

BUILDING THINKING CLASSROOMS



- Liljedahl, P. (2014). The affordances of thinking classrooms. In Y. Li, E. Silber, & E. Silber (Eds.), *Mathematical Practices*. (pp. 1-10)

- Liljedahl, P. (2015). Thinking classrooms. In P. Kilpatrick, & E. Silber (Eds.), *Mathematical Perspectives*. (pp. 1-10)

- Liljedahl, P. (2016). Thinking classrooms. *Proceedings of the 40th International Conference of the International Association for the Development of Mathematics Education*, Szeged, Hungary.

- Liljedahl, P. (2017). *The 1st International Conference on Mathematics Education*. Korea.

- Liljedahl, P. (2018). Thinking classrooms. In S. Carreira, N. Carreira, & K. Jordano (Eds.), *Mathematical problem solving: A focus on technology*. (pp. 1-10)

- Liljedahl, P. (2019). *Teaching and learning in a thinking classroom*. New York: Routledge.

- Liljedahl, P. & Alla B. Pieronkiewicz (2020). *Towarzystwo Autorytetu*. (pp. 1-10)

- Liljedahl, P. (in press). Thinking classrooms. Thousand Oaks, CA: Sage.

- Mike Pruner, MSc (2019). Thinking classrooms. (pp. 1-10)

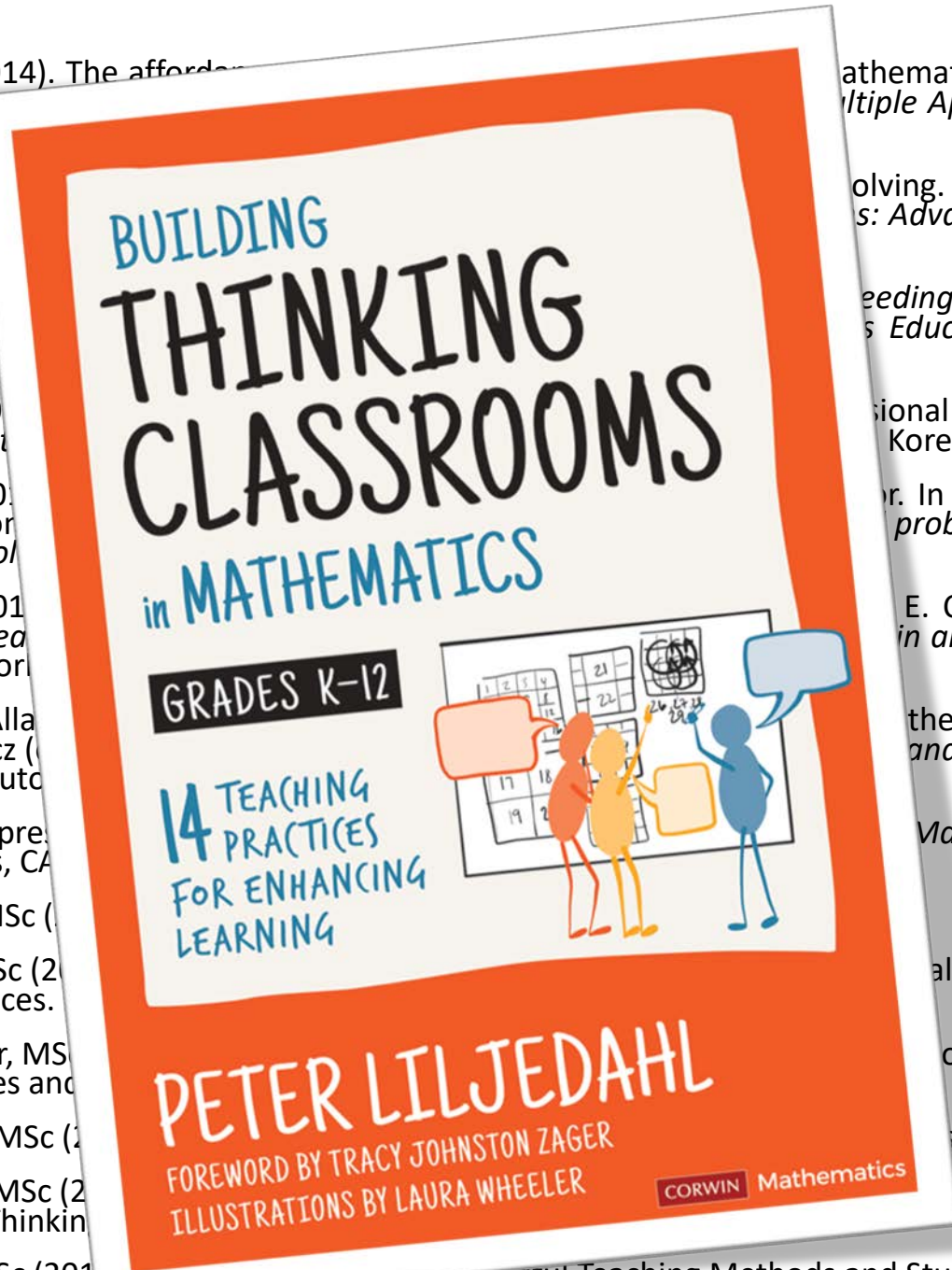
- Oana Chiru, MSc (2019). Thinking classrooms in Common Places. (pp. 1-10)

