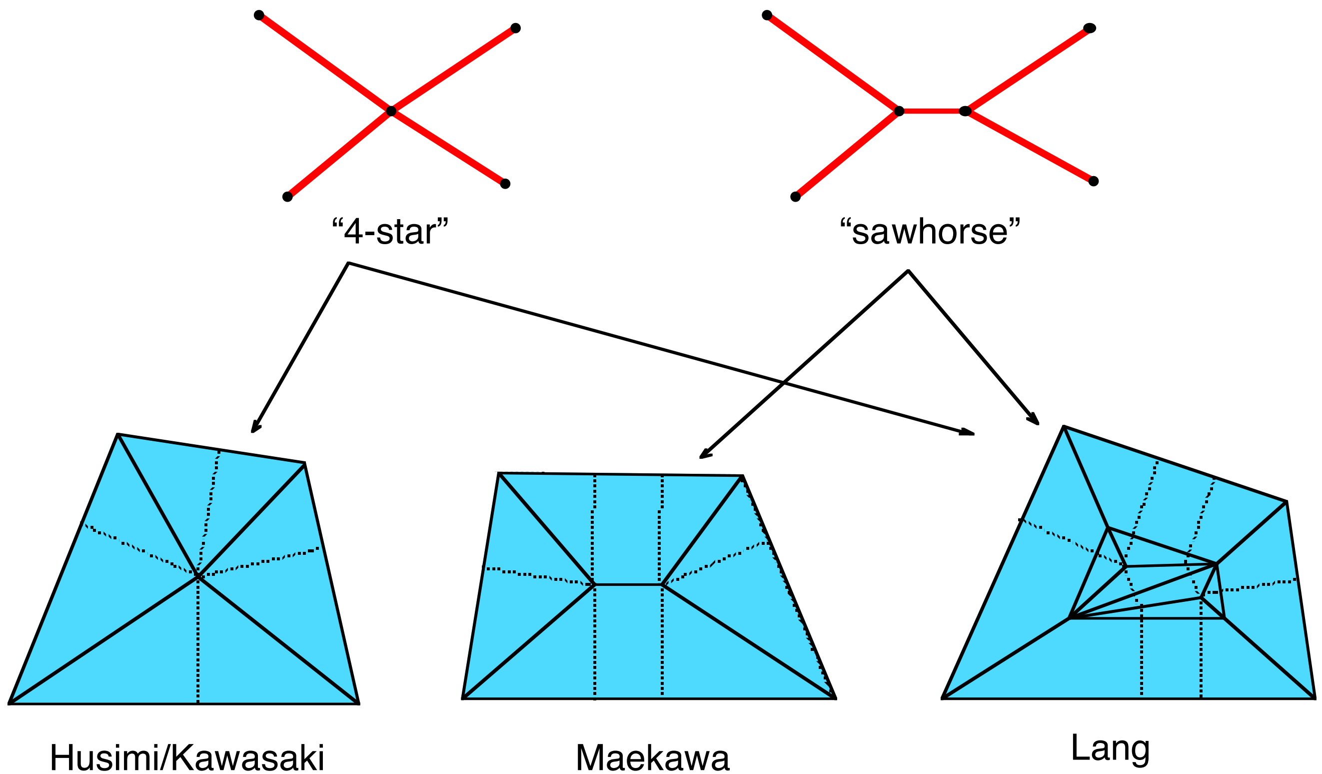
- Crease patterns that collapse a polygon so that its edges form a stick figure are called “bun-shi,” or molecules (Meguro)
- Different molecules are known from the origami literature.
- Triangles have only one possible molecule.




the “rabbit ear” molecule

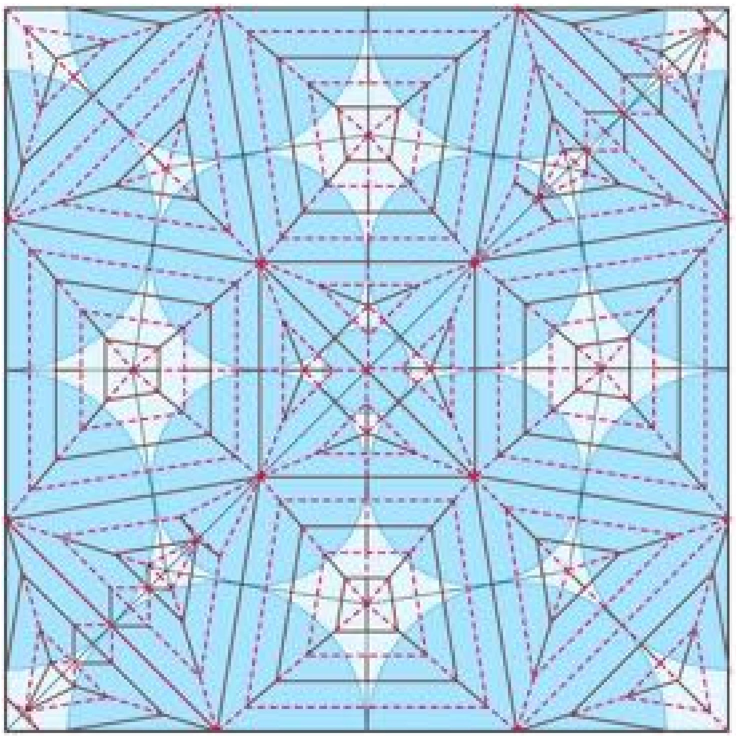
# Quadrilateral molecules

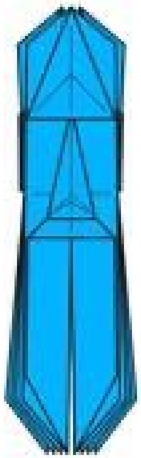
- There are two possible trees and several different molecules for a quadrilateral.
- Beyond 4 sides, the possibilities grow rapidly.

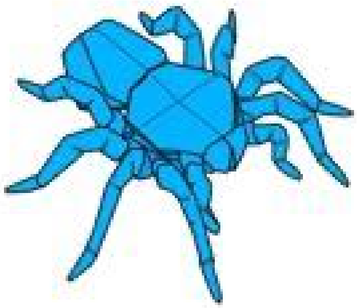


# Circles and Rivers

- Pack circles, which represent all the body parts.
- Fill in with molecular crease patterns.
- Fold!





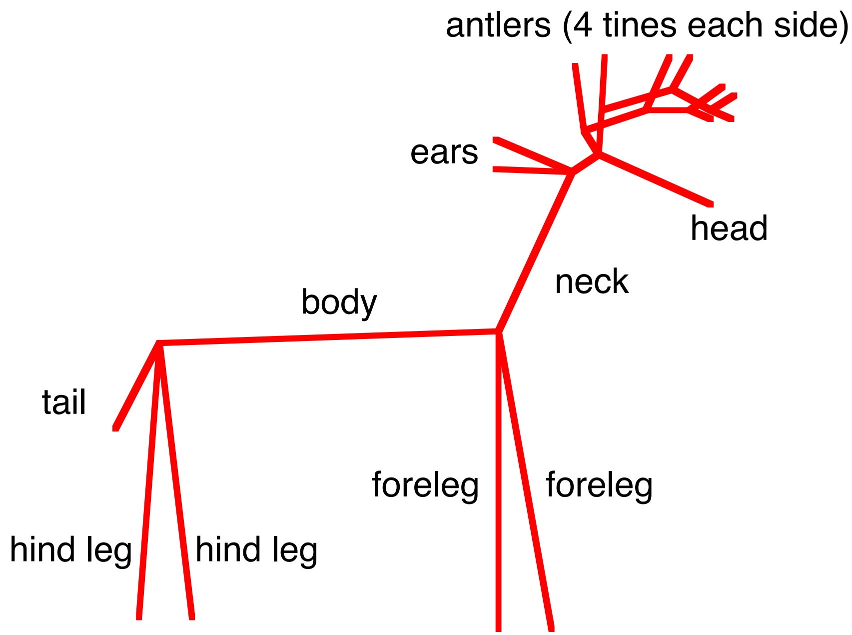


# Circle-River Design

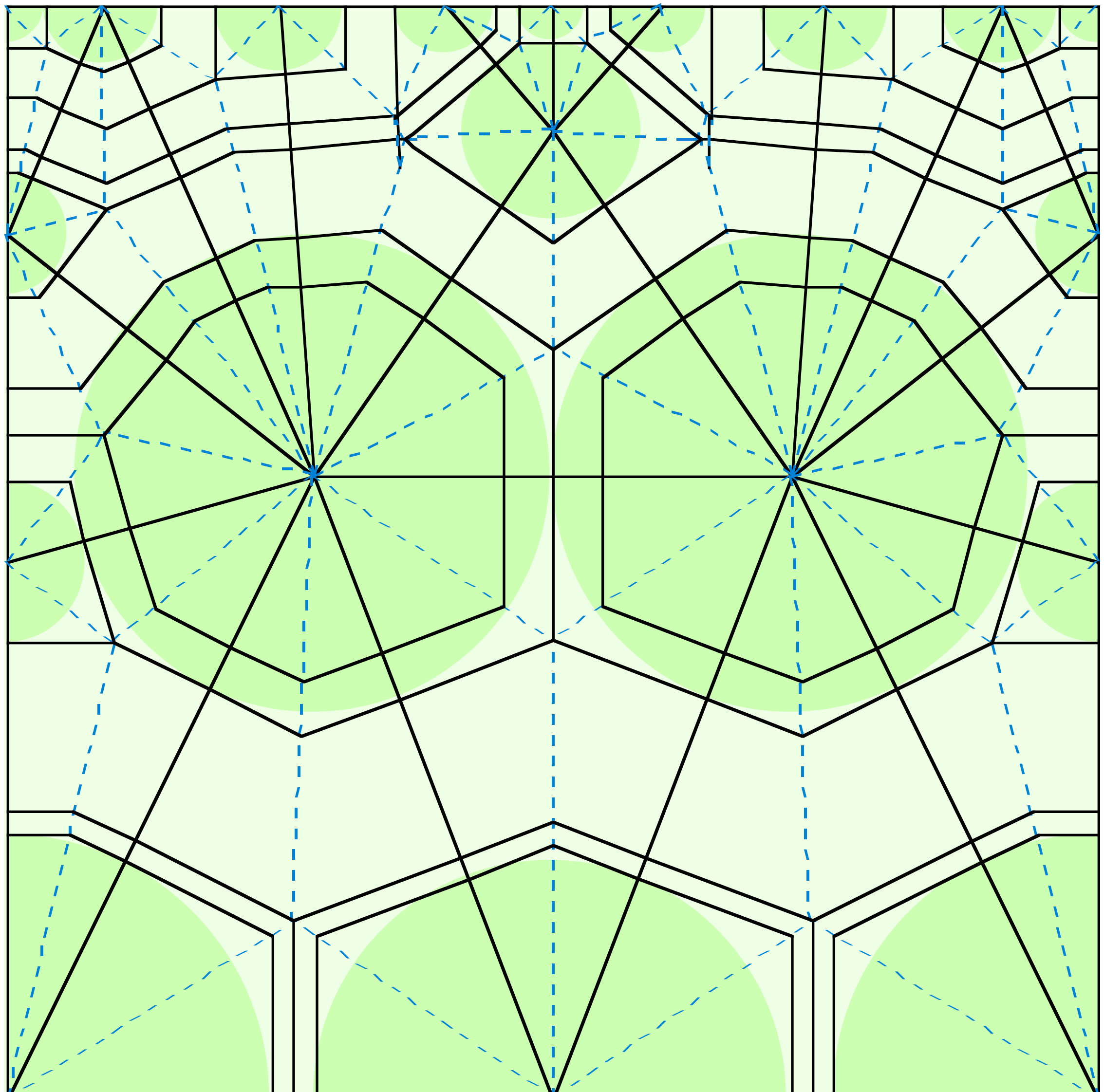
- The combination of circle-river packing and molecules allows an origami composer to construct bases of great complexity using nothing more than a pencil and paper.
- But what if the composer had more…
- Like a computer?

# Computer-Aided Origami Design

- 16 circles (flaps)
- 9 “rivers “ (connections)
- 200 equations!



# The crease pattern



# Roosevelt Elk

# Bull Moose

# TreeMaker Software

- Algorithms are described in
  - R. J. Lang, "A Computational Algorithm for Origami Design," *12th ACM Symposium on Computational Geometry*, 1996
  - R. J. Lang, *Origami Design Secrets* (A K Peters, 2003)
- Macintosh/Linux/Windows binaries and source available (free!) from
  - http://www.langorigami.com/treemaker.htm

# The Bug Wars

- 1970s: “Insects are impossible”
- 1980s: “Insects are possible”
- 1990s: Battle of the species!

Tarantula

Ka

“Samurai Helmet” Beetle

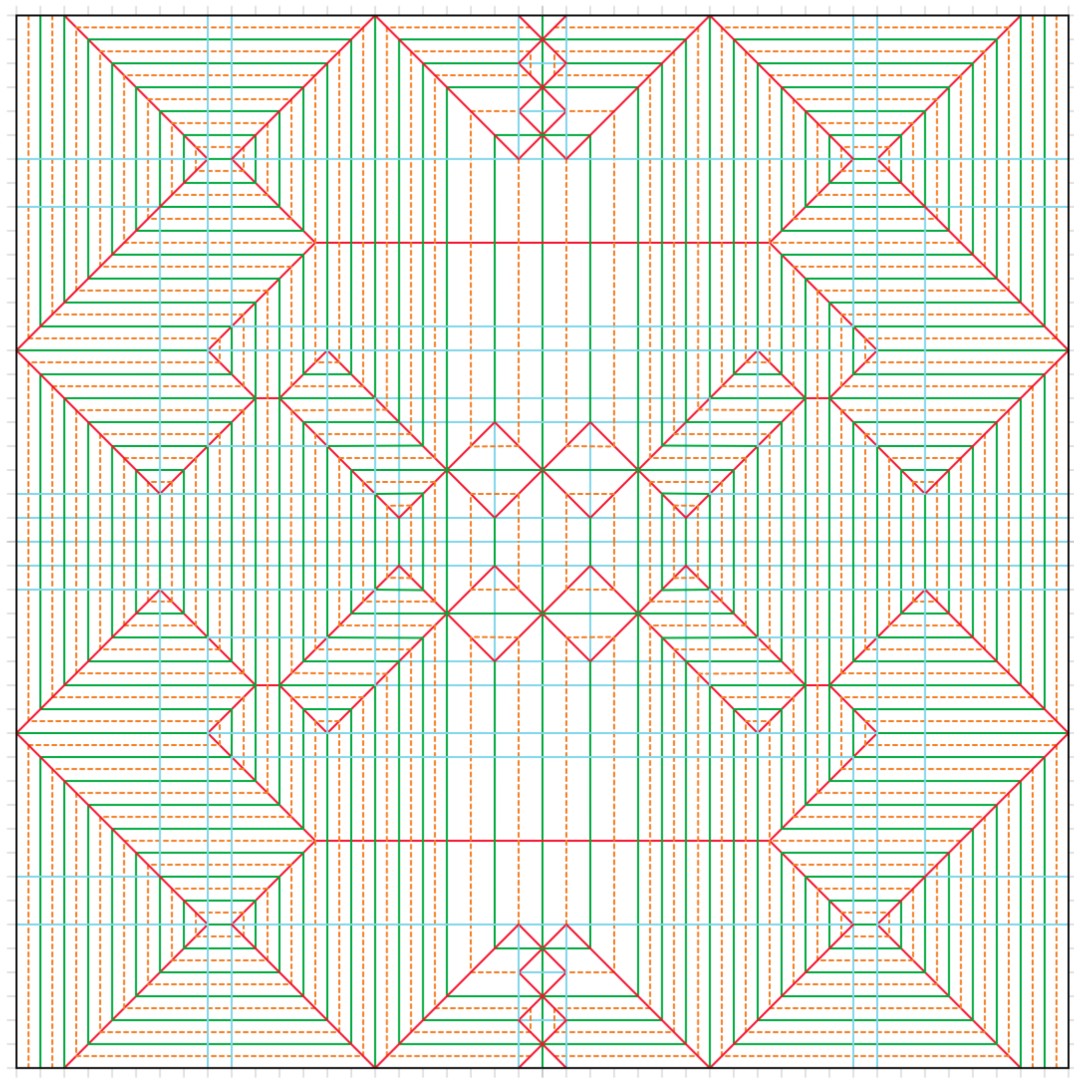
# Eupatorus gracilicornis

Euthysanius Beetle

Praying Mantis

# Two Praying Mantises

# Crease Patterns



Centipede

Tick

Tick on Quarter

# Representational

Dancing Crane

Barn Owl

Grizzly Bear

Tree Frog


# Origami on Demand

- Create “almost anything”
- graphics, advertisements, commercials


Its legend is well known.

# Assembly

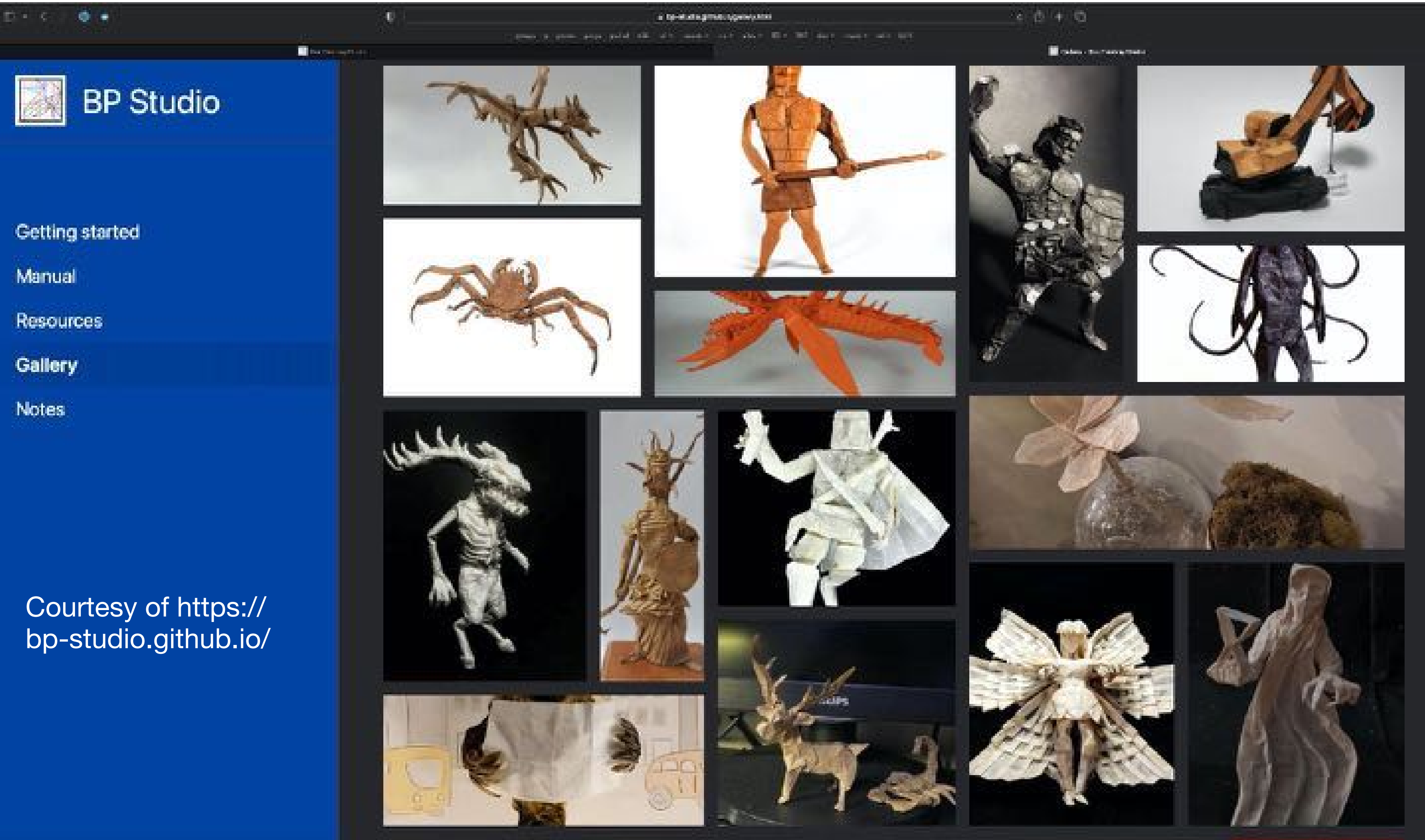
# Origami Software

- TreeMaker (Lang) — shapes with appendages
- Origamizer (Tachi) — arbitrary surfaces
- ReferenceFinder (Lang) — finds folding sequences
- Tess (Bateman) — constructs origami tessellations
- Rigid Simulator (Tachi) — flexible surface linkages
- Oripa (Jun Mitani) — crease pattern folder
- BP Studio (Mu-Tsun Tsai) — shapes with appendages
- …and more!

# Box-pleating

- Origami design on a grid (pack squares, not circles)
- No computer needed! (Just use graph paper)
- But if you have a computer…
  - BP Studio, written by Mu-Tsun Tsai

# BP-Studio Gallery

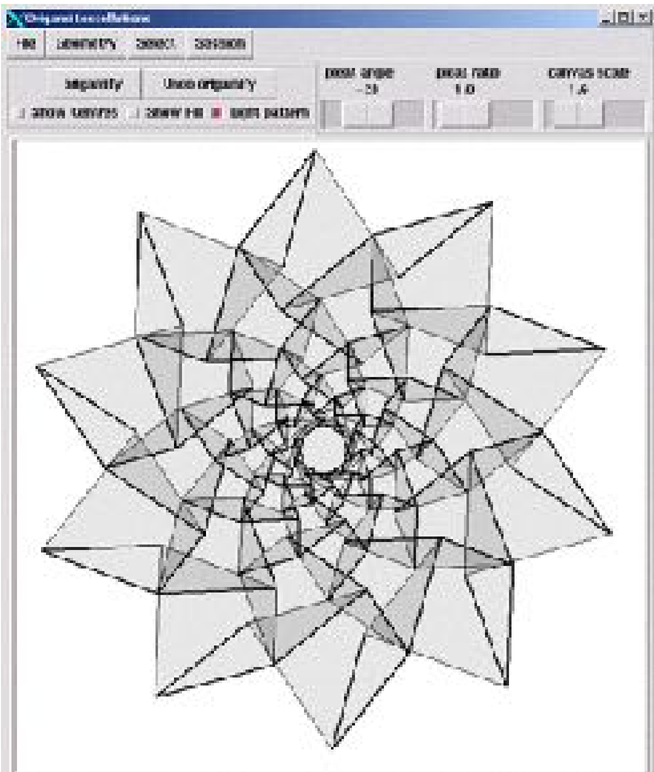


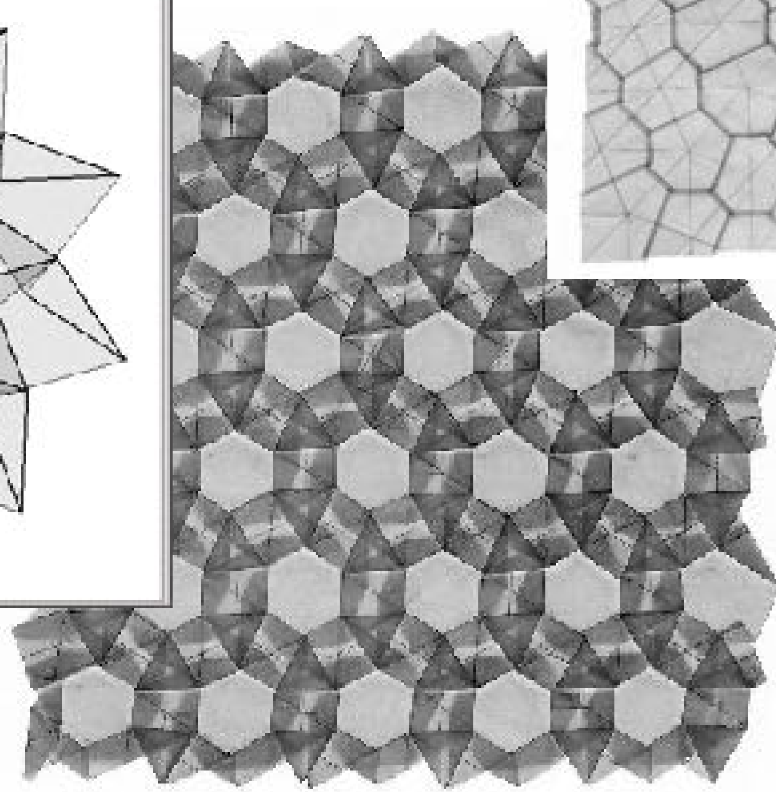
# Tachi's Teapot (via *Origamizer*)

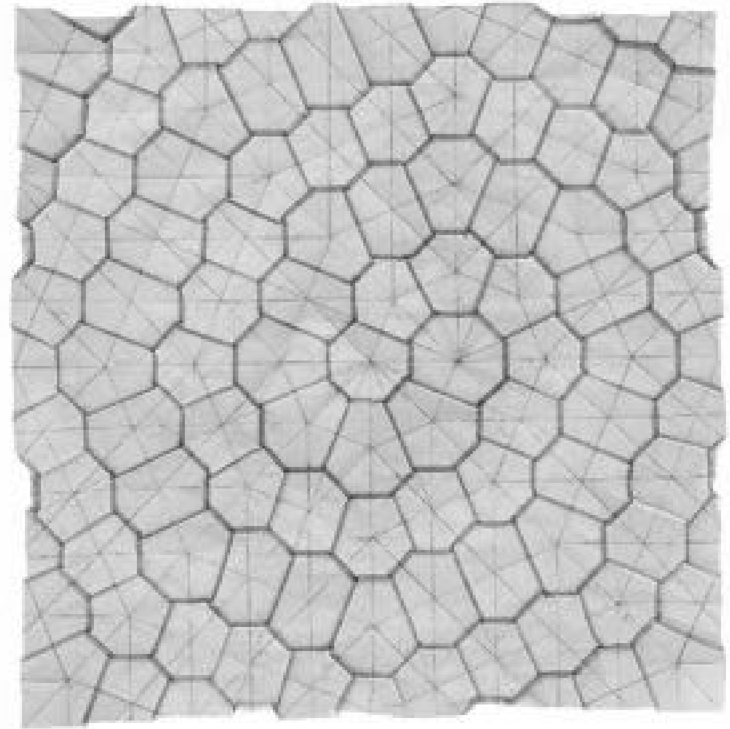
The "Utah teapot"

Computed crease pattern

# Bateman's Tess







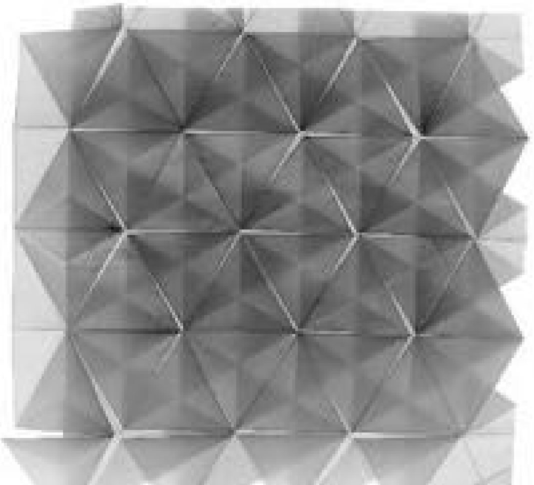
Images courtesy Alex Bateman

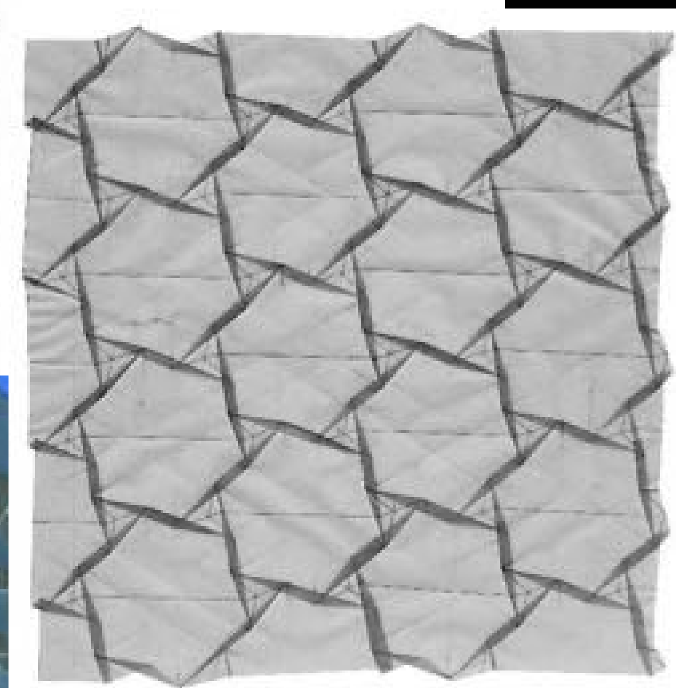
# Geometric Origami

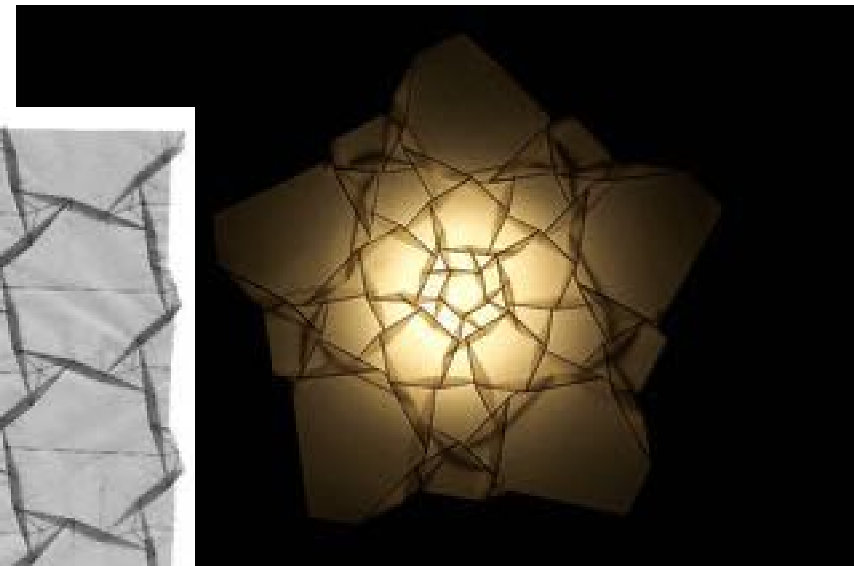
- Mathematical descriptions have permitted the construction of elaborate geometrical objects from single-sheet folding:
  - Flat Tessellations (Resch, Palmer, Bateman, Cooper, Gjerde)
  
  3-D faceted tessellations (Fujimoto, Huffman)
  
  Curved surfaces (Huffman, Mosely)
  
  …and more!

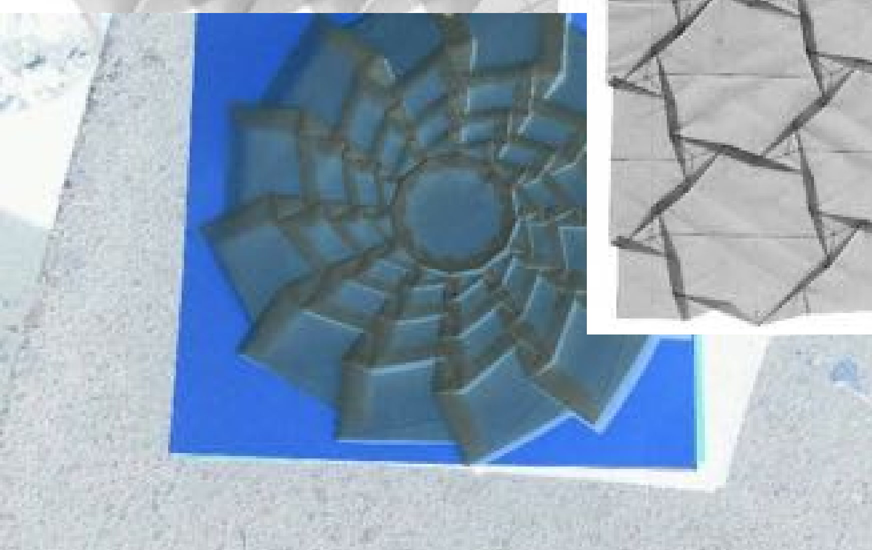
# Tessellation Examples

- Examples from www.papermosaics.co.uk





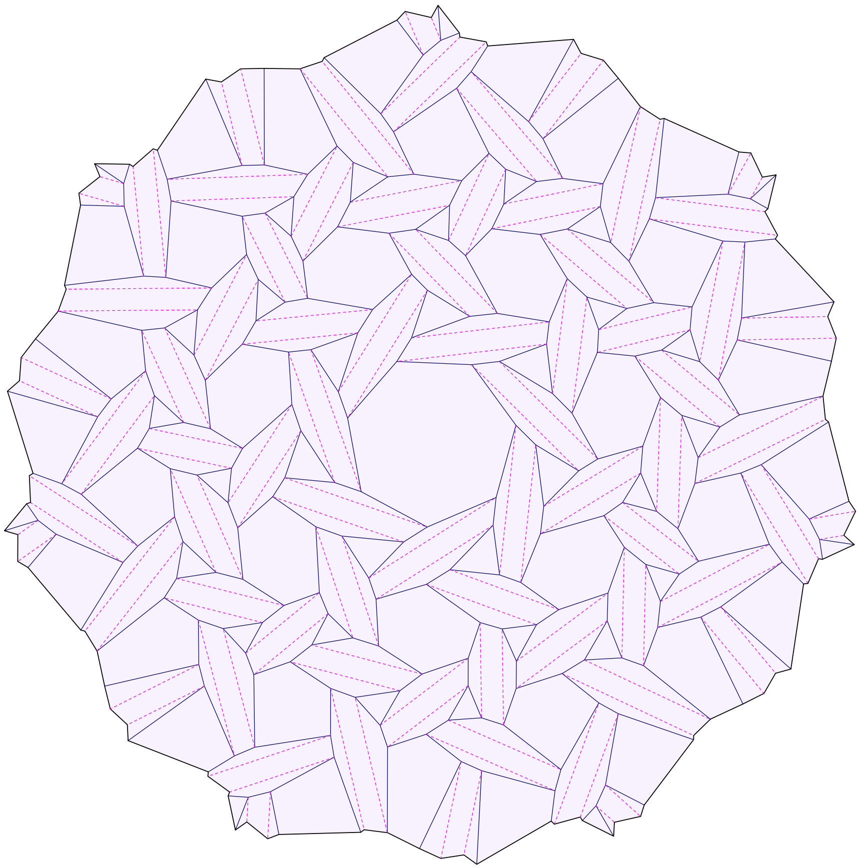


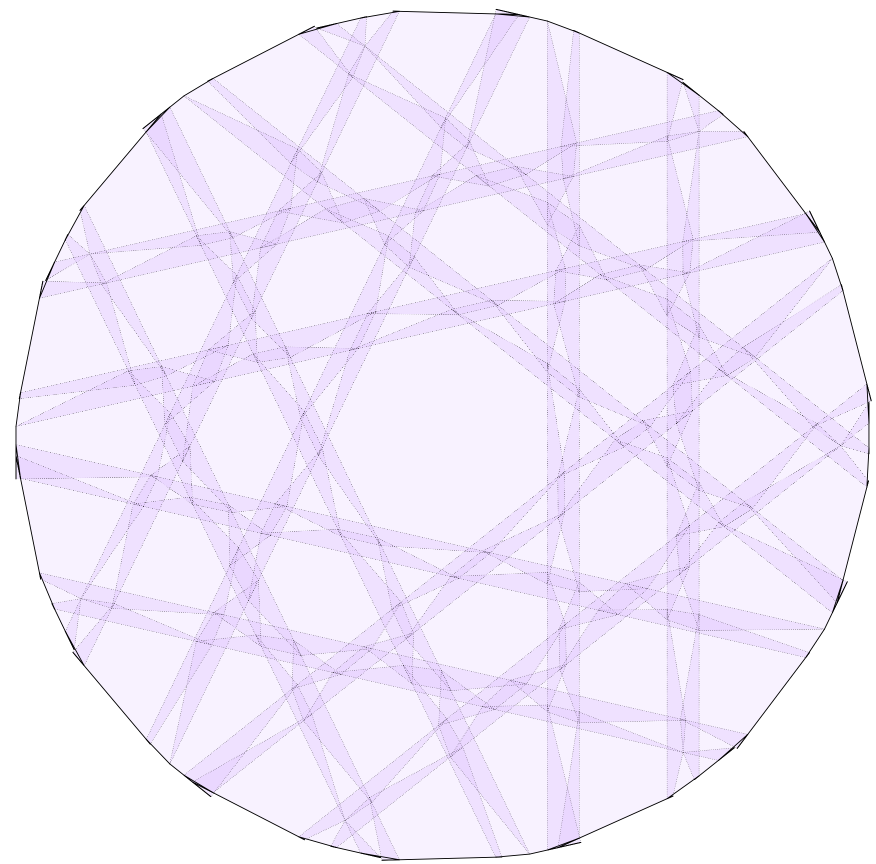


# Spiral Tessellation

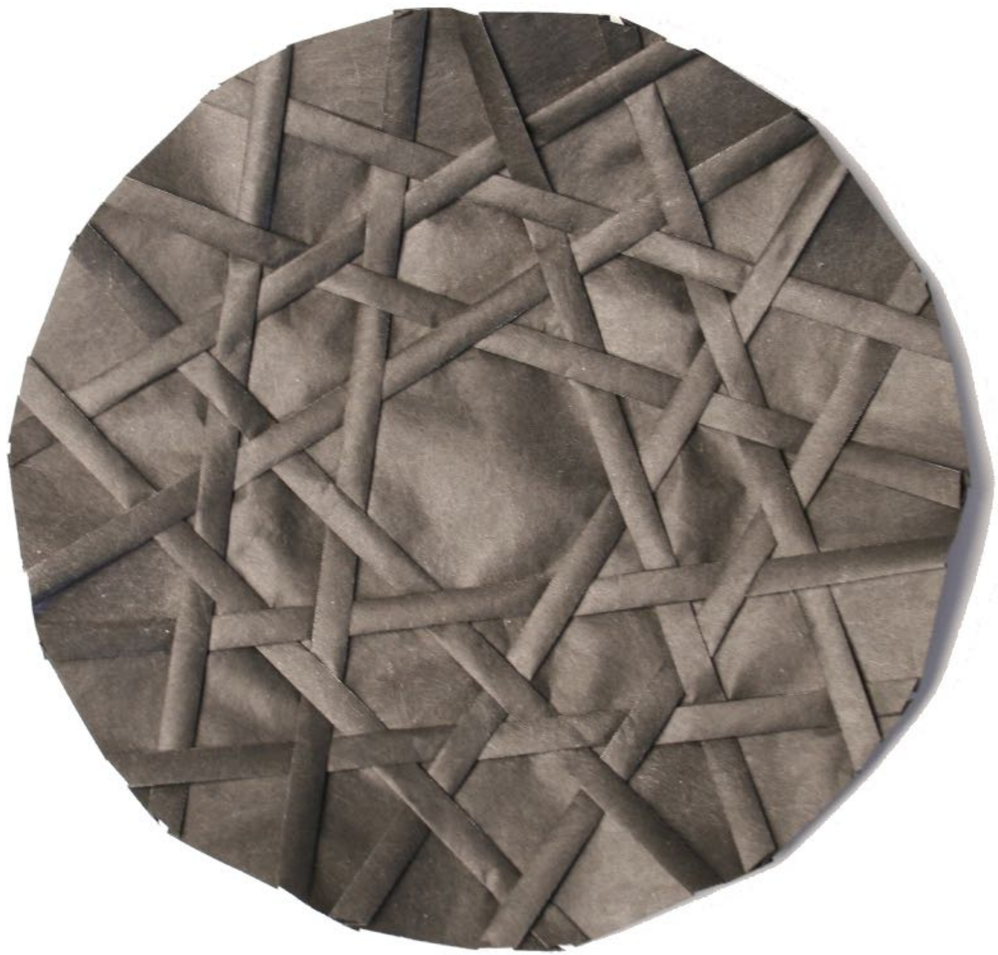
# Egg17 Tessellation

# 7-fold 2-layer CP & FF





# 7-fold 2-layer

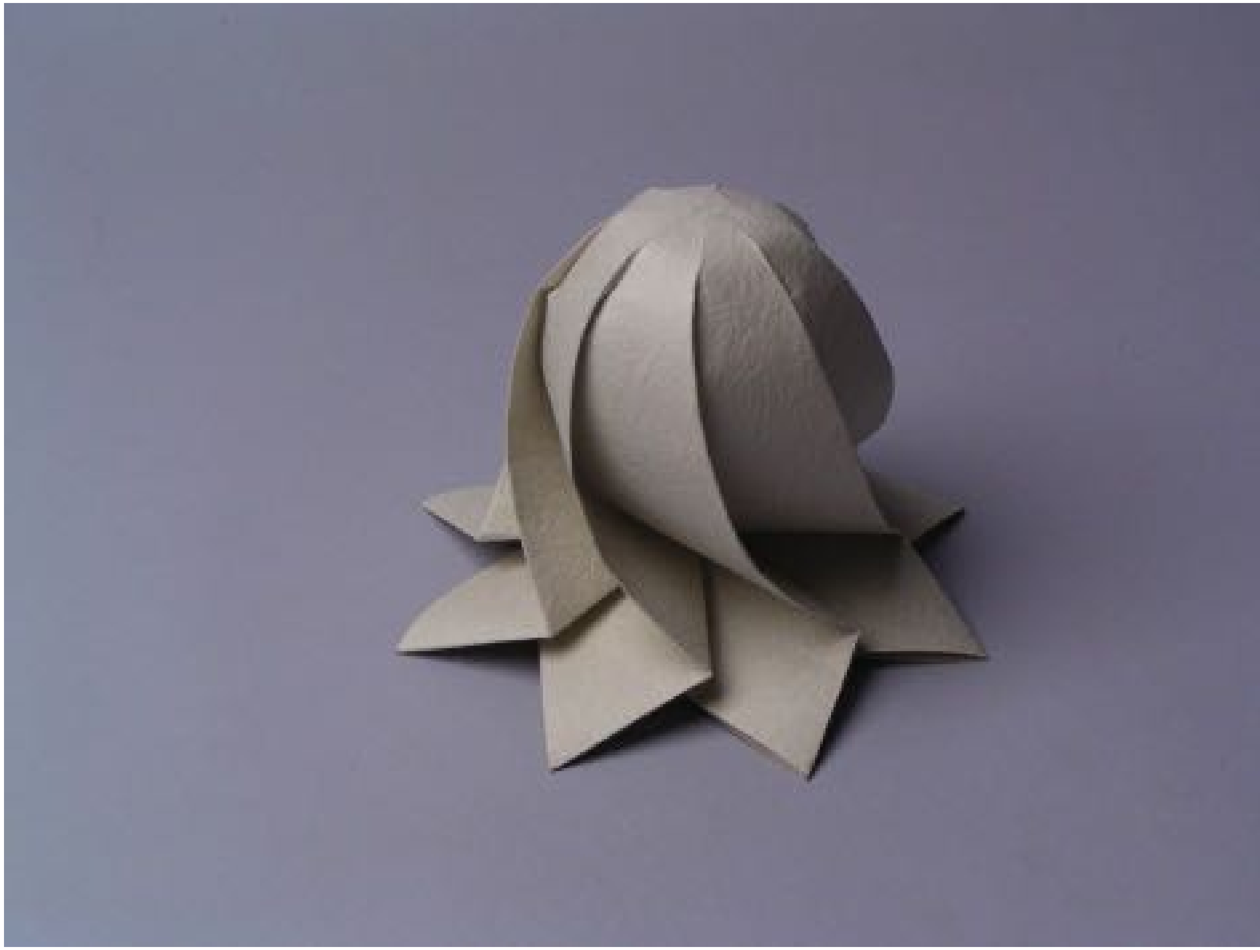


# Generalizations

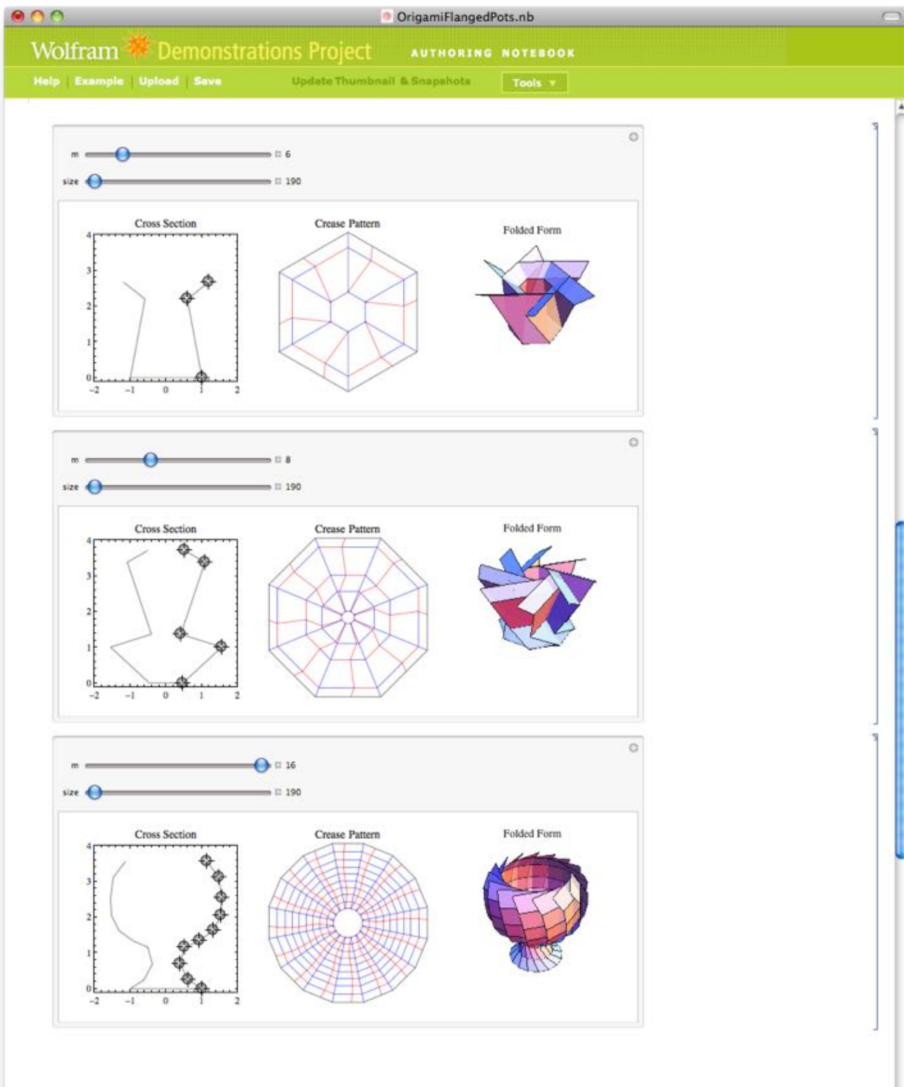
- 3D (non-flat)
- Curved folds
- Thick folds
- Stretching/deformation
- Needed for the Real World!