- Chris McGregor, MSc (2019). Thinking classrooms on Vertical Surfaces and Walls. (pp. 1-10)

- Maria Kerkoff, MSc (2019). Thinking classrooms. (pp. 1-10)

- Beth Baldwin, MSc (2019). Thinking classrooms: Efficacies in a Thinking Classroom. (pp. 1-10)

- Nikki Mann, MSc (2019). Thinking classrooms: Perception of their Reception of Mathematical Knowledge. (pp. 1-10)



mathematics classroom. *Multiple Approaches and*

solving. In P. Felmer, J. ... *s: Advances and New*

eedings of the 40th s Education, Szeged,

sional Development. Korea.

r. In S. Carreira, N. ... *problem solving: A*

E. Chernoff (eds.) *in an international*

the possibility. In *and its education*.

Math Learning.

al Experiences

on-Permanent

ks.

nd Group

... Mindful Teaching Methods and Student

Perception of their Reception of Mathematical Knowledge

18 YEARS AGO ...



If 6 cats can kill 6 rats in 6 minutes, how many cats are required to kill 100 rats in 50 minutes?

- Lewis Carroll





If 6 cats can kill 6 rats in 6 minutes, how many cats are required to kill 100 rats in 50 minutes?

- Lewis Carroll

If 6 cats can kill 6 rats in 6 minutes, how many cats are required to kill 100 rats in 50 minutes?