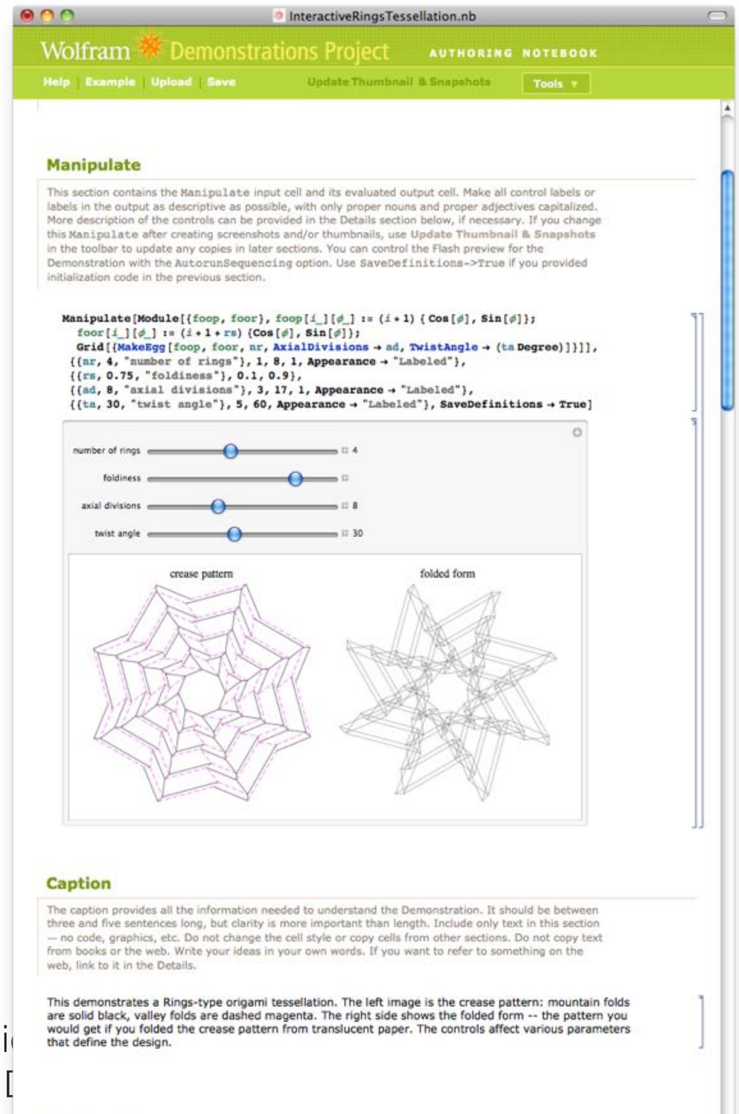
# Flanged sphere

- Concept demo'd by Palmer in 2000
- Inspiration for my generalization



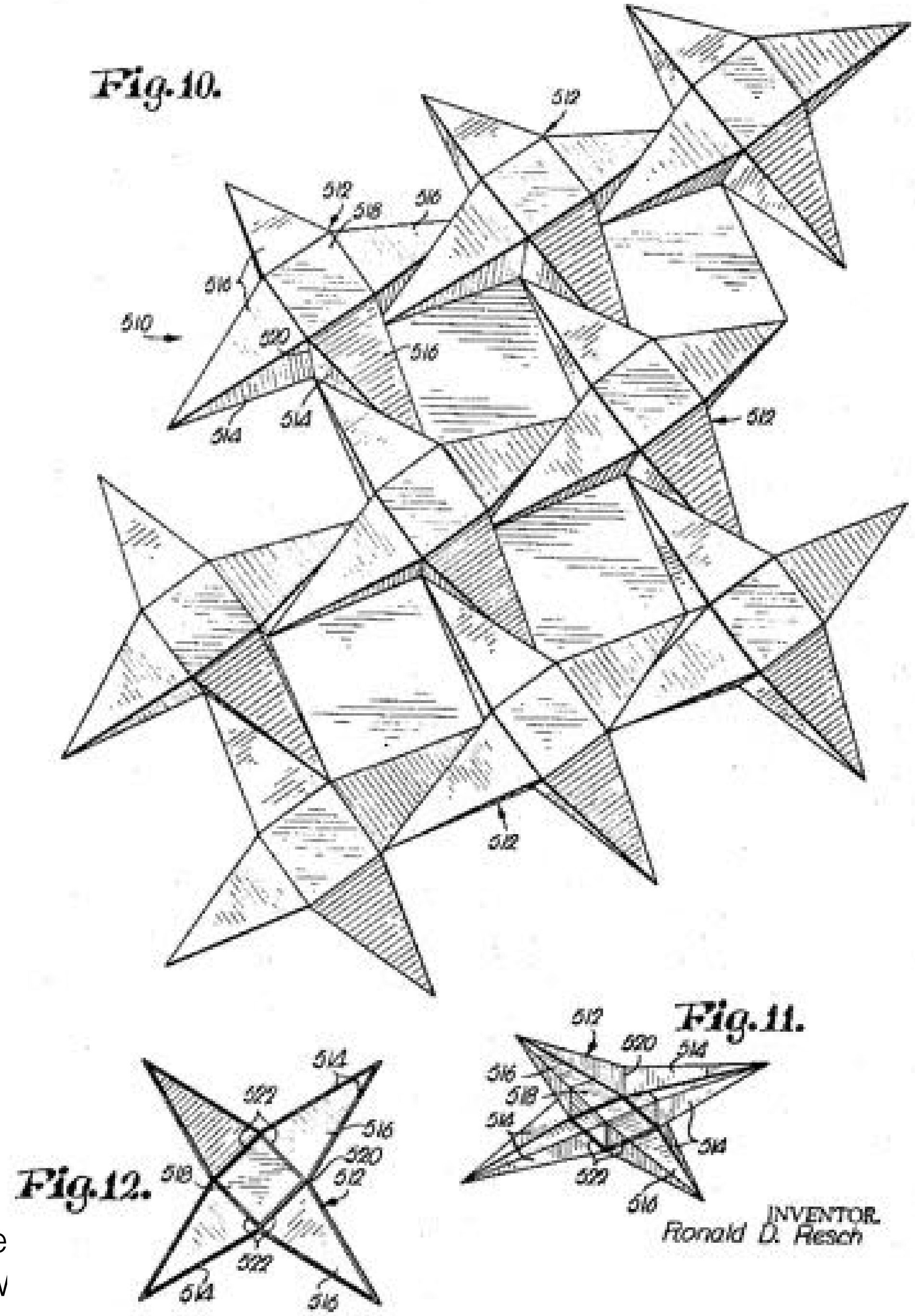
# Mathematica Package



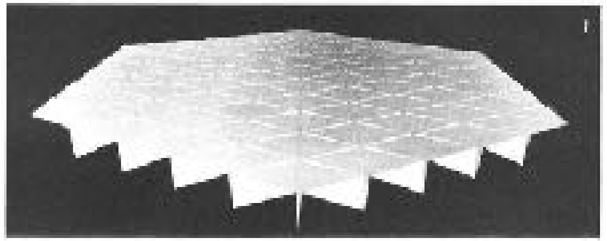



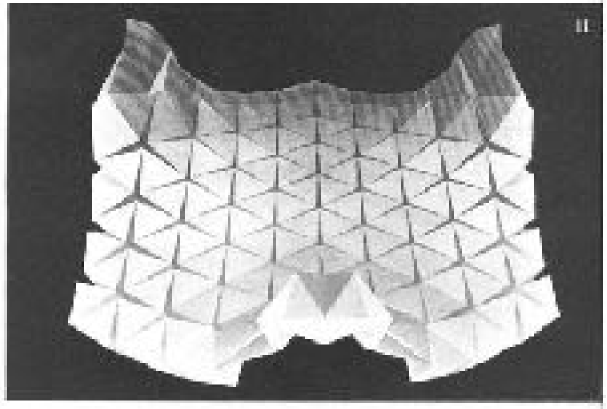
# Ron Resch

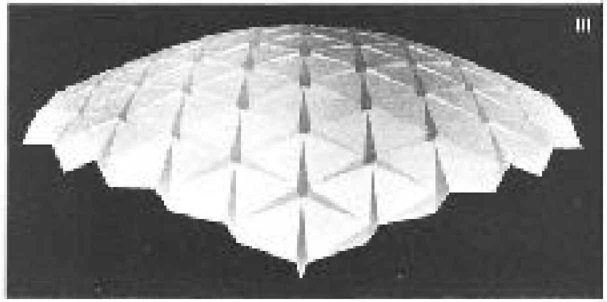
- Computer scientist and artist Ron Resch designed (and patented) 2- and 3-D tessellations back in the 1960s
- See US Patent 3,407,588.



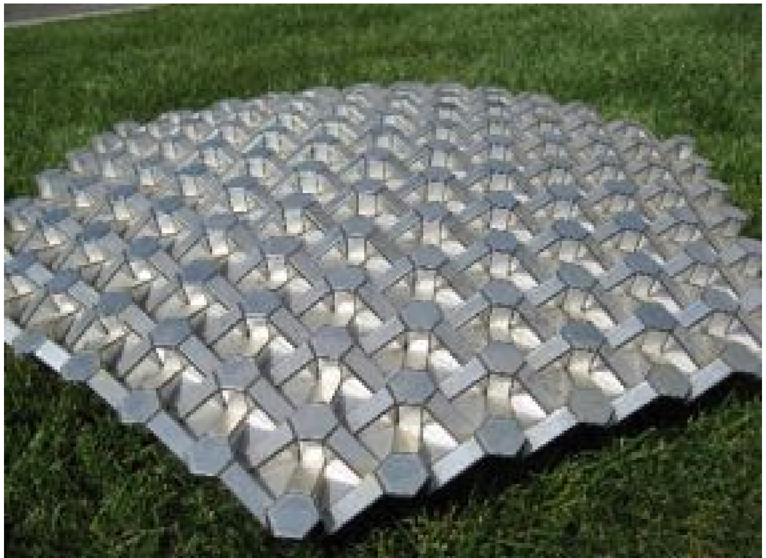
# Ron Resch

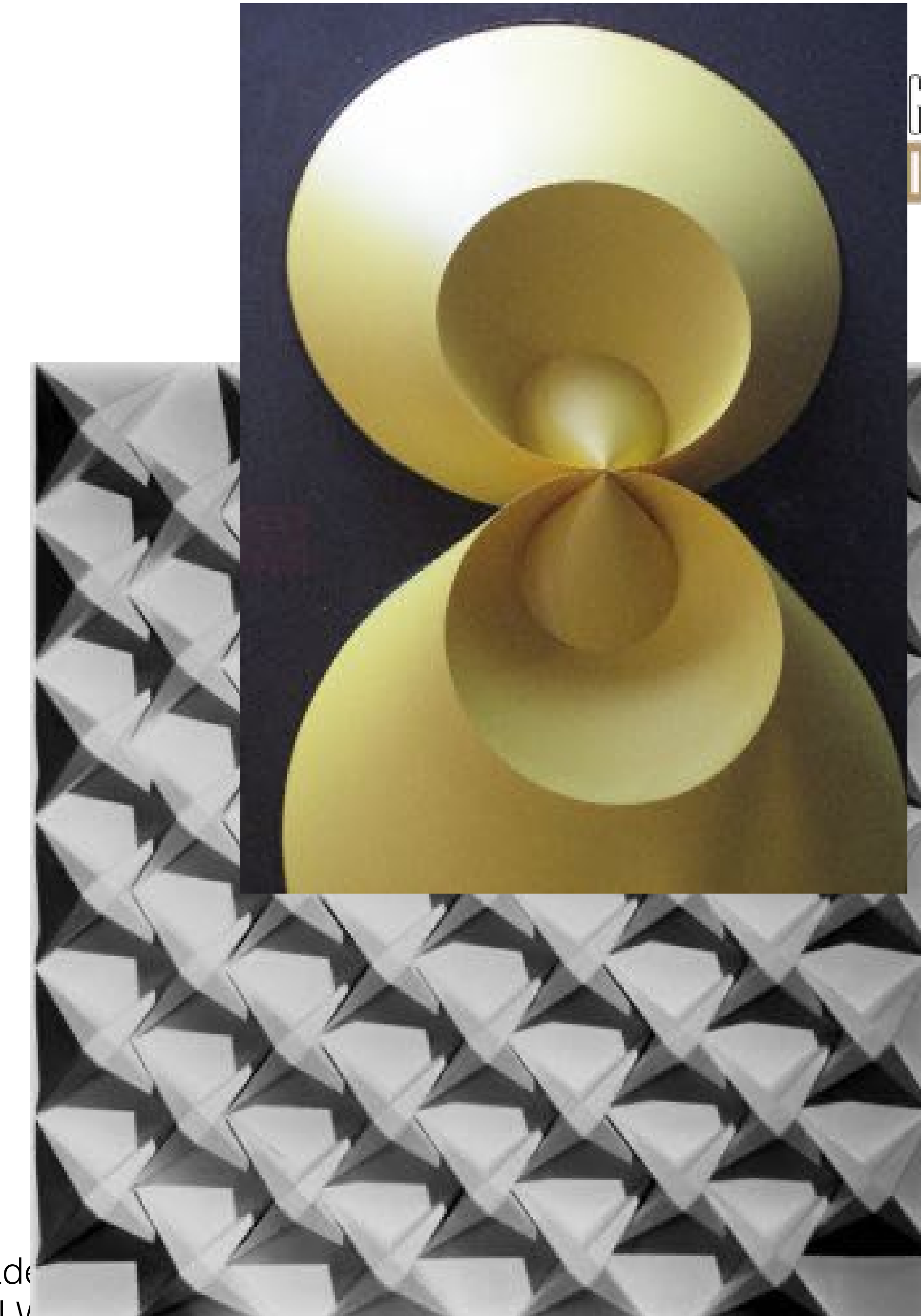


# Ron Resch





# David Huffman

- Developed both faceted and curved forms
- Is almost unknown in origami
- But is famous in computers (Huffman Codes)

- reconstruction of original David Huffman design

# Applications in the Real World

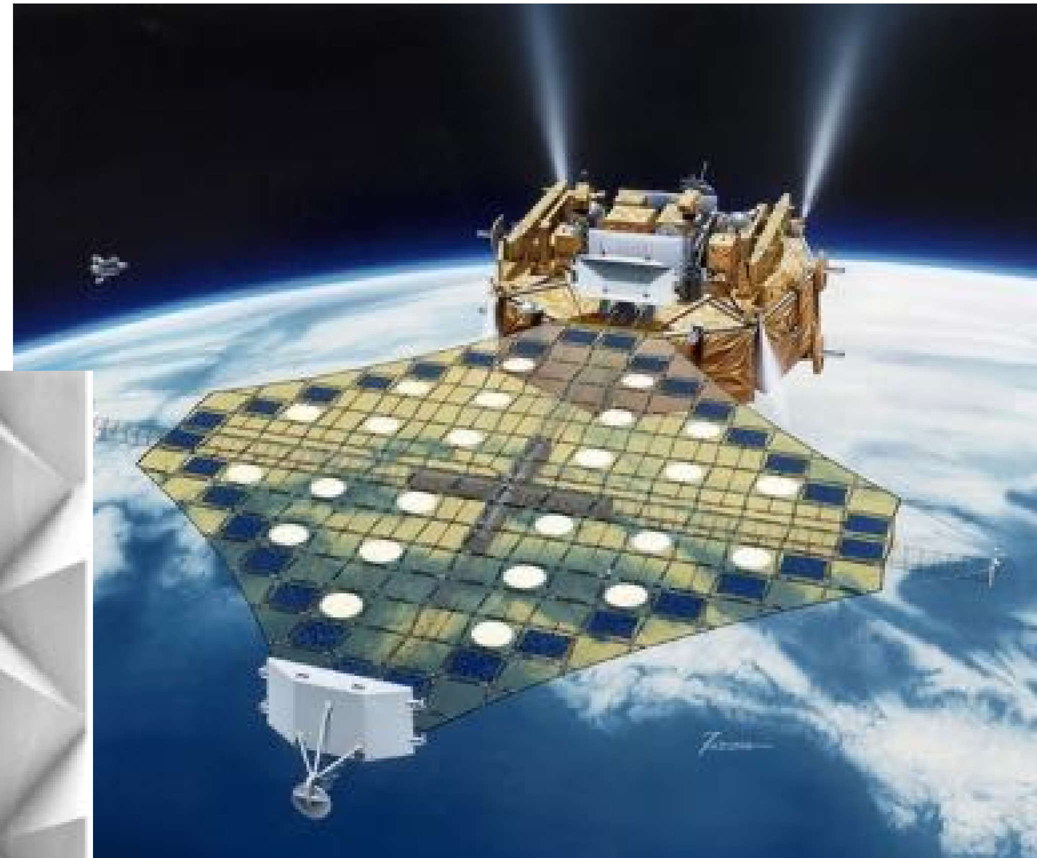
- Mathematical origami has found many applications in solving real-world technological problems, in:
  - Space exploration (telescopes, solar arrays, deployable antennas)
  - Automotive (air bag design)
  - Medicine (sterile wrappings, implants)
  - Consumer electronics (fold-up devices)
  - …and more.

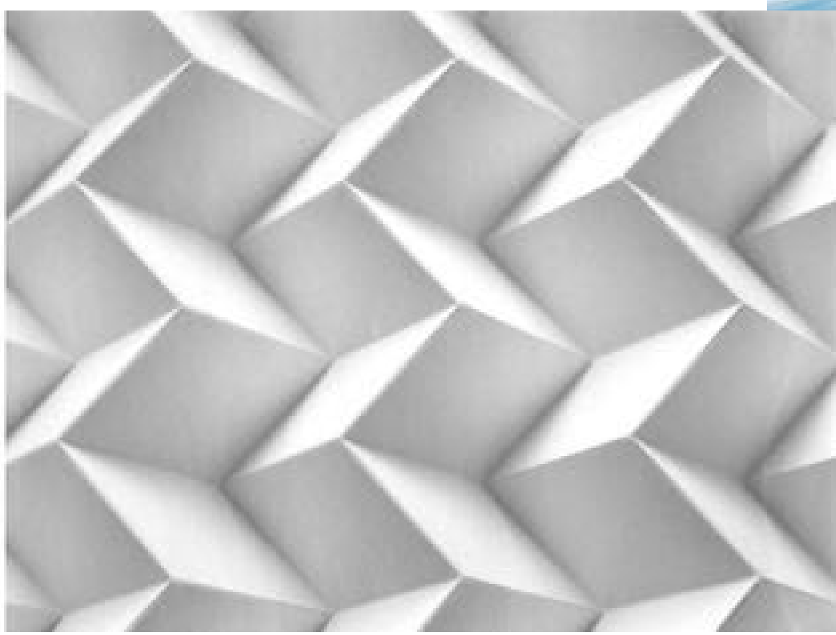
# Miura "map-fold"

- A map of Venice with one degree of freedom

# Miura-Ori, by Koryo Miura

- First "origami in space"
- Solar array, flew in 1995

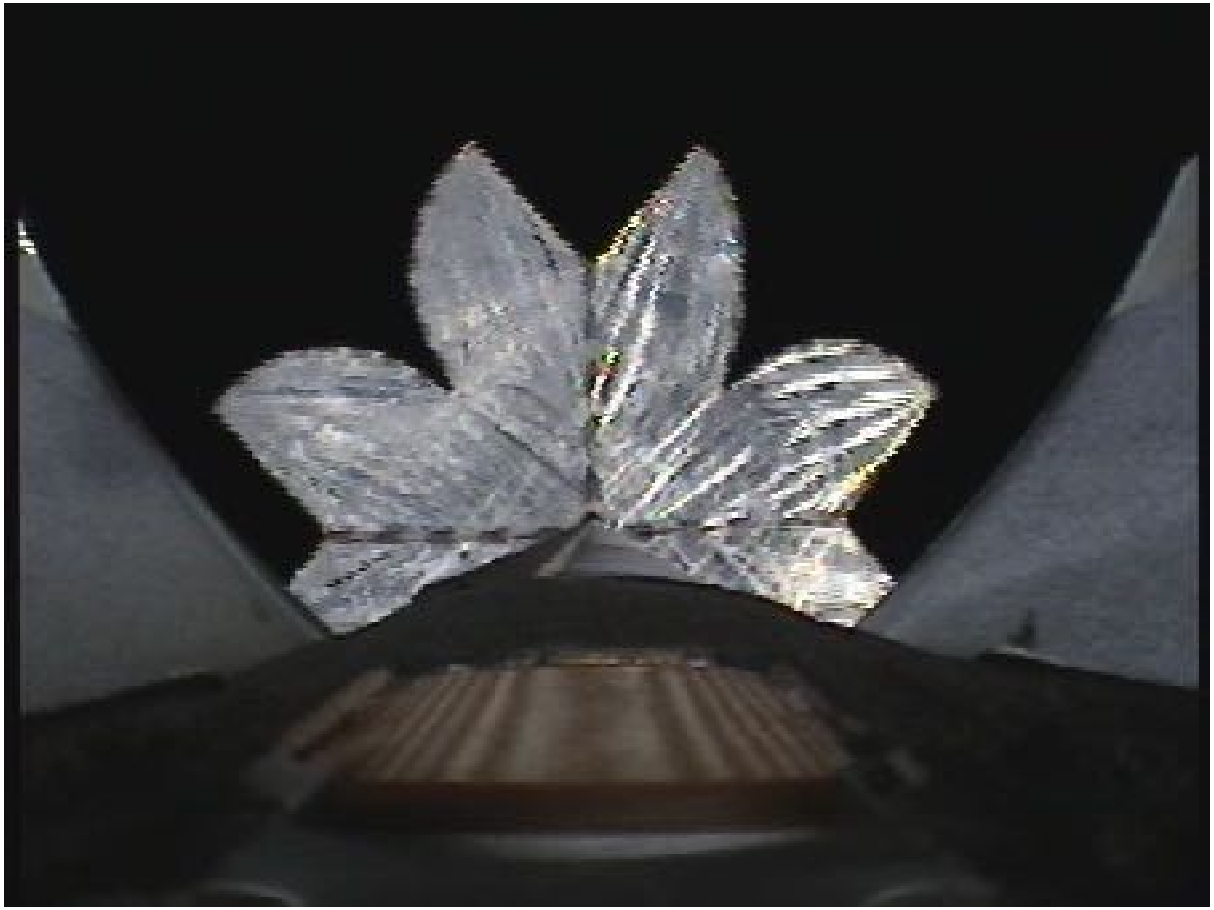




# Solar Sail

- Japanese Aerospace Exploration Agency
- Mission flown in August 2004
- First deployment of a solar sail in space

# Solar Sail



http://www.isas.jaxa.jp/e/snews/2004/0809.shtml

# James Webb Space Telescope

- Multiply segmented mirror folds into thirds

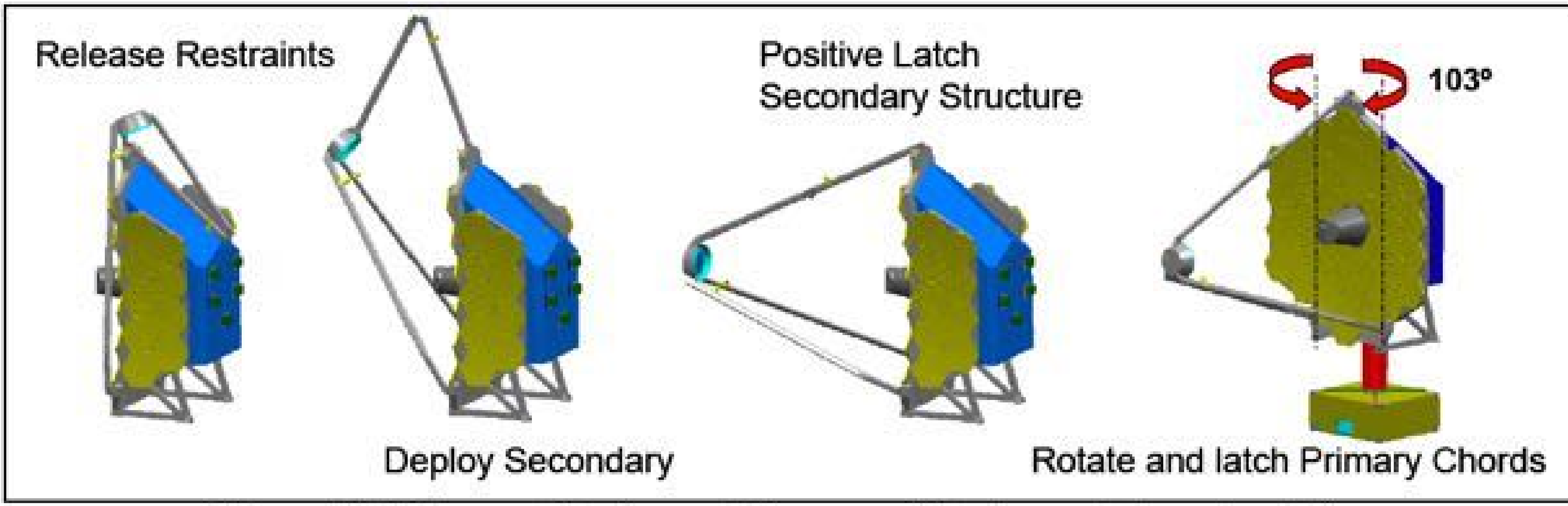
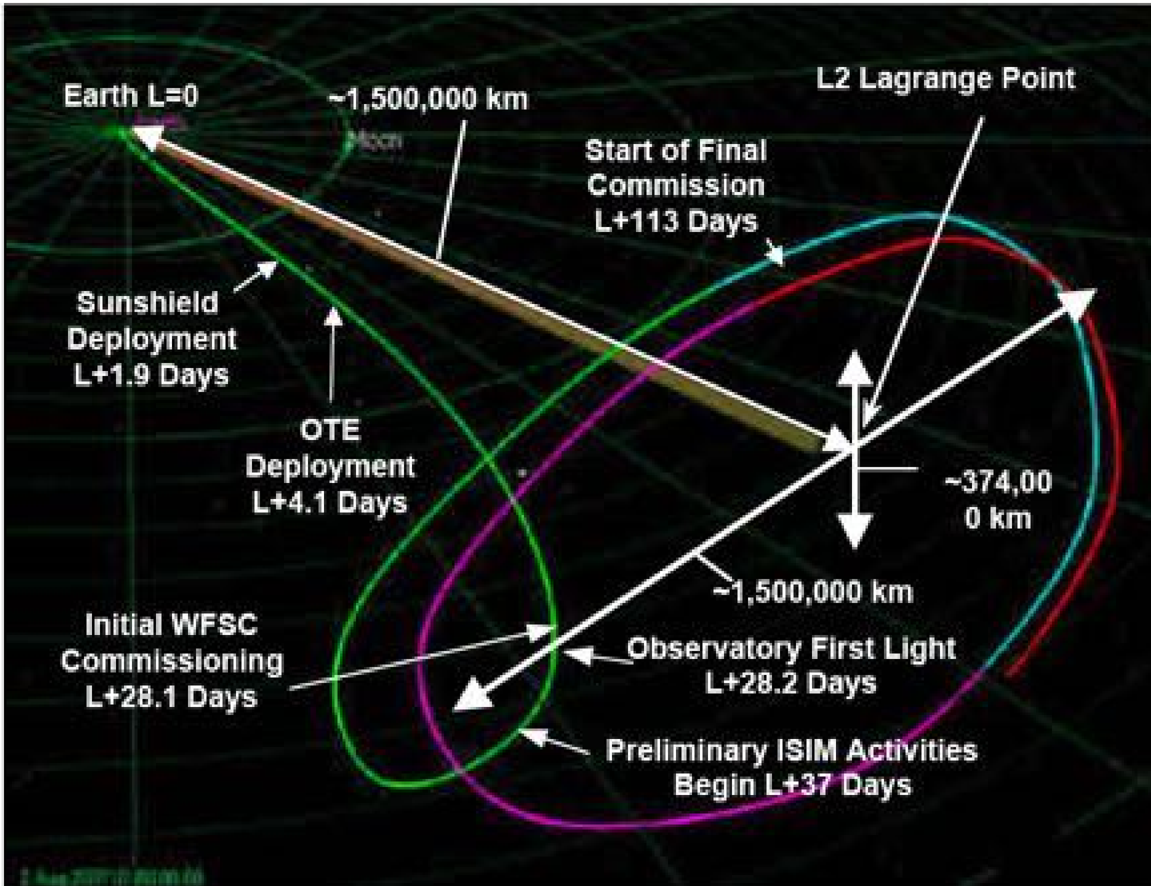
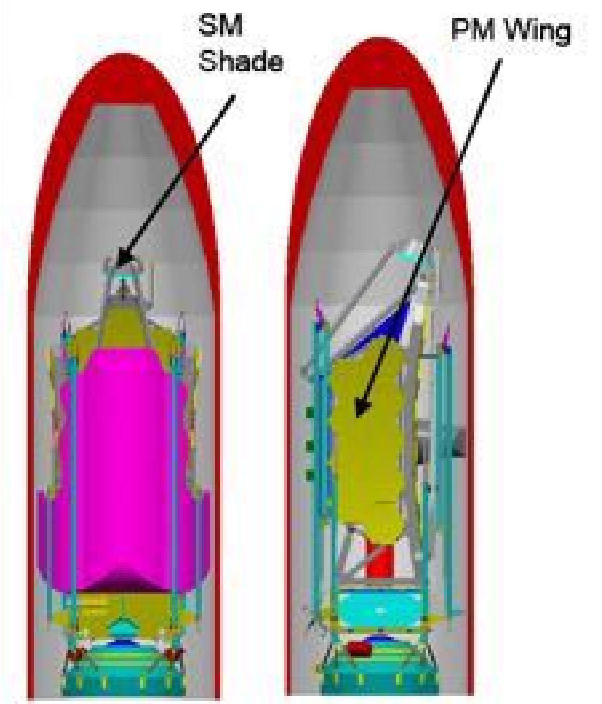


Figure 10. Telescope Deployment Sequence (Deployment steps 4 and 5)

# JWST Stowage

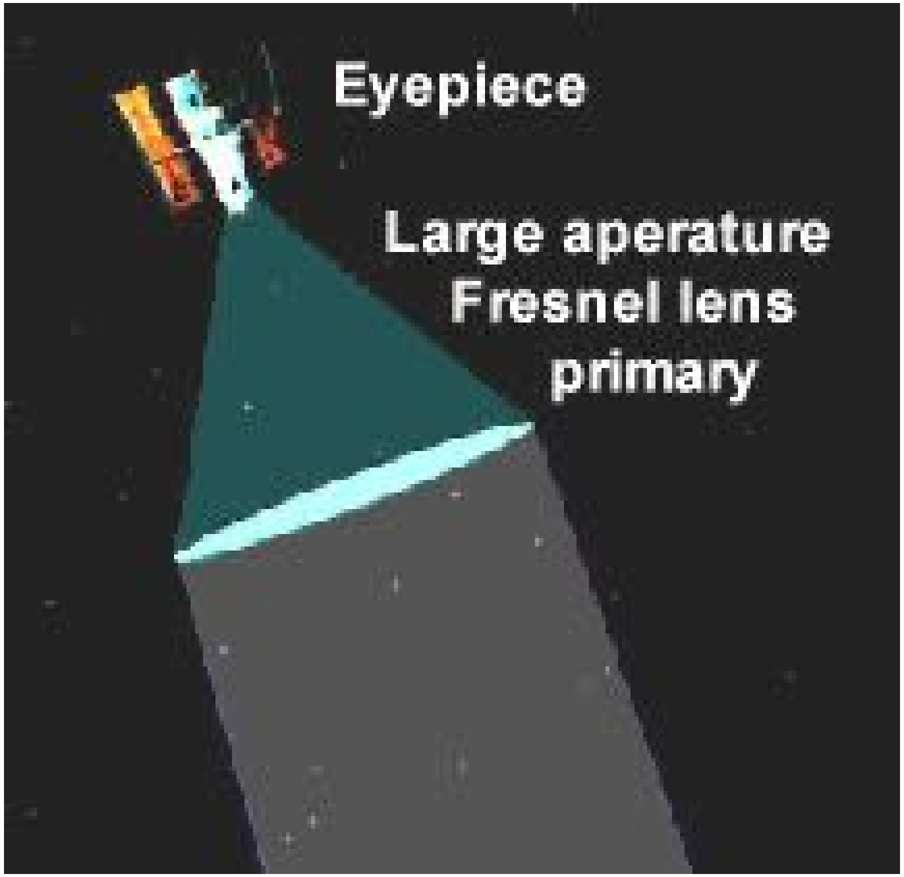



Stowed Observatory in the Atlas V 541

# The “Eyeglass” Telescope

- Lawrence Livermore National Laboratory
- Geostationary orbit
- 100 meter diameter (a football field)

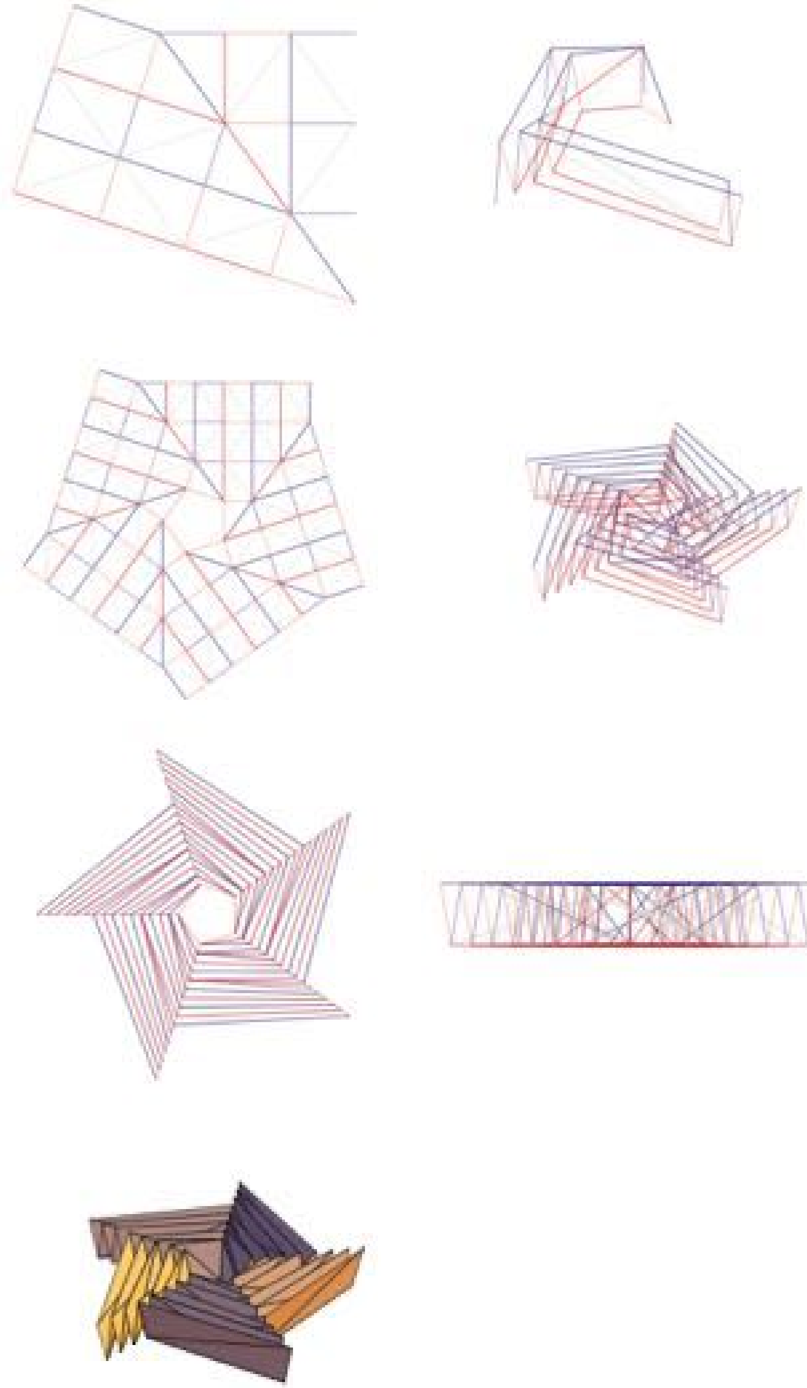


# The lens and the problem

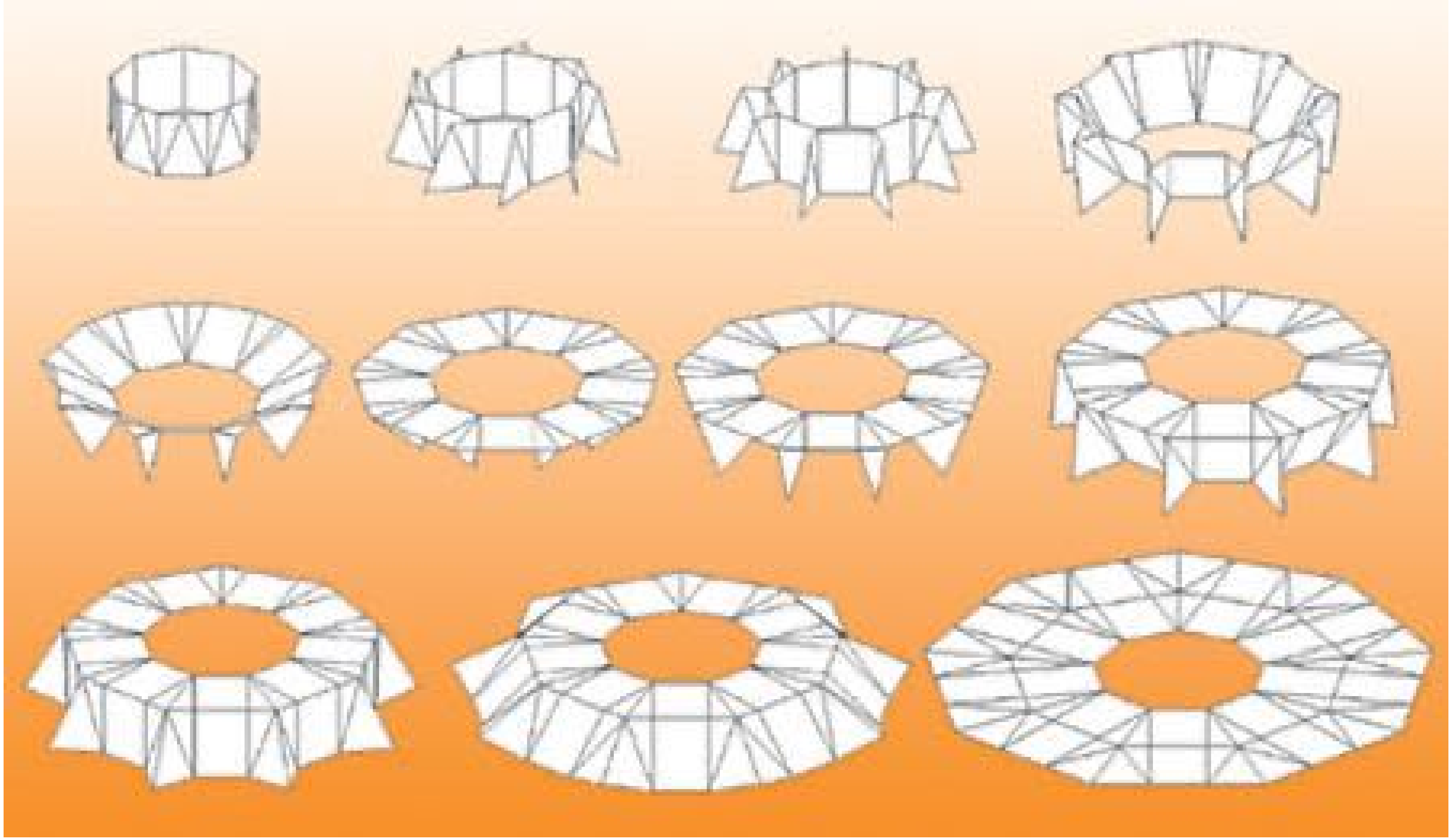
- The 100-meter lens must fold down to 3 meters

# Analysis

- Analyzed several families of collapsing structures, including “flashers” and umbrella-liked patterns
- Initial modeling in Mathematica™ solving NLCO that enforce isometry between folded and unfolded state, followed by 3D modeling at LLNL



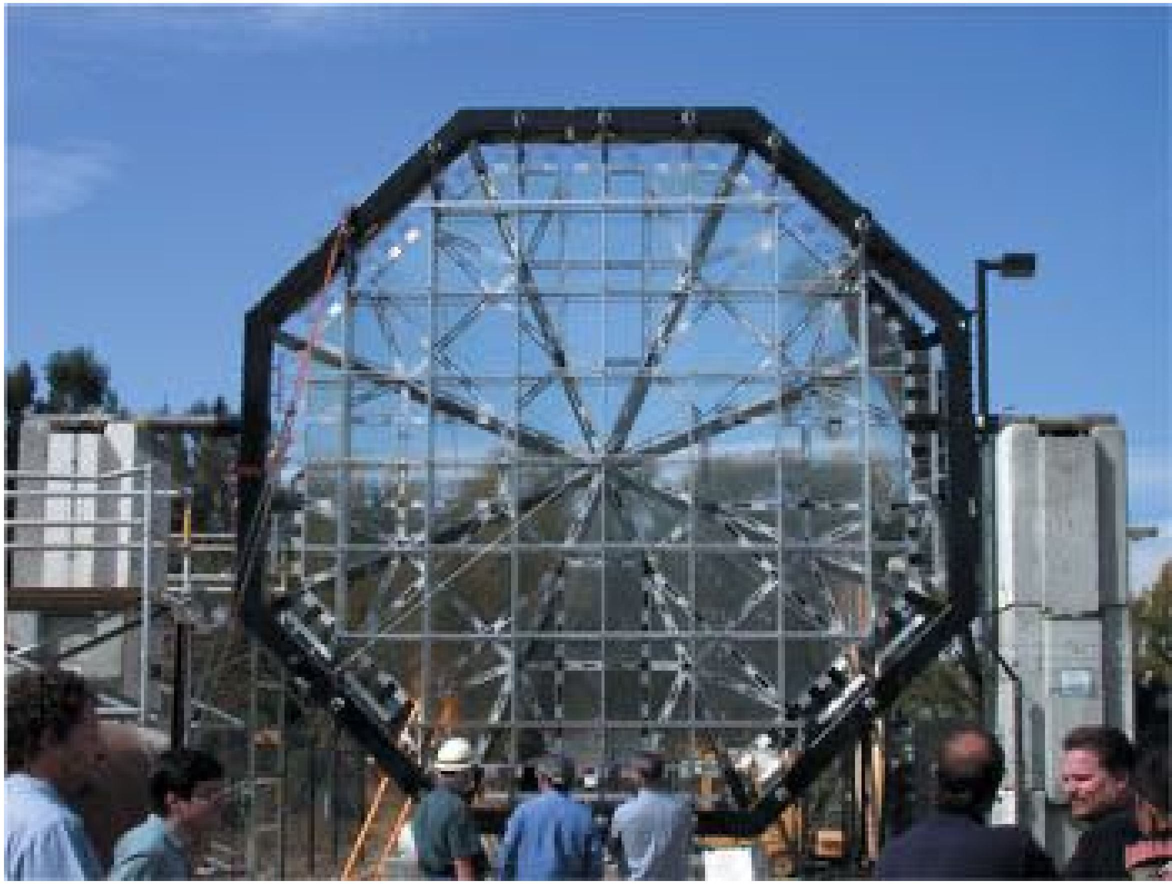
# Umbrella



# Foldable 3.7 meter Eyeglass

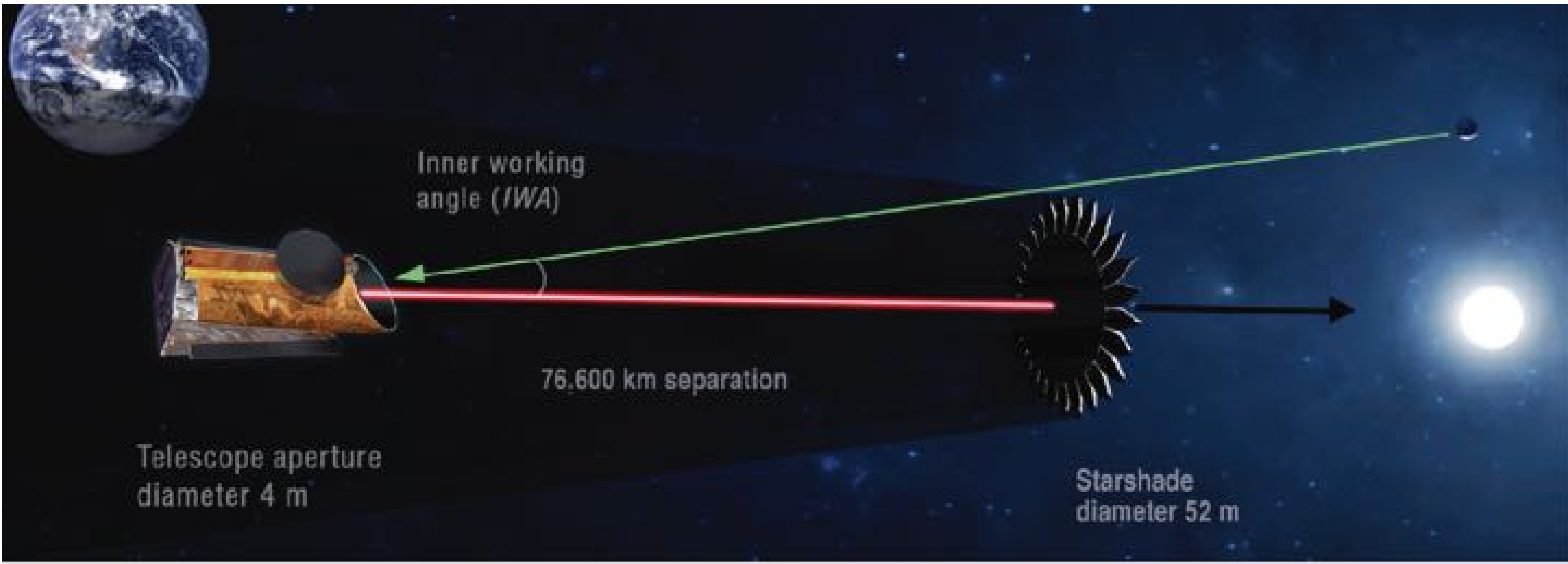
# 5-meter prototype

- The 5-meter prototype folds to about 1.5 meter.