- *Lewis Carroll*

STUDENTING

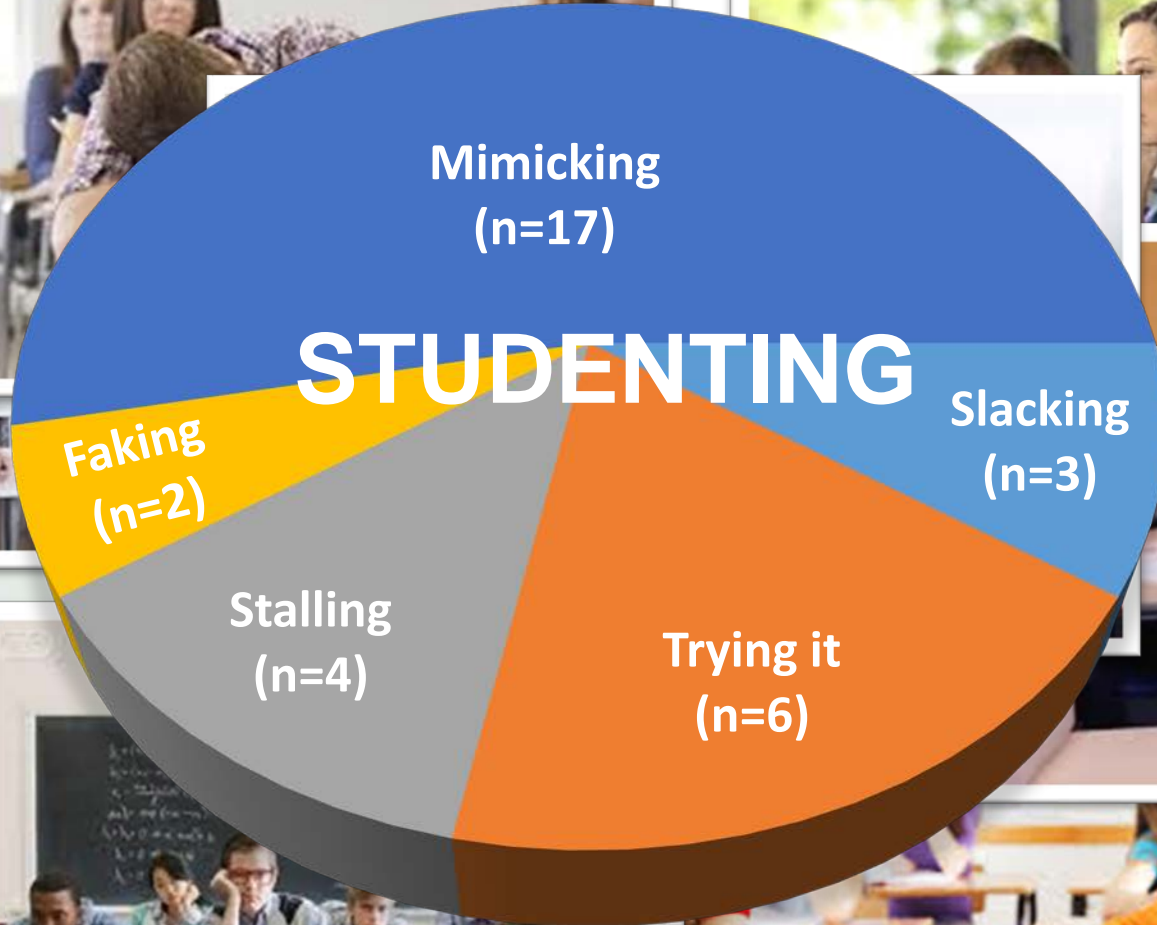
Mimicking
(n=17)

Slacking
(n=3)

Trying it
(n=6)

Stalling
(n=4)

Faking
(n=2)










INSTITUTIONAL NORMS



NON-NEGOTIATED NORMS





**INSTITUTIONAL
NORMS**

**STUDENTS
NOT THINKING**



400+ TEACHERS | 15 YEARS | 2 WEEK CYCLES



RENEGOTIATING THE NON-NEGOTIATED NORMS

400+ TEACHERS | 15 YEARS | 2 WEEK CYCLES

CLASSROOM PRACTICES

1	What are the types of tasks we use?	
2	How we form collaborative groups?	
3	Where students work?	
4	How we arrange the furniture in our classroom?	
5	How we answer questions?	
6	When, where, and how tasks are given?	
7	What homework looks like?	
8	How we foster student autonomy?	
9	How we use hints and extensions?	
10	How we consolidate a lesson?	
11	How we give notes?	
12	What we choose to evaluate?	
13	How we use formative assessment?	
14	How we grade?	

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1 What are the types of tasks we use?	
2 How we form collaborative groups?	
3 Where students work?	
4 How we arrange the furniture in our classroom?	
5 How we answer questions?	
6 When, where, and how tasks are given?	
7 What homework looks like?	
8 How we foster student autonomy?	
9 How we use hints and extensions?	
10 How we consolidate a lesson?	
11 How we give notes?	
12 What we choose to evaluate?	
13 How we use formative assessment?	
14 How we grade?	

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1 What are the types of tasks we use?	Use thinking tasks
2 How we form collaborative groups?	Form frequent visibly random groupings
3 Where students work?	Use vertical non-permanent surfaces
4 How we arrange the furniture in our classroom?	<i>Defront</i> the classroom
5 How we answer questions?	Only answer keep thinking questions
6 When, where, and how tasks are given?	Give tasks early, standing, and verbally
7 What homework looks like?	Give check your understanding questions
8 How we foster student autonomy?	Be intentionally less helpful
9 How we use hints and extensions?	Create and manage <i>flow</i>
10 How we consolidate a lesson?	Consolidate from the bottom
11 How we give notes?	Use meaningful notes
12 What we choose to evaluate?	Evaluate what you value
13 How we use formative assessment?	Communicate to students where they are and where they are going
14 How we grade?	Report out based on data (not points)

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1 What are the types of tasks we use?