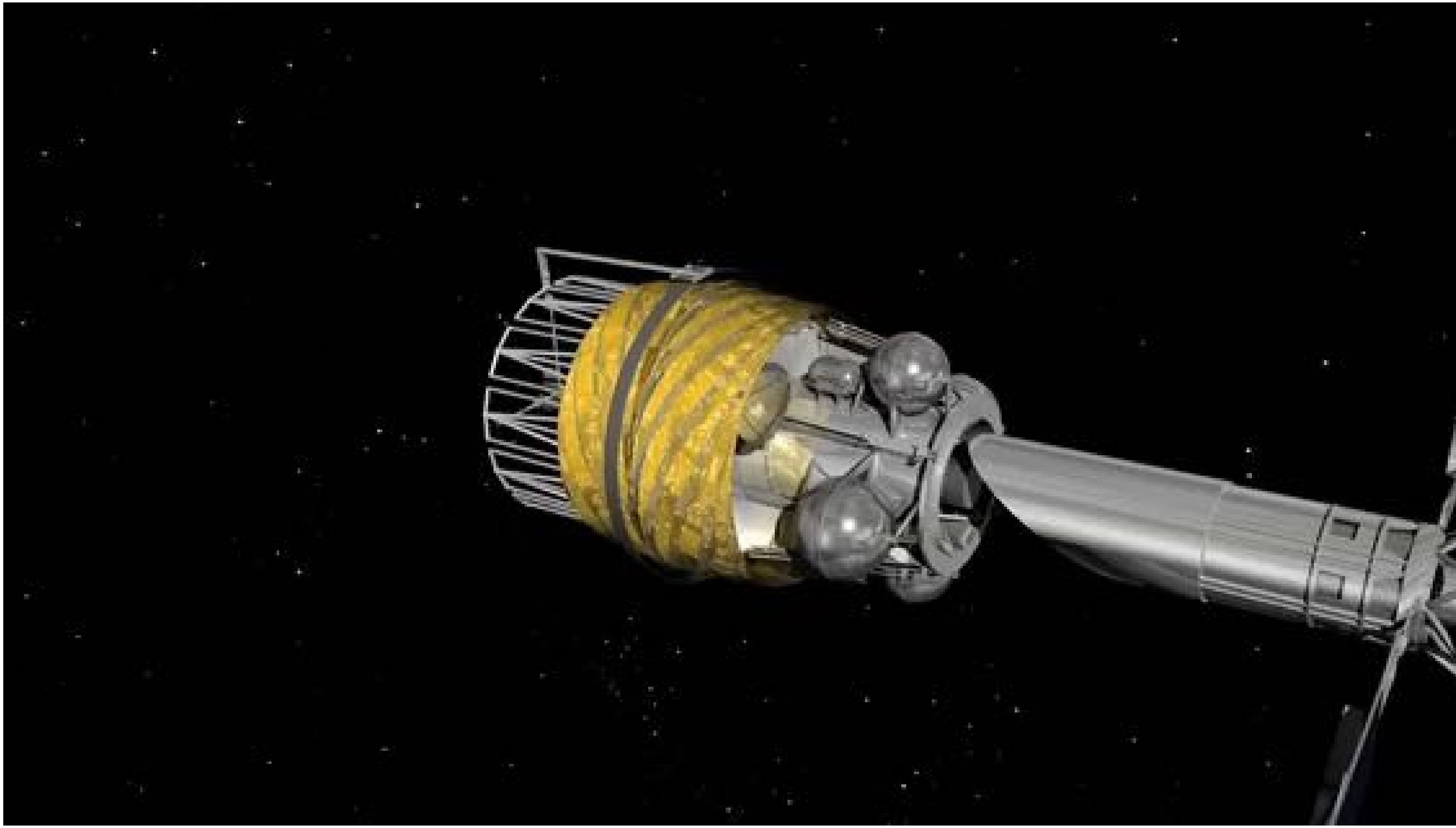


# Starshade

- A space-based telescope to seek extrasolar planets



# Deployment



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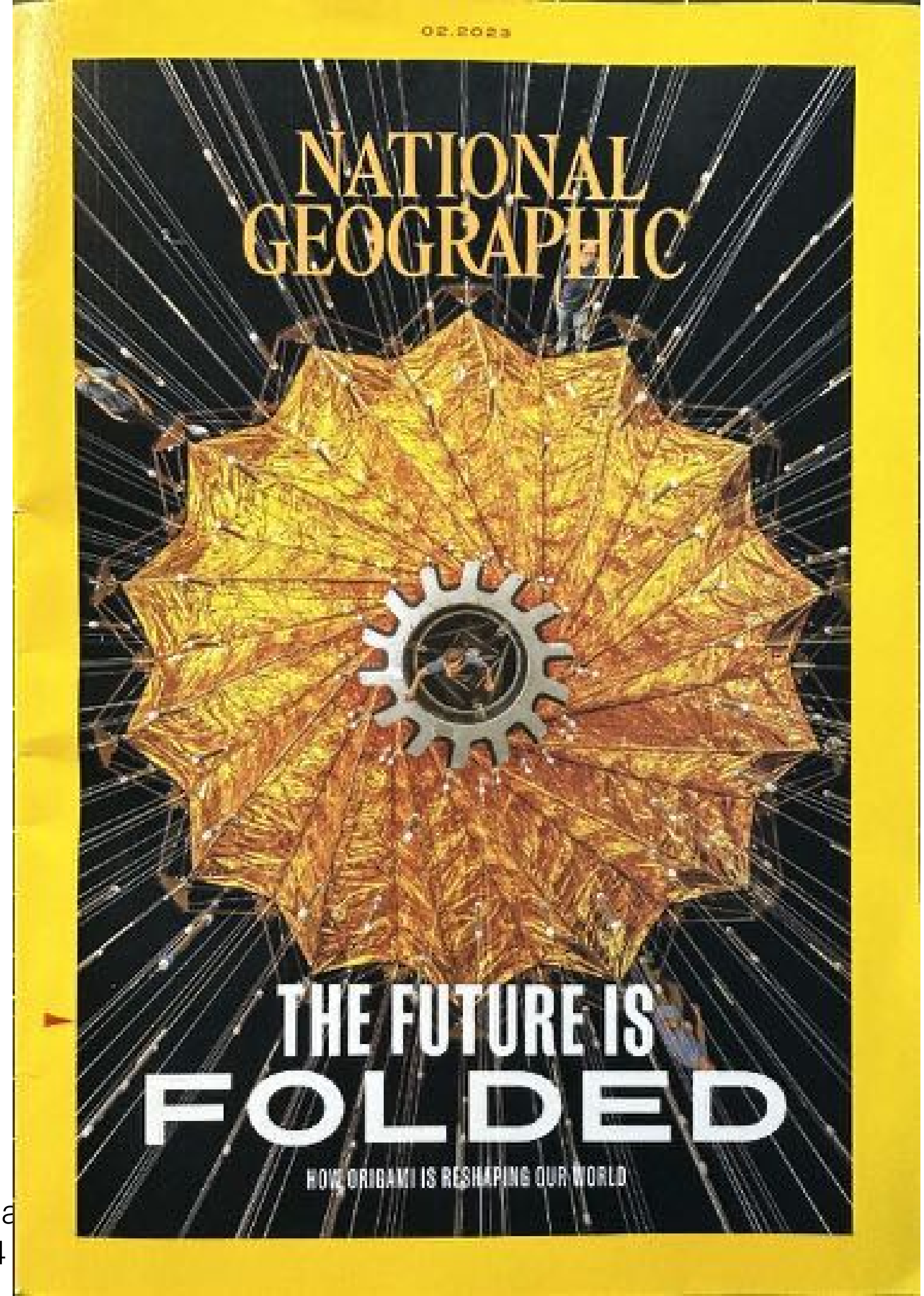
# JPL - Starshade

- 52 m occulter to fly in formation with a telescope to search for extra-solar planets

Courtesy of Manan Arya, JPL

Video courtesy of Manan Arya, NASA/JPL

- February 2023 National Geographic cover feature



# Inflatable Mast : Origami

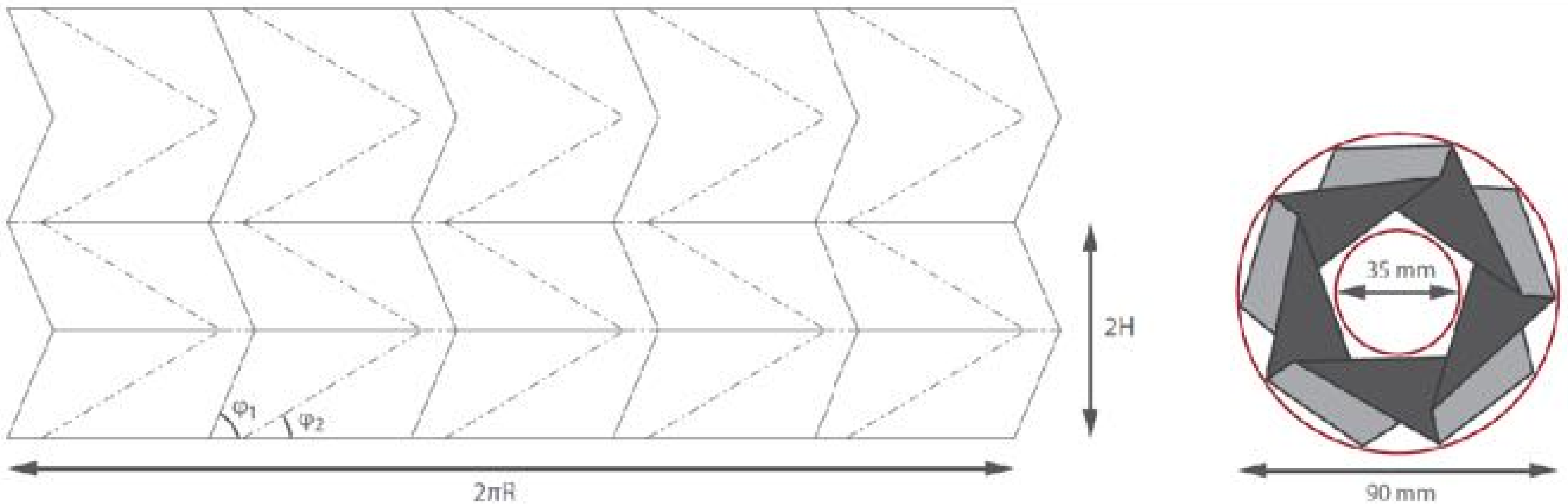
Work by Mark Schenk, Surrey Space Center, Surrey, UK

- **Selected Fold Pattern**

The selected fold pattern for the inflatable boom:

$$D = 90\ mm \qquad n = 5 \qquad \varphi_1 = 67^\circ \qquad H/R = 0.67$$

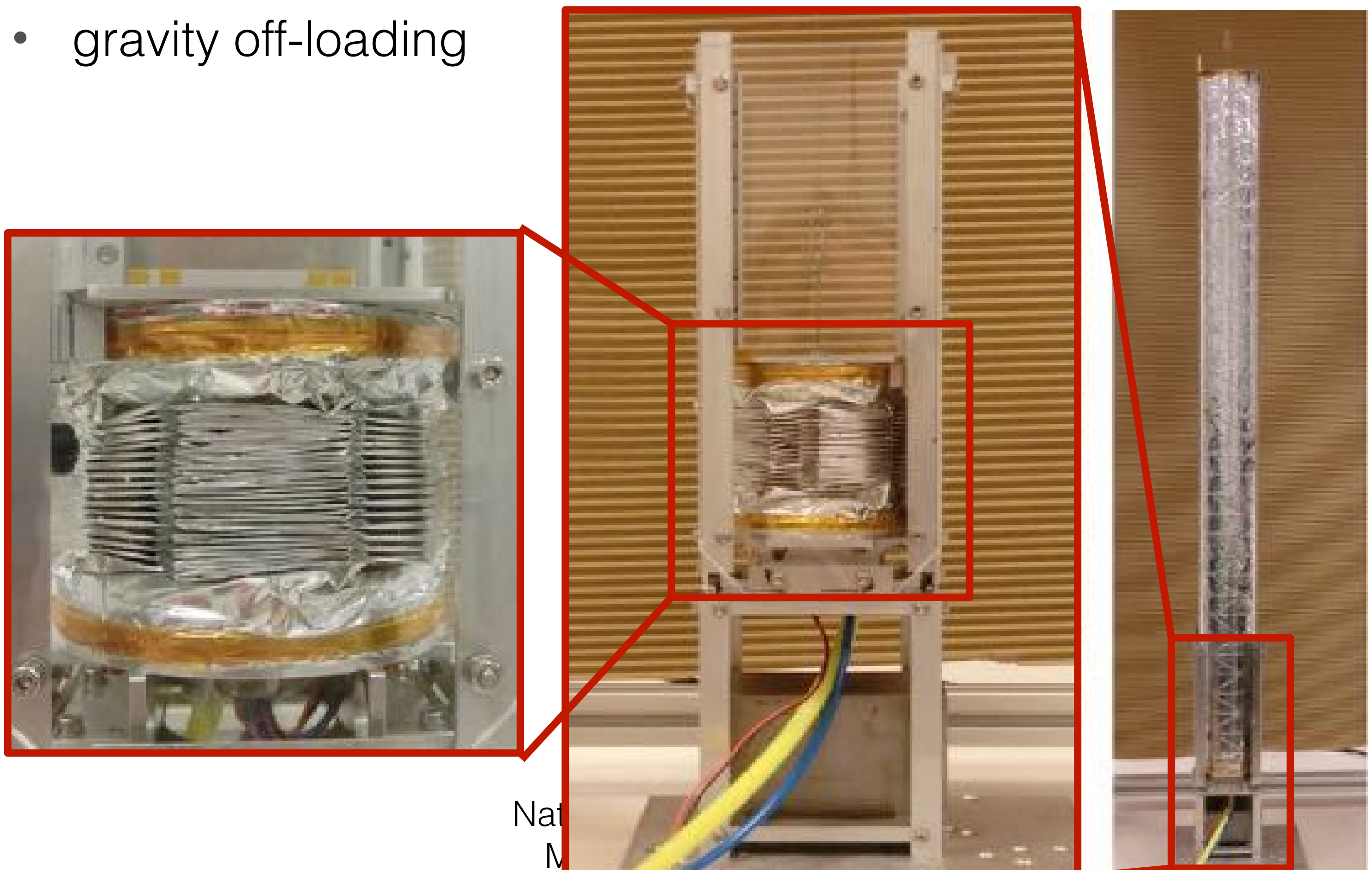
The pattern was selected for minimal material deformation, small outer diameter, large open cross-section, and foldability.



# Inflatable Mast : Manufacture & Testing

**Boom Deployment Testing**

- reliable deployment in ambient & vacuum conditions
- gravity off-loading



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# Oru Kayak

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Images courtesy of orukayak.com

# Twist Bottle

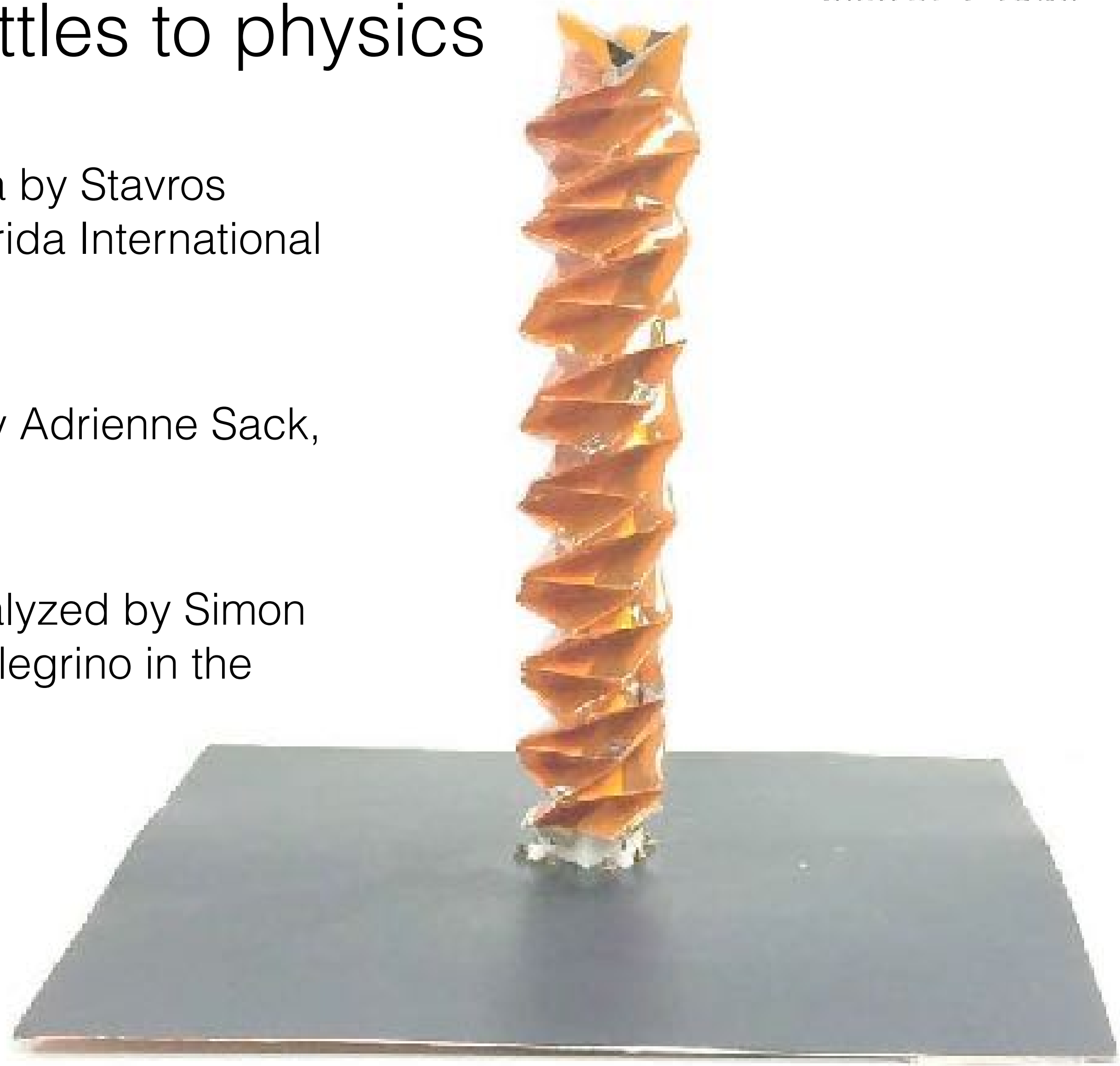
- Twist bottle by James Hart (2009)

# From bottles to physics

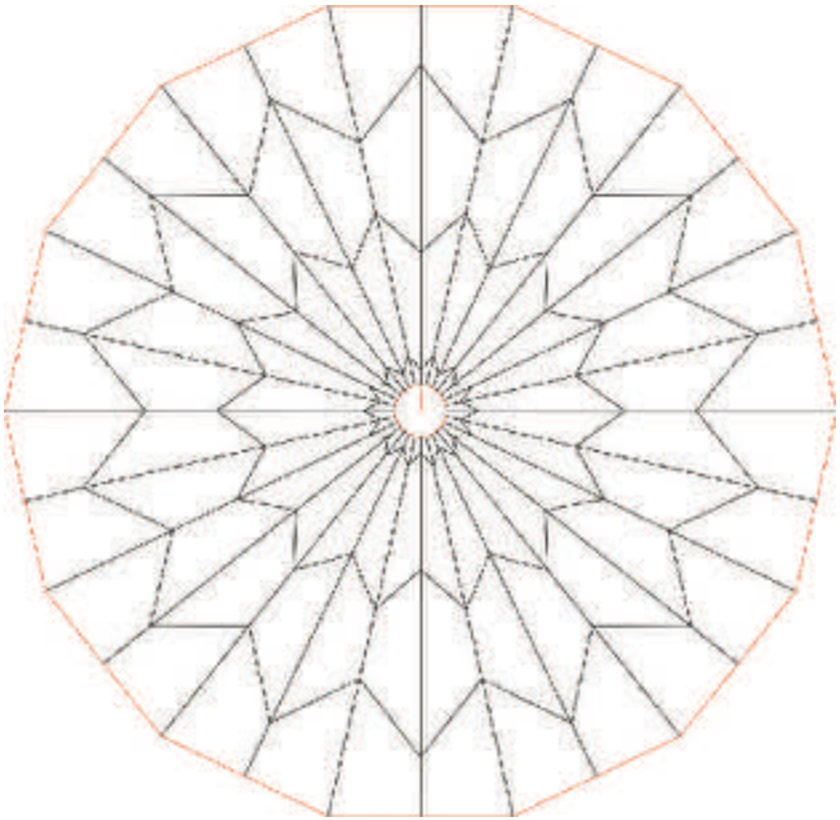
- —Microwave antenna by Stavros Georgakopoulos, Florida International University

- —Fashion designs by Adrienne Sack, Uyen Nguyen

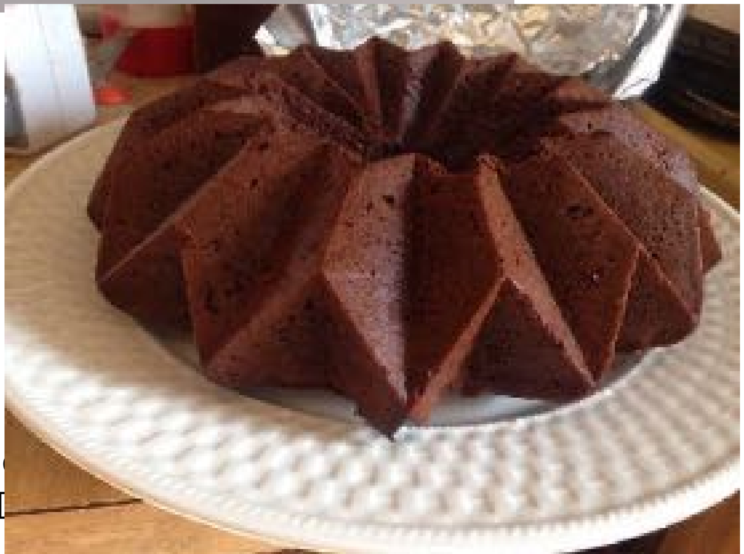
- —Mathematically analyzed by Simon Guest and Sergio Pellegrino in the 1990s



# …to food

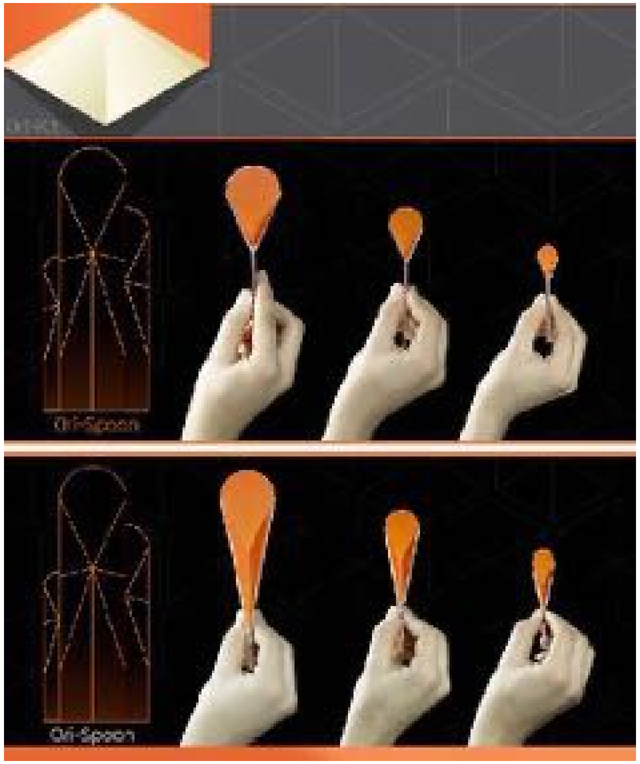


Courtesy of David Morgan,
Brigham Young University



# Ori-Kit folding utensils

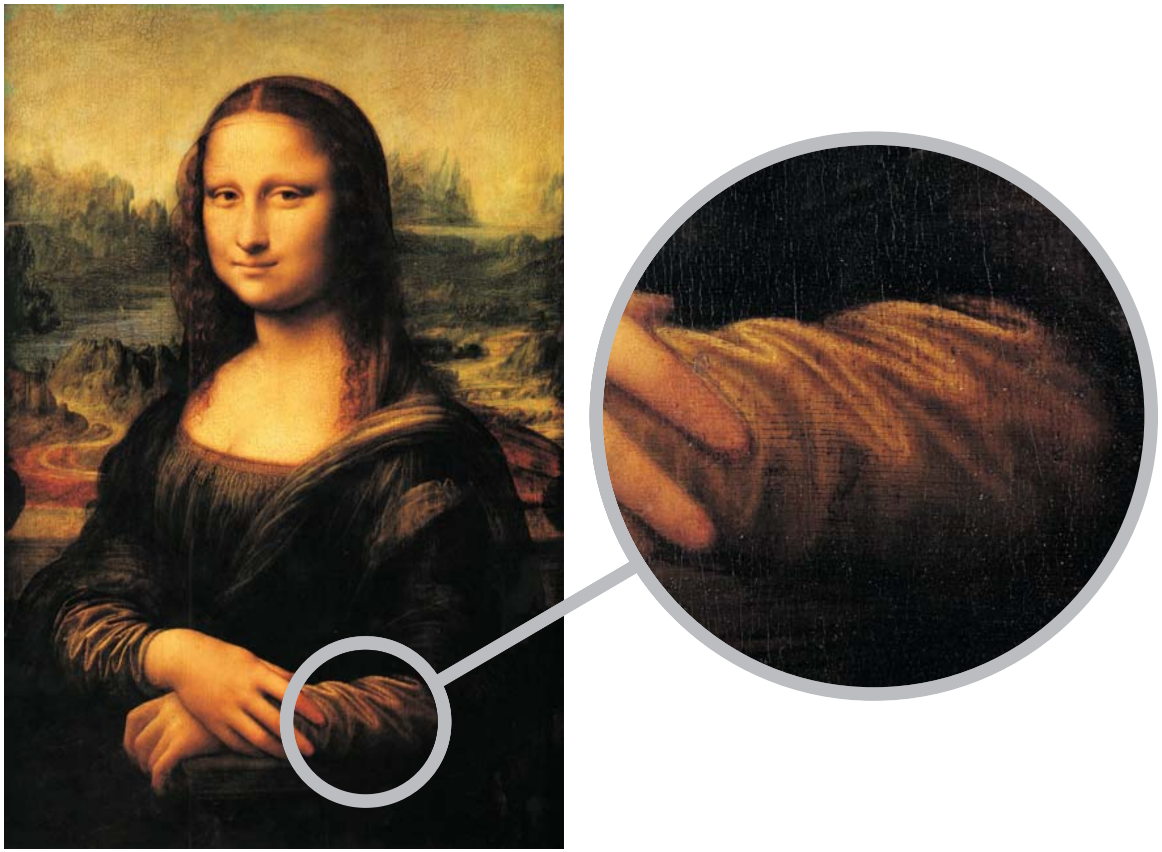
- Fold-up measuring spoons, colander
- “Small for storage, large at destination”





Courtesy of CrowdCreate, http://crowdcreate.us/ori-kit

# Folded concepts are timeless

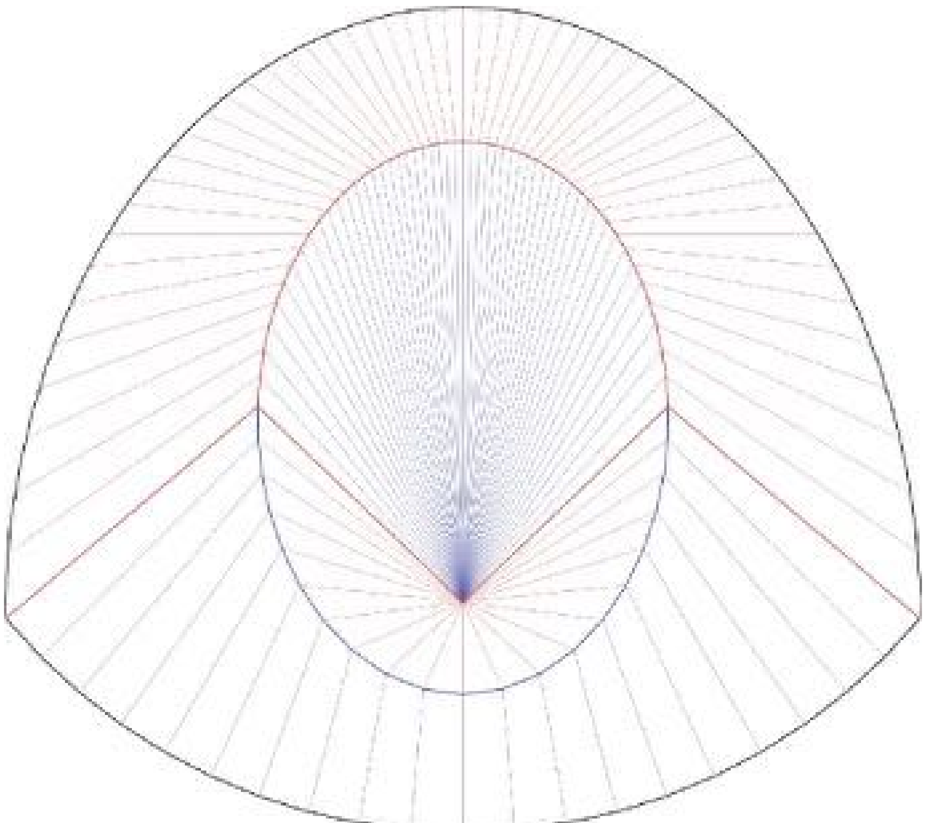


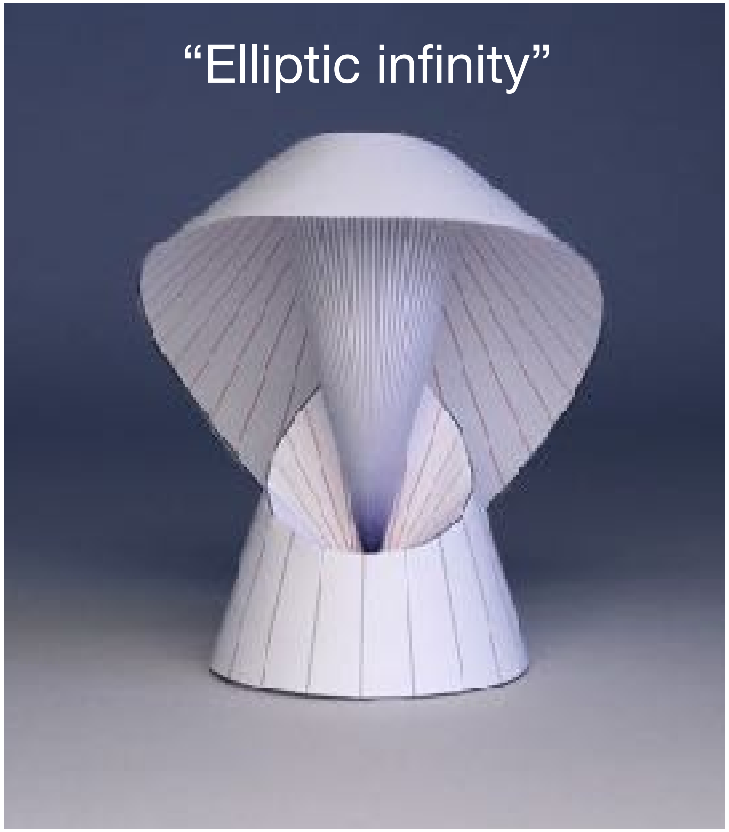

Soft drink can design by Koryo Miura

# Lamps

# Curved folding optics

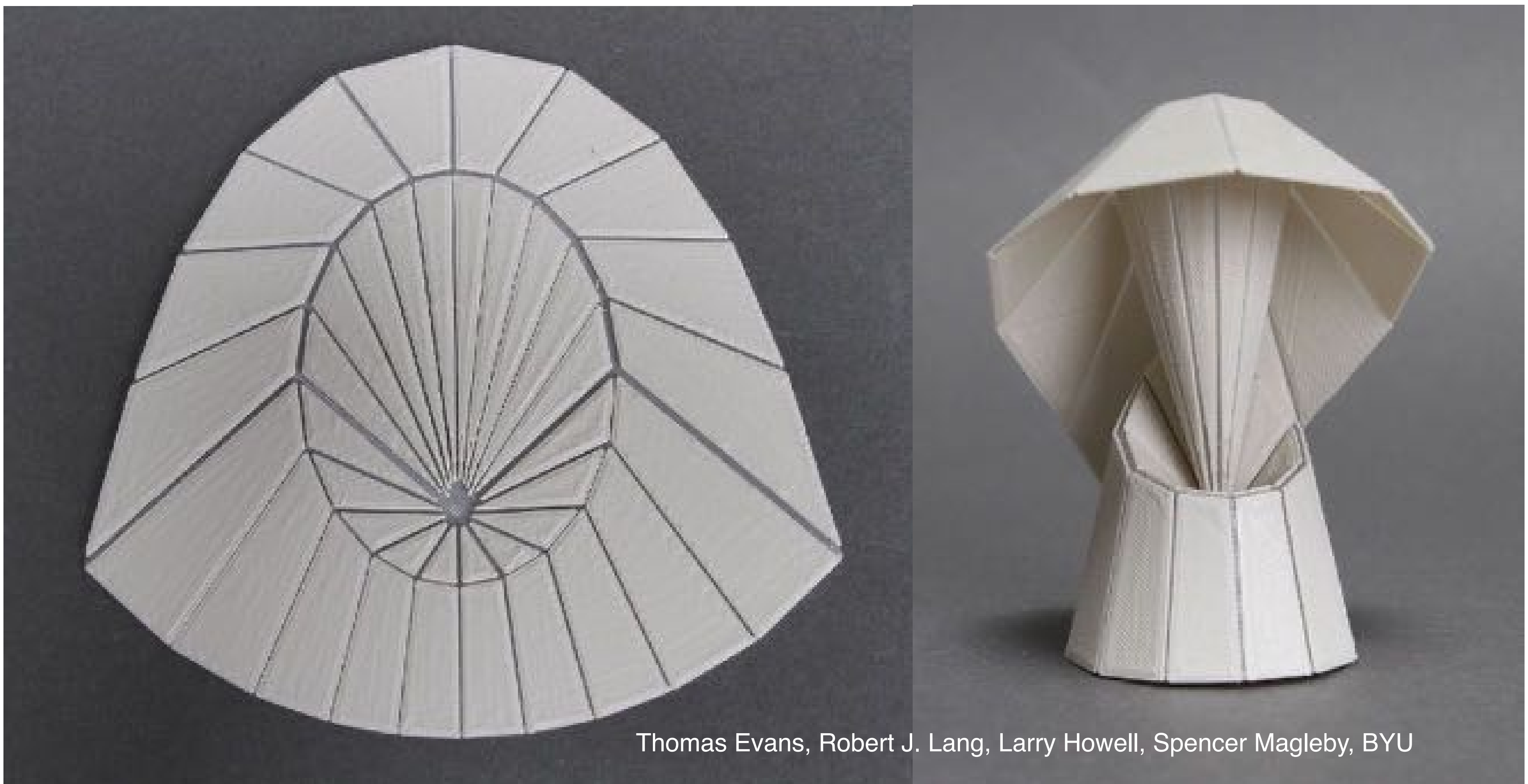
- Conic-section curved folding first explored by David Huffman
- Ruling lines refract like rays of light





# Elliptic infinity

- A discrete approximation of a curved-fold design



Thomas Evans, Robert J. Lang, Larry Howell, Spencer Magleby, BYU

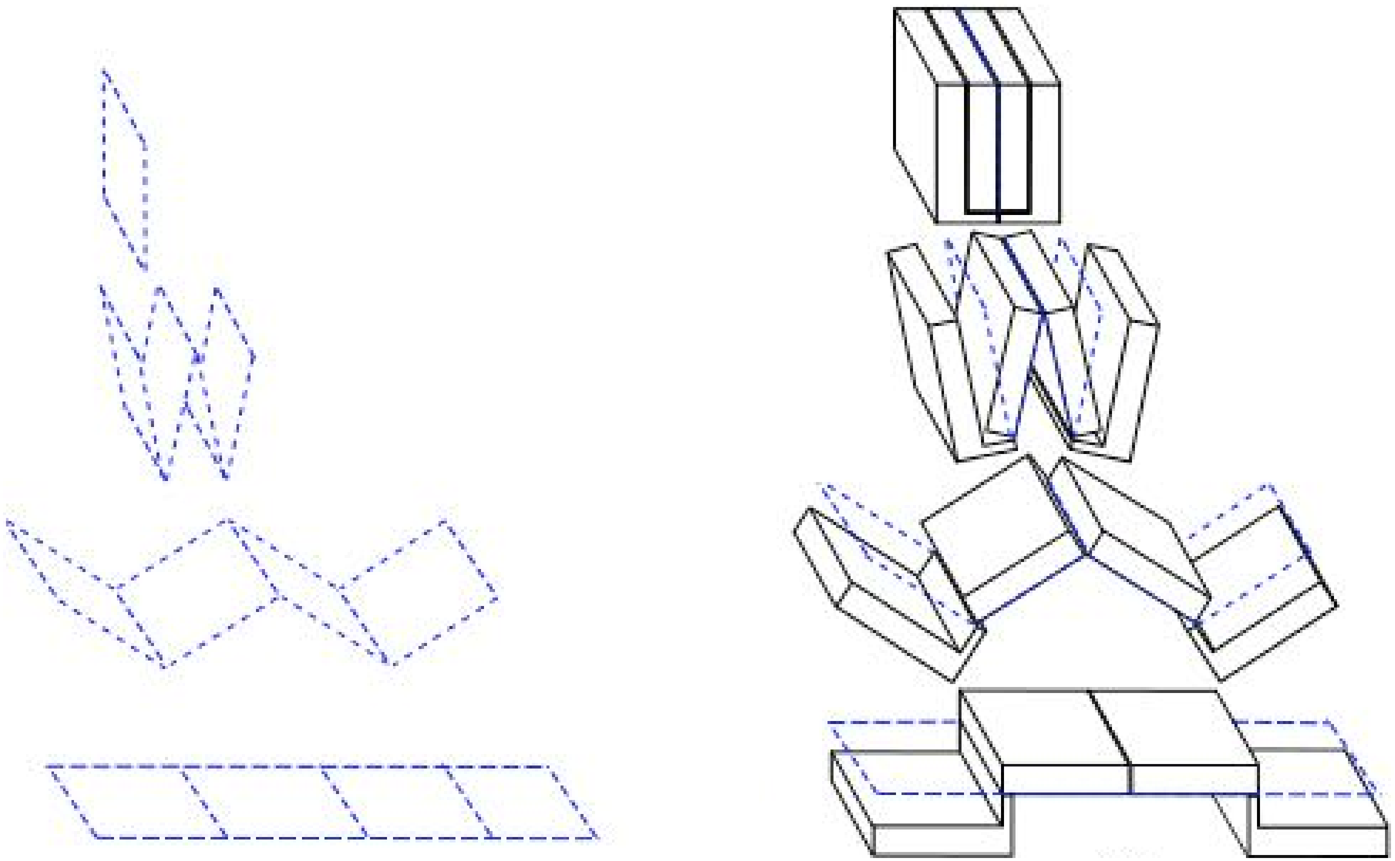
# The Origami Lamp

# Foldcore Panels

Images courtesy of Yves Klett,
Institute for Aircraft Design,
University of Stuttgart

# Thick origami

- Paper thickness usually negligible, but real-world applications must account for it



schematic courtesy of CMR Group, Brigham Young University

# Thick origami mechanisms

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images courtesy of CMR Group,
Brigham Young University

# Kinematics

- Current research at U. Tokyo, BYU, and Oxford extends origami to thick materials while preserving motion

video courtesy of CMR Group, Brigham Young University

# Application of thick origami

- Bulletproof barrier for law enforcement
- Begin with paper prototype

- Then develop “thick-origami” version

Imagery courtesy of CMR Group, Brigham Young University, and PBS

# Deployment

- Multi-layer Kevlar

- Single-person deployable

Images courtesy of CMR Group, Brigham Young University

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Ma

# Testing

Video courtesy of CMR Group, Brigham Young University

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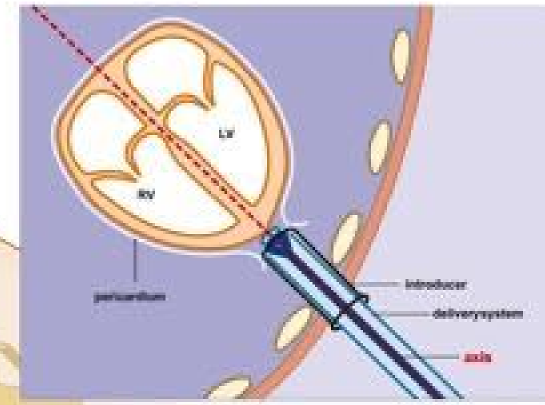
# Heart Implant

- Developed by Paracor Medical, Inc.
- The implant unfolds from delivery tube
- “Small for the journey, large at the destination”



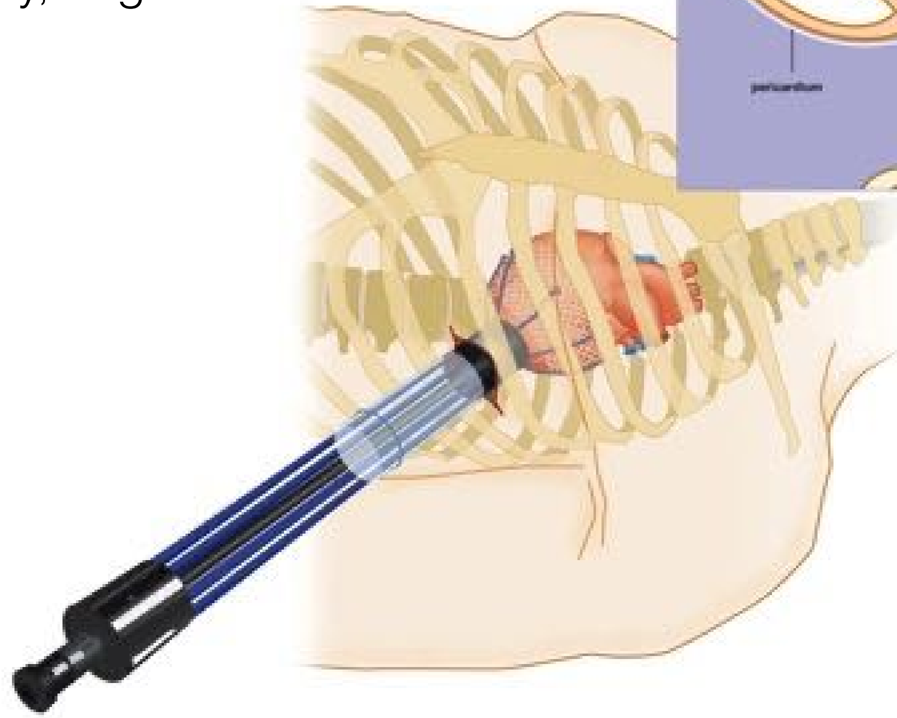



Image usage courtesy of Paracor Medical, Inc.

# Stents

- Origami Stent graft developed by Zhong You (Oxford University) and Kaori Kuribayashi

An origami stent made from stain-less steel. Its diameter expends from 12 mm to 23 mm.

www.tulane.edu/~sbc2003/pdfdocs/0257.PDF

# Artificial liver

- Concept: seed a sheet with liver cells,
- then fold into “lobule” groupings

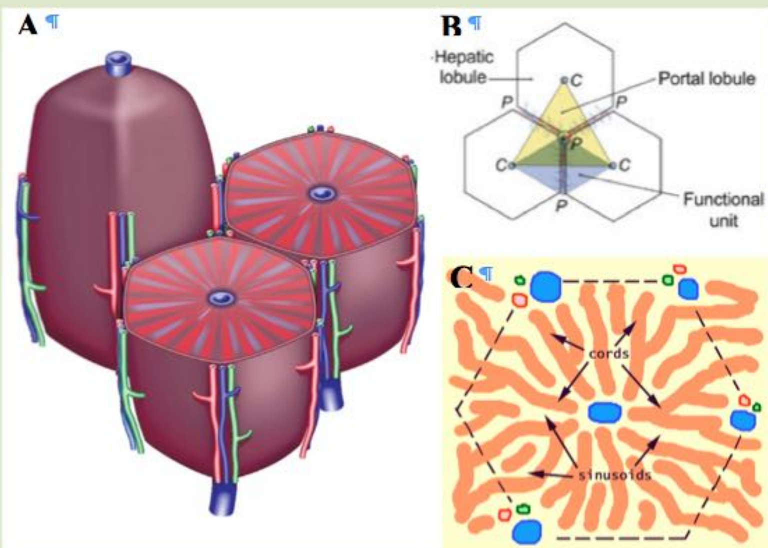
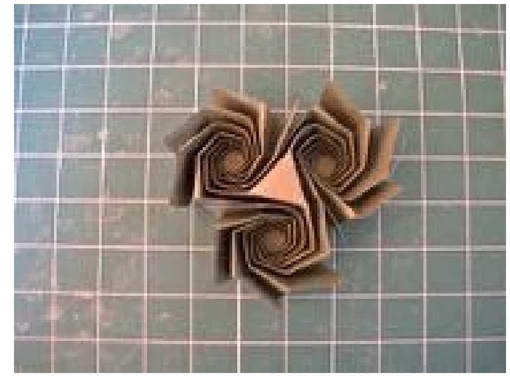
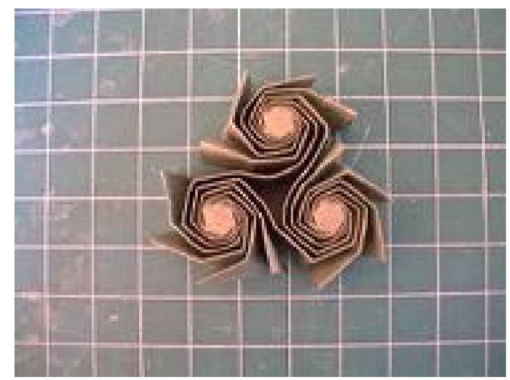


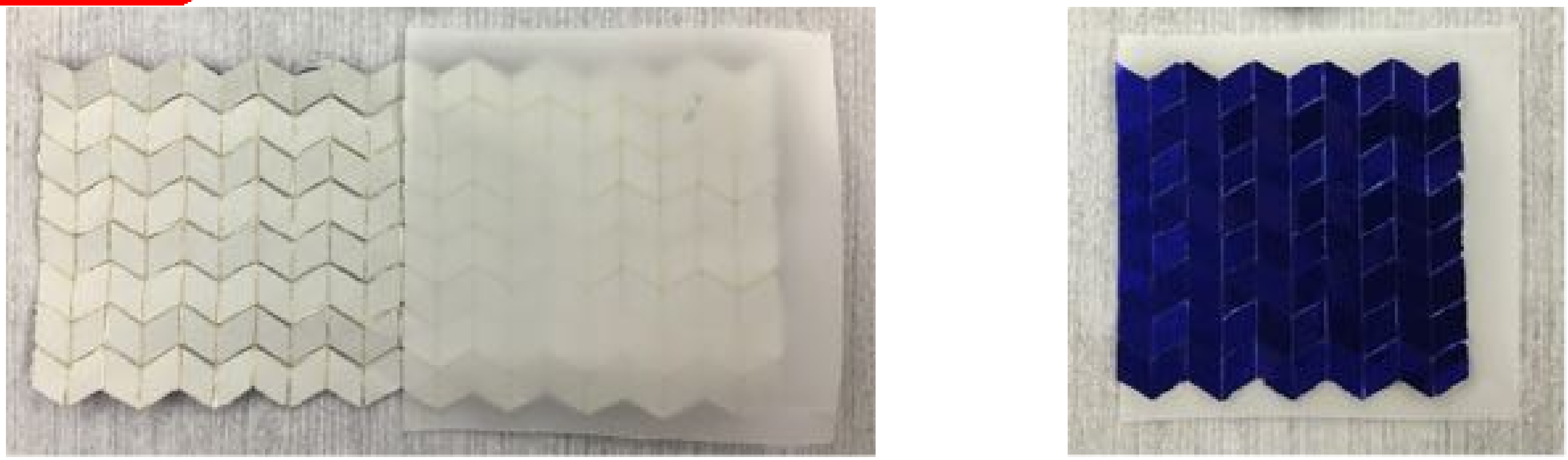

**Figure 4. (A) Schematic 3D rendering of arrayed liver lobules [*], (B) hepatic lobule, portal lobule, and functional unit as alternate subunits [*], and (C) schematic structure of a single lobule showing hepatic cords (orange), sinusoids (yellow), vasculature (blue and red), and bile ducts (green) [*].** ¶



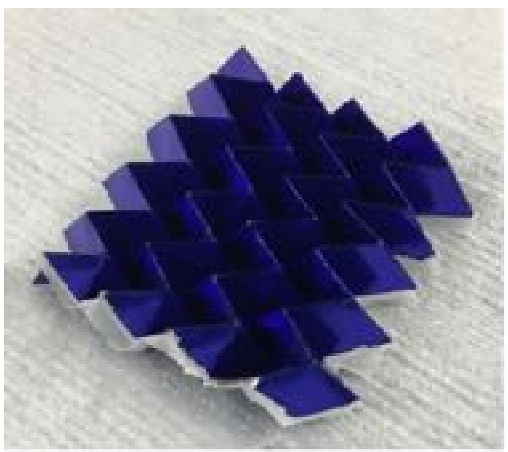


Carol Livermore et al, Northeastern University, MIT, San Jose State University
NSF EFRI-ODISSEI project

# Miura-folding parallel channels



1. Insert nanoporous membrane sheet between two laser-patterned foils

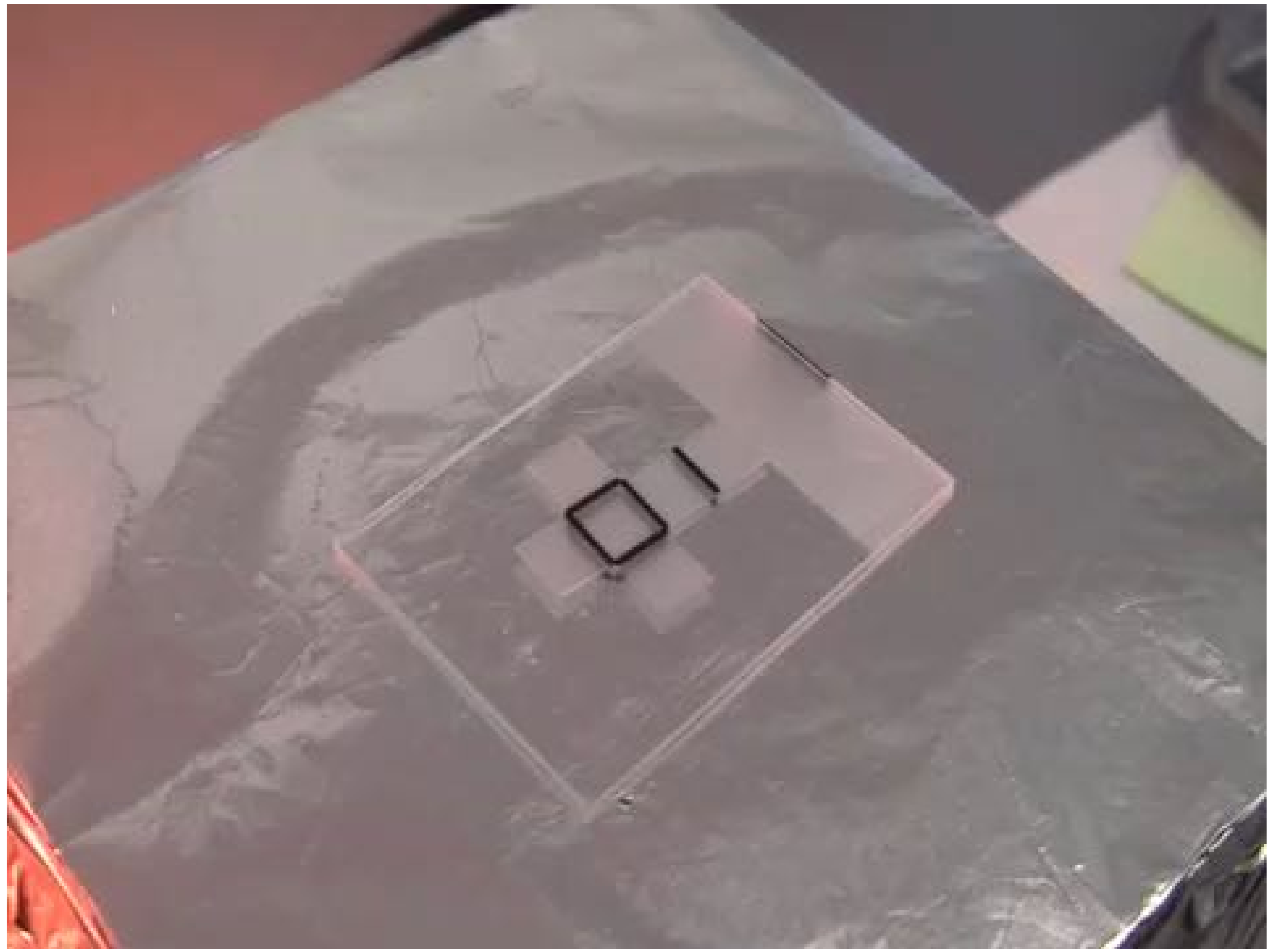


2. Fold the structure into Miura fold along patterned hinges

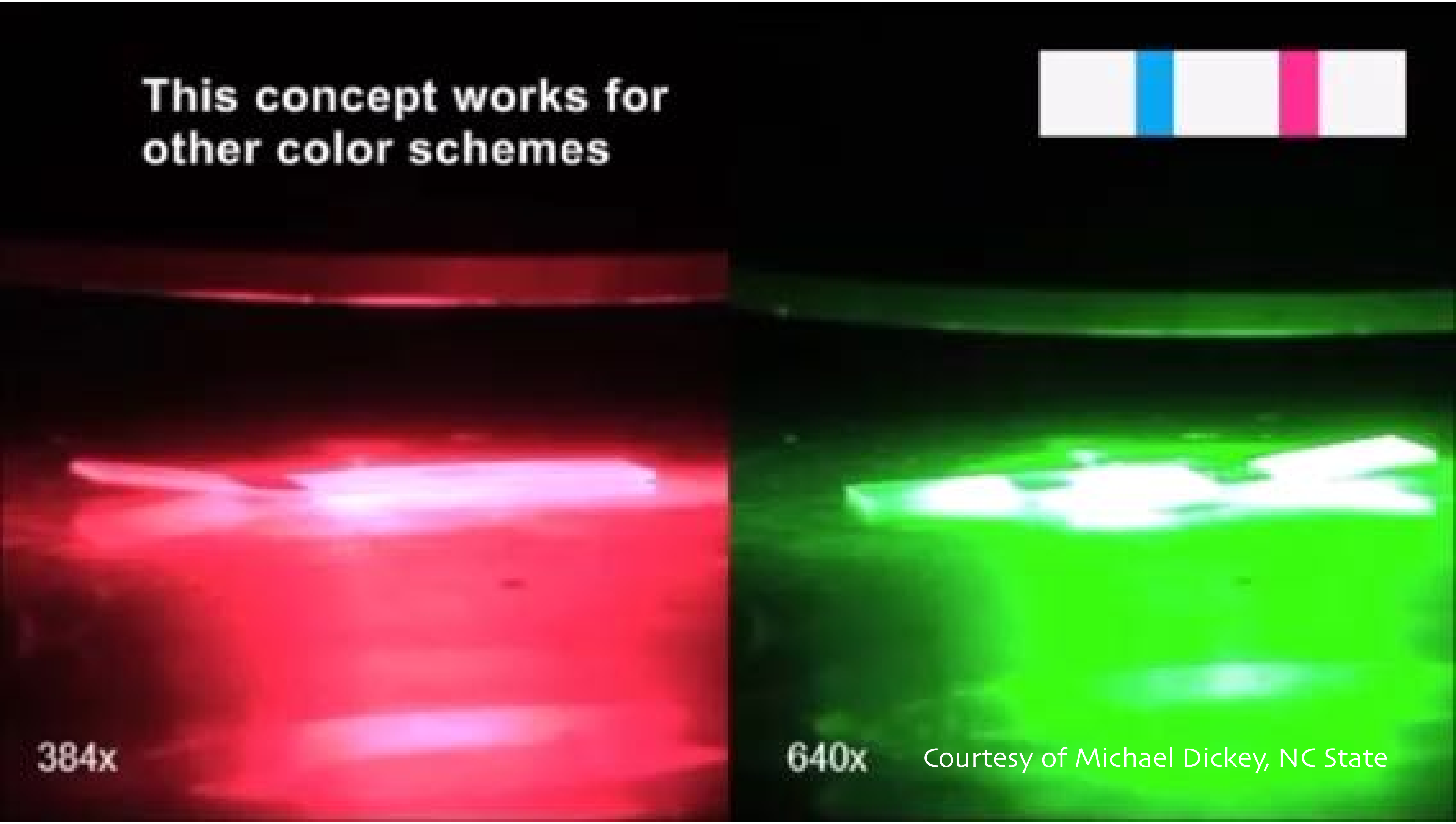
3. Release the nanoporous membrane

# Actuation

- Self-folding using local light absorption, Dickey et al., NC State University



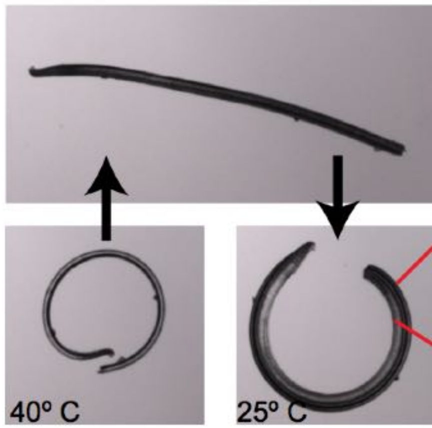
# Order by color

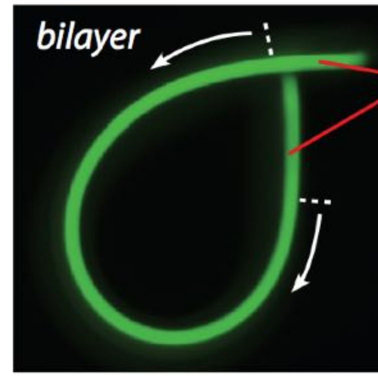


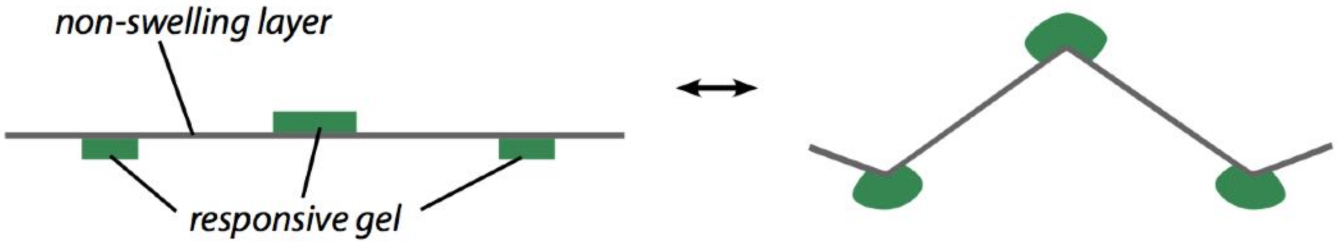
# Magnetic actuation

Courtesy Mary Frecker, Penn State

# Hydrogel programmed folding

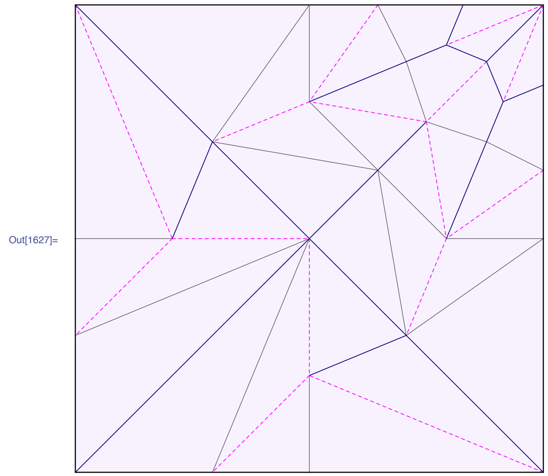


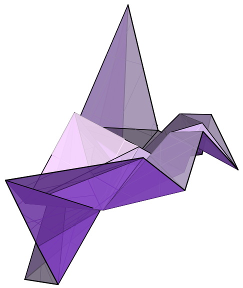



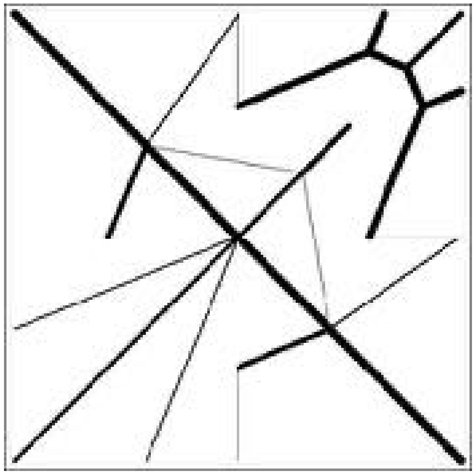

Ryan Hayward, Chris Santangelo (UMass), Itai Cohen (Cornell), Tom Hull (WNEU)
NSF EFRI-ODISSEI funded project

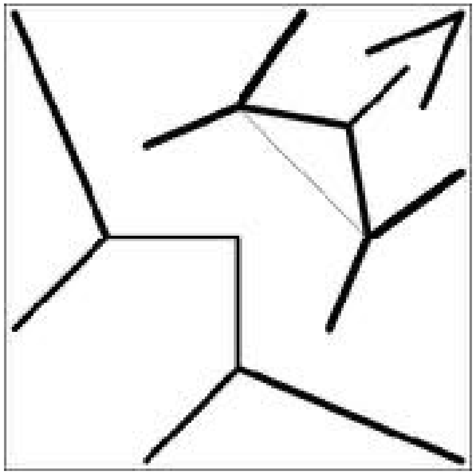
# Programmed folds



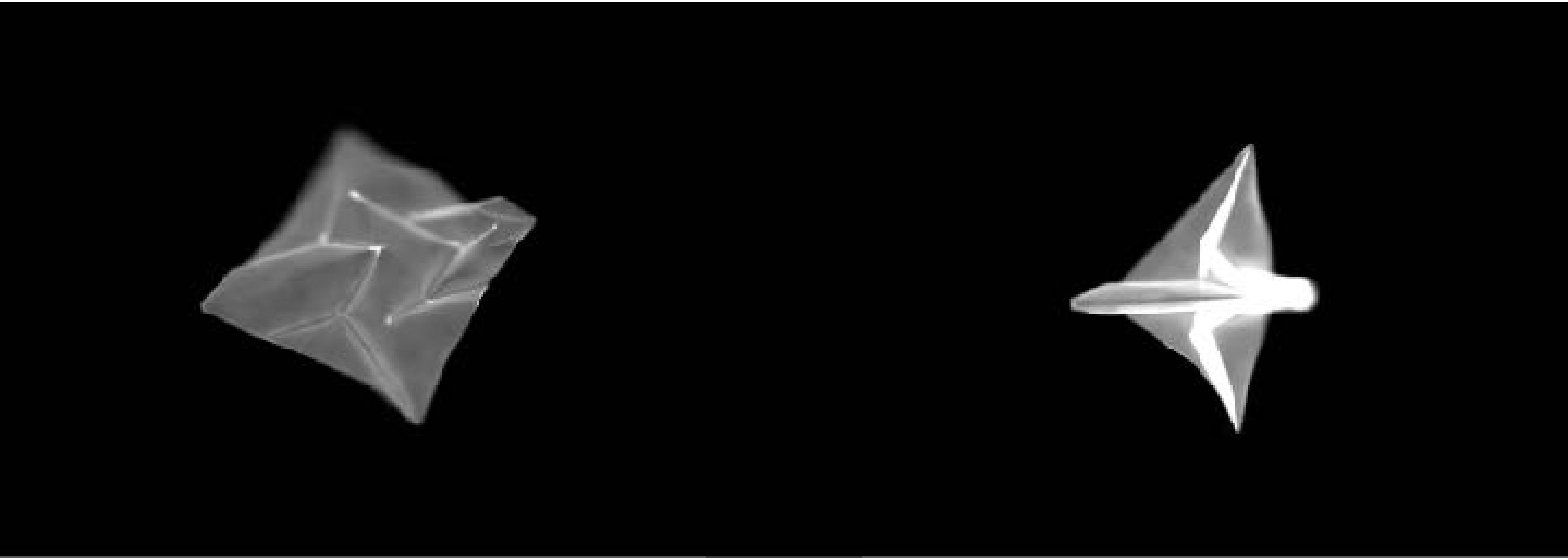
mountains





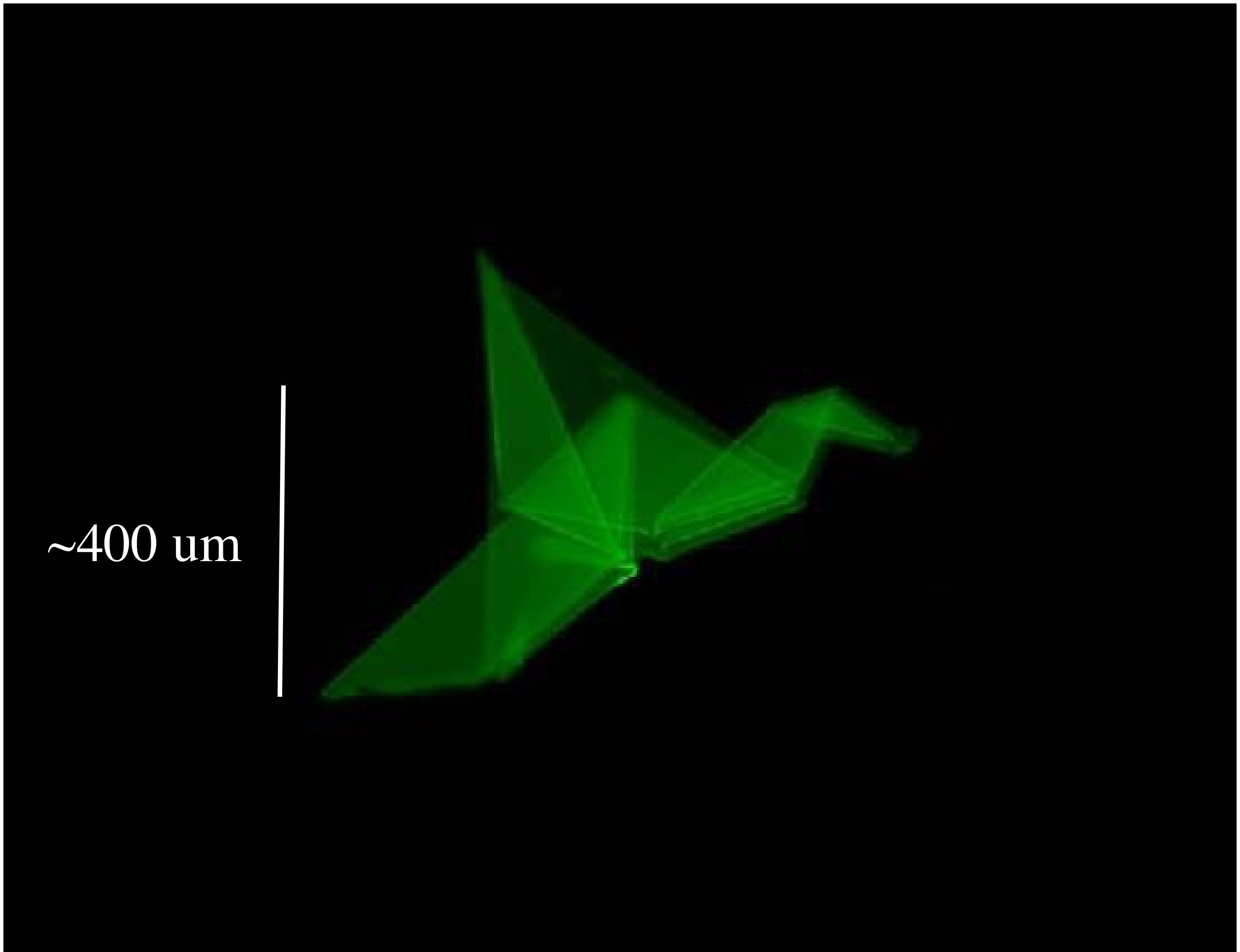
valleys

# Look, no hands



Ryan Hayward, Chris Santangelo, Jun-hee Na (UMass), Itai Cohen (Cornell), Tom Hull (WNEU)
NSF EFRI-ODISSEI funded project

# World’s smallest flapping bird



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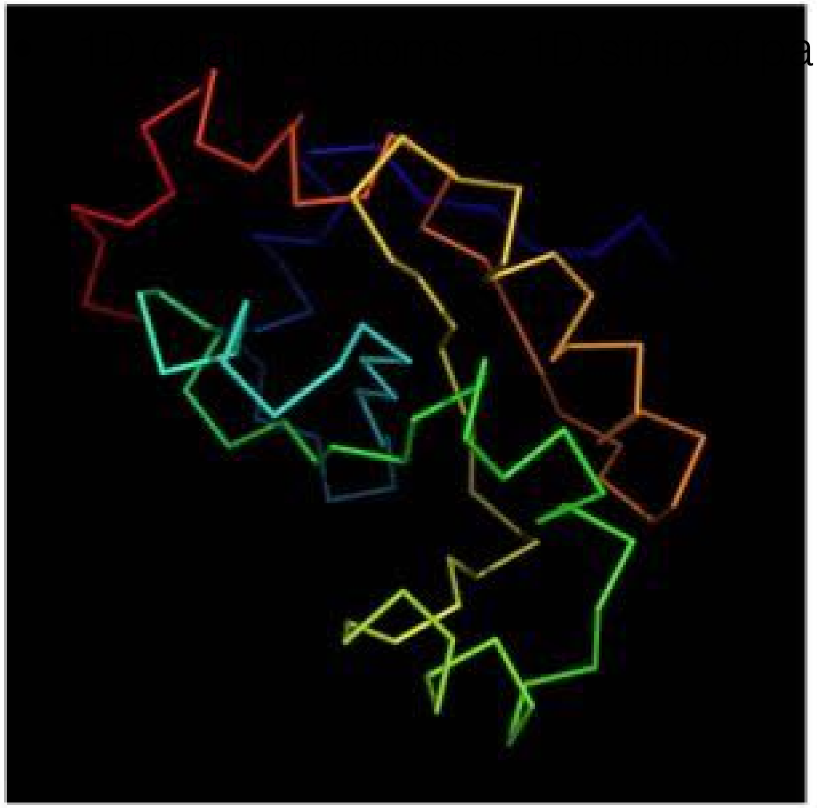
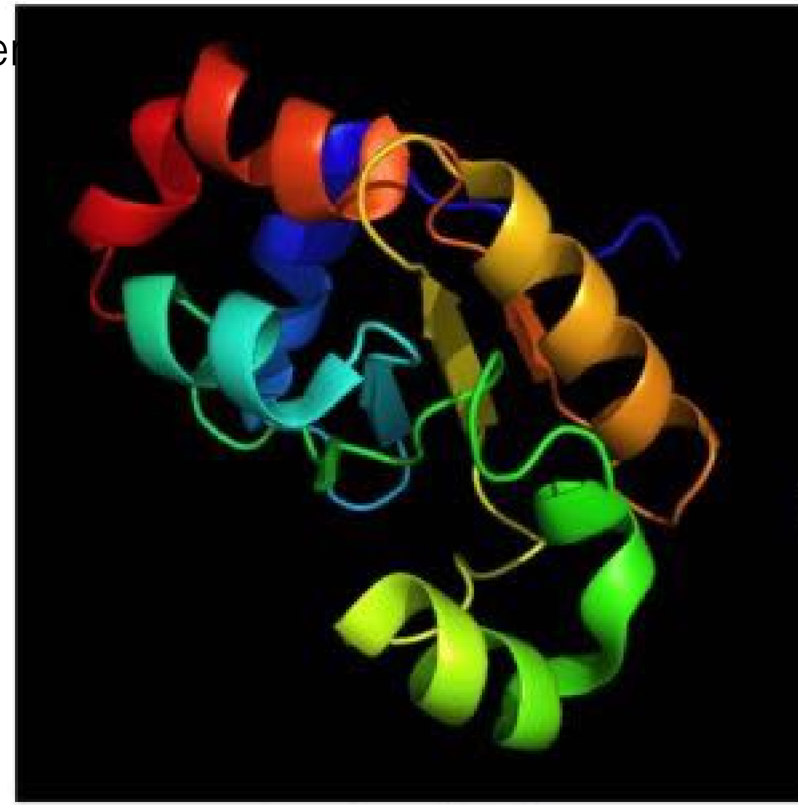
# Protein Folding





# Folds = Function

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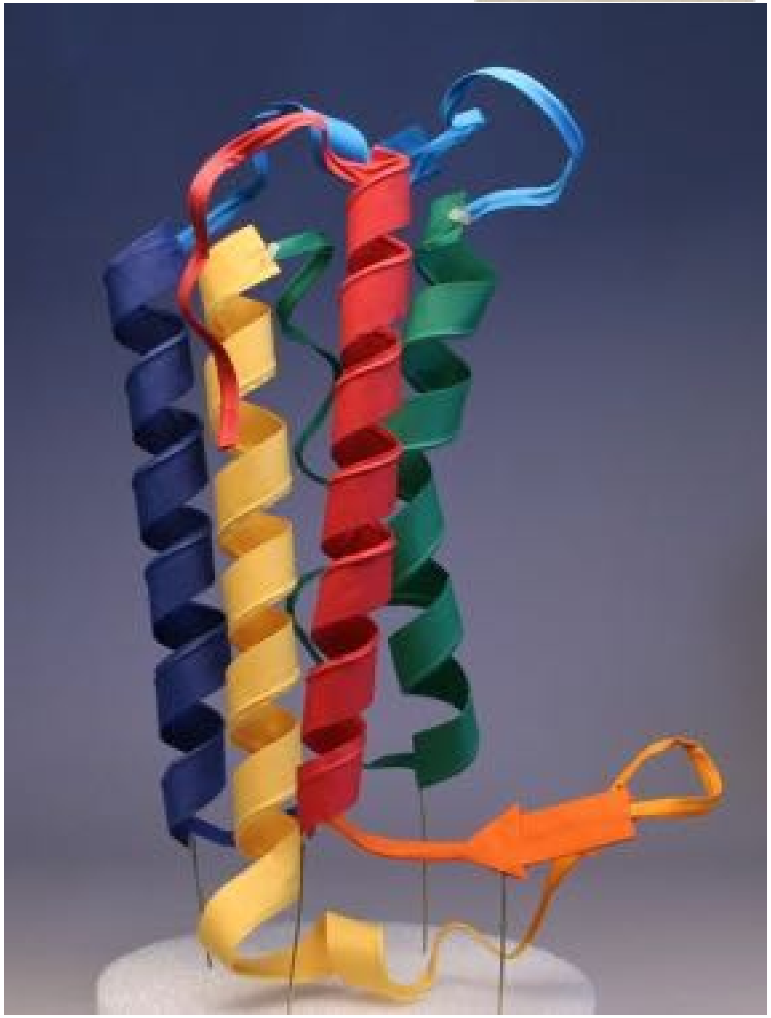
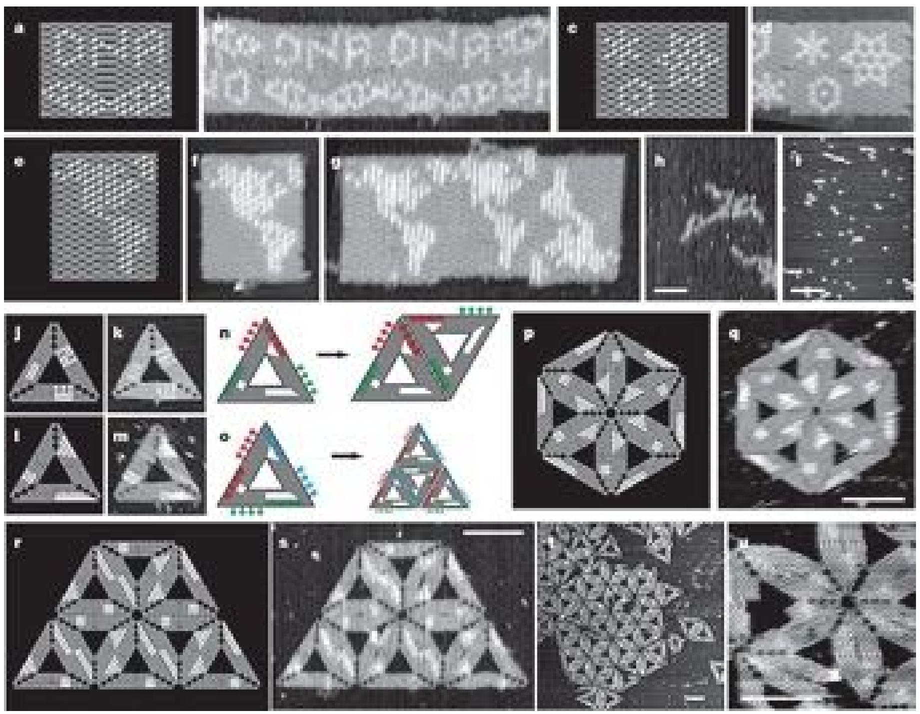
- The Grand Question of medicine and biology:
- How does a protein chain fold up?
- Fundamental laws of folding apply at any scale

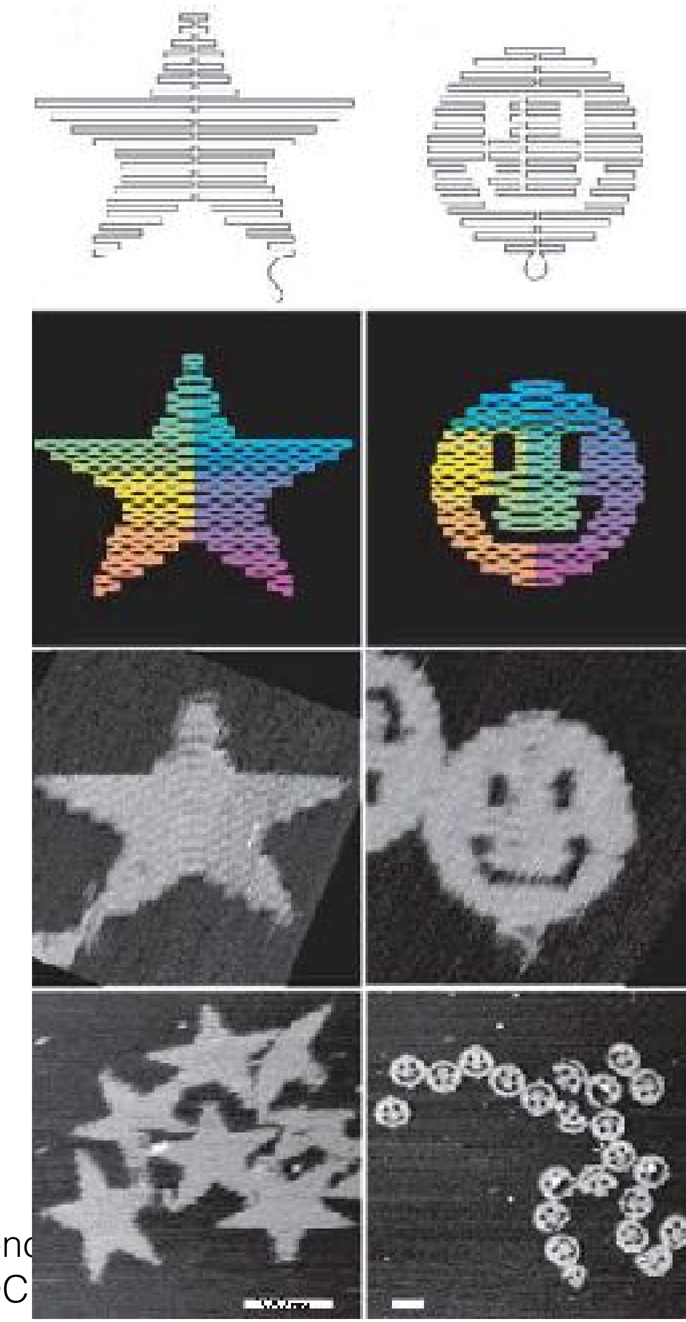


Origami "DinB-like protein (NP_980948.1) from Bacillus cereus"

# Folding DNA

- Paul Rothemund at Caltech developed techniques to fold DNA into origami shapes





Paul Rothemund, “Folding DNA to create nanoscale shapes and patterns,” Nature, 2006

# DNA Origami for Leukemia

- DNA origami cloaks anti-cancer drug (daunorubicin)
- Cancer cells swallow the “pill”, unfold the DNA, and die

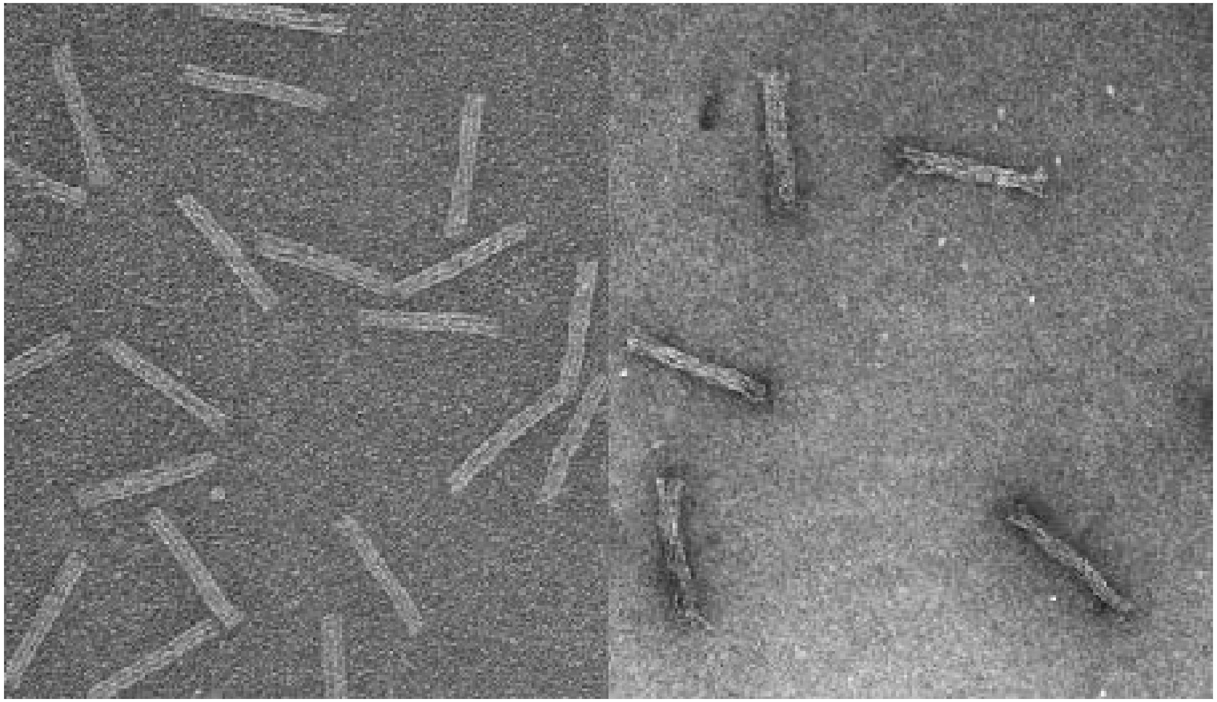
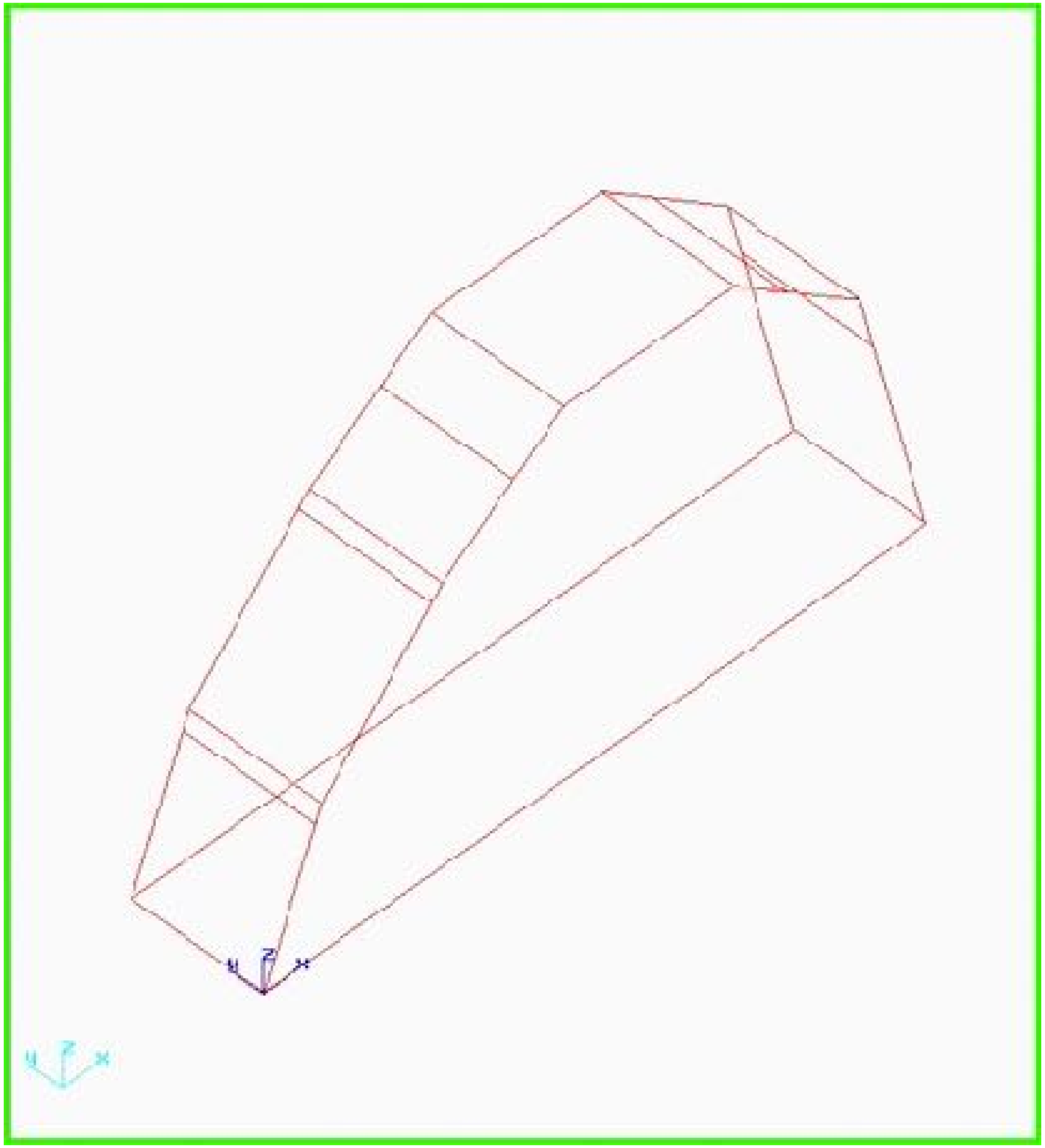


Image courtesy of John Byrd & Carlos Castro, Ohio State University (*Small*, March 2, 2016)

# Airbags

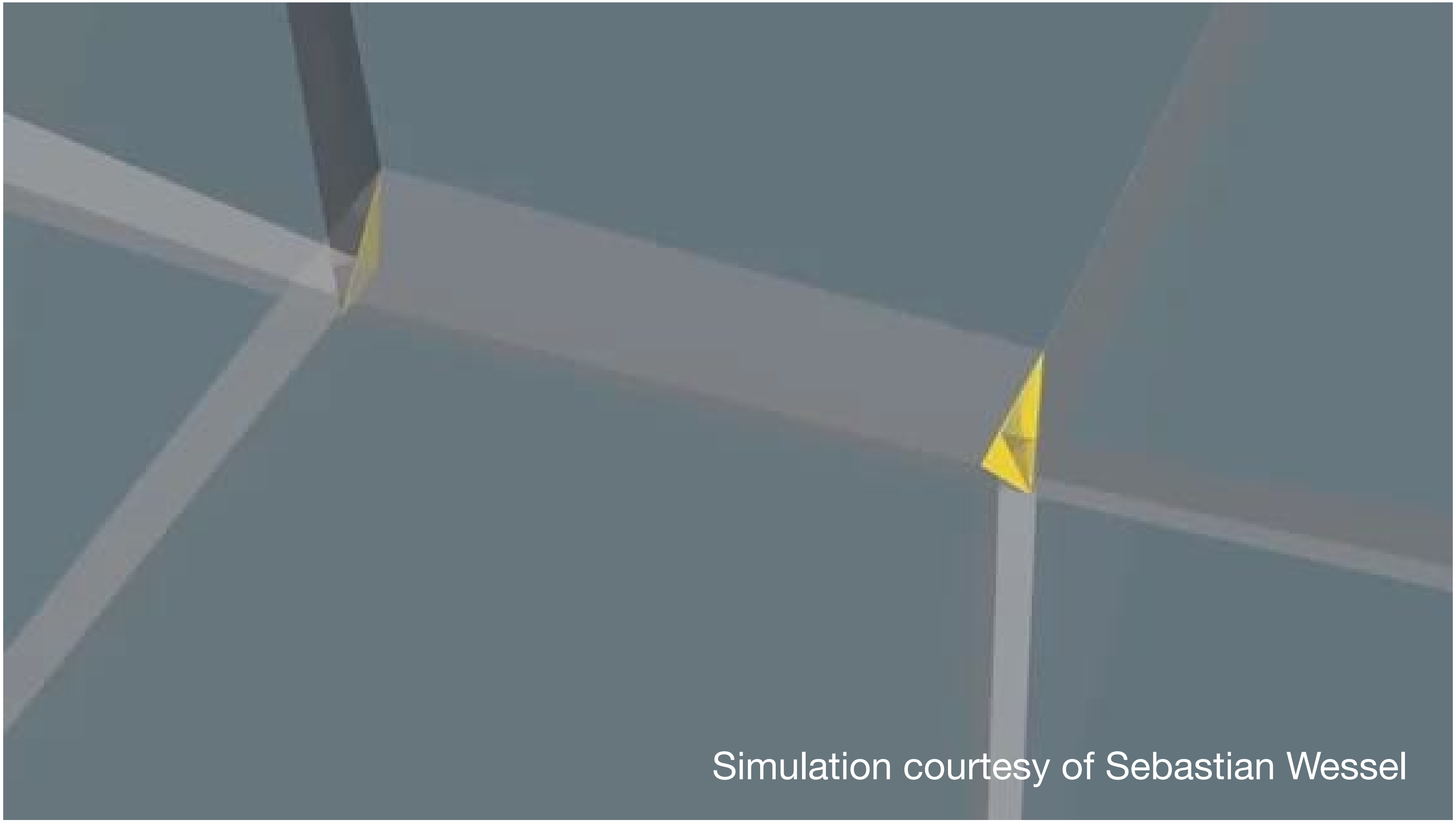
- Origami algorithms used in simulation



Animation courtesy EASi Enginering GmbH

# Origami in higher dimensions

- Simulations of dark matter reveal origami folding of 3D “sheets”



Simulation courtesy of Miguel Aragon-Calvo

# Resources

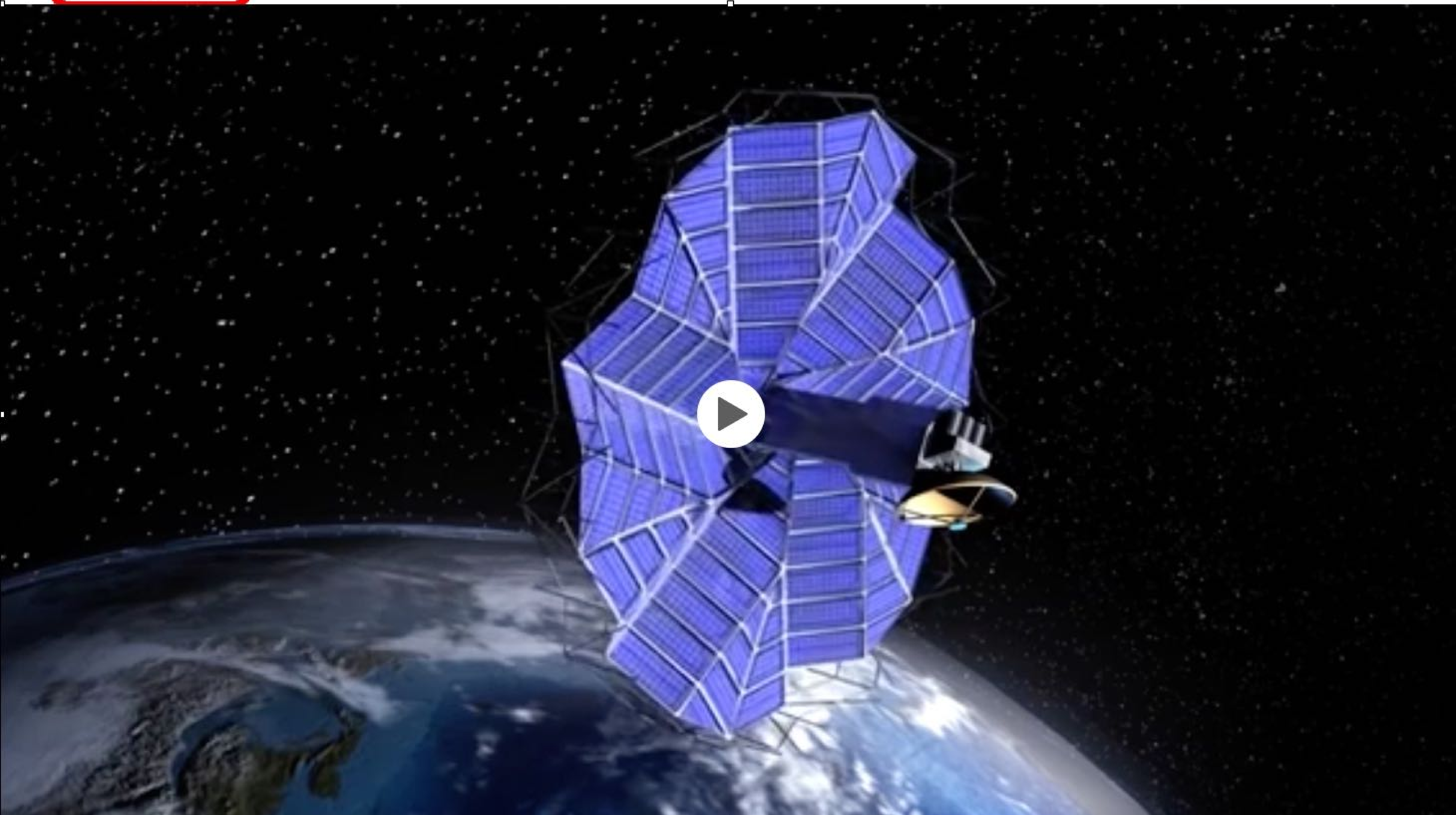
- Further information may be found at
  - http://www.langorigami.com
- "Between the Folds," a Peabody-award-winning origami documentary
  - http://www.betweenthefolds.com
- "The Origami Revolution"
  - PBS NOVA, streaming at http://www.pbs.org/video/2365955827/



# From the everyday...

- “TrailerTail” deployable flaps -- cut wind resistance

# To the future...



“Flying Peace,” by Kevin Box and Robert J. Lang, White Rock, NM