2 How we...

3 W...

4 Gol...

5 You...

6 hor...

7 pa...

8 an...

9 W...

10 ho...

11 ho...

12 ho...

13 ho...

14 ho...

15 ho...

16 ho...

17 ho...

18 ho...

19 ho...

20 ho...

21 ho...

22 ho...

23 ho...

24 ho...

25 ho...

26 ho...

27 ho...

28 ho...

29 ho...

30 ho...

31 ho...

32 ho...

33 ho...

34 ho...

35 ho...

36 ho...

37 ho...

38 ho...

39 ho...

40 ho...

41 ho...

42 ho...

43 ho...

44 ho...

45 ho...

46 ho...

47 ho...

48 ho...

49 ho...

50 ho...

51 ho...

52 ho...

53 ho...

54 ho...

55 ho...

56 ho...

57 ho...

58 ho...

59 ho...

60 ho...

61 ho...

62 ho...

63 ho...

64 ho...

65 ho...

66 ho...

67 ho...

68 ho...

69 ho...

70 ho...

71 ho...

72 ho...

73 ho...

74 ho...

75 ho...

76 ho...

77 ho...

78 ho...

79 ho...

80 ho...

81 ho...

82 ho...

83 ho...

84 ho...

85 ho...

86 ho...

87 ho...

88 ho...

89 ho...

90 ho...

91 ho...

92 ho...

93 ho...

94 ho...

95 ho...

96 ho...

97 ho...

98 ho...

99 ho...

100 ho...

FURTHER PRACTICE

5. Factorize each of the following.

- (a) $2x^2 - 12x + 18$
- (b) $45m^2 + 60mn + 20n^2$
- (c) $5 - 125n^2$
- (d) $x^2y - 22xy + 121y$
- (e) $72m^2 - 98n^2$
- (f) $9a^3 - 12a^2b + 4ab^2$
- (g) $3x^3 - 48x$
- (h) $18y^2z^2 - 2y^4$

6. Factorize each of the following.

- (a) $(x + y)^2 - 14(x + y) + 49$
- (b) $(a + 3b)^2 - (2c + d)^2$
- (c) $25 - (u - v)^2$
- (d) $1 - 12xy + 36x^2y^2$

MATH WORK

7. Harry has $(9x^2 + 24xy + 16y^2)$ marbles, where x and y are positive integers. He arranges the marbles as a square array.

- (a) Express, in terms of x and y , the number of marbles on each side of the array.
- (b) When $x = 2$ and $y = 5$, find the number of marbles on a side of the array.

BRAINWORKS

8. Factorize each of the following.

- (a) $y^4 - 81$
- (b) $z^4 - 625$

9. (a) Expand $(a + b + c)^2$.
(b) Factorize $a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac$.
(c) Let $E = a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac$.
(i) What is the minimum value of E ?
(ii) Find two possible sets of values of a , b , and c such that the value of E is the minimum.

10. Alex was asked to factorize

$$x^2y^2 + 36 - 4x^2 - 9y^2.$$

He tried some ways of grouping terms as shown below.

$$\begin{aligned} x^2y^2 + 36 - 4x^2 - 9y^2 &= (x^2y^2 + 36) - (4x^2 + 9y^2) \\ x^2y^2 + 36 - 4x^2 - 9y^2 &= (x^2y^2 + 36 - 4x^2) - 9y^2 \\ x^2y^2 + 36 - 4x^2 - 9y^2 &= x^2y^2 + (36 - 4x^2 - 9y^2) \end{aligned}$$

As he could not carry out factorization with the above groupings, he concluded that the expression could not be factorized. Do you agree with him? Why or why not?

Share

10

Moves

2 1+1

4 1+3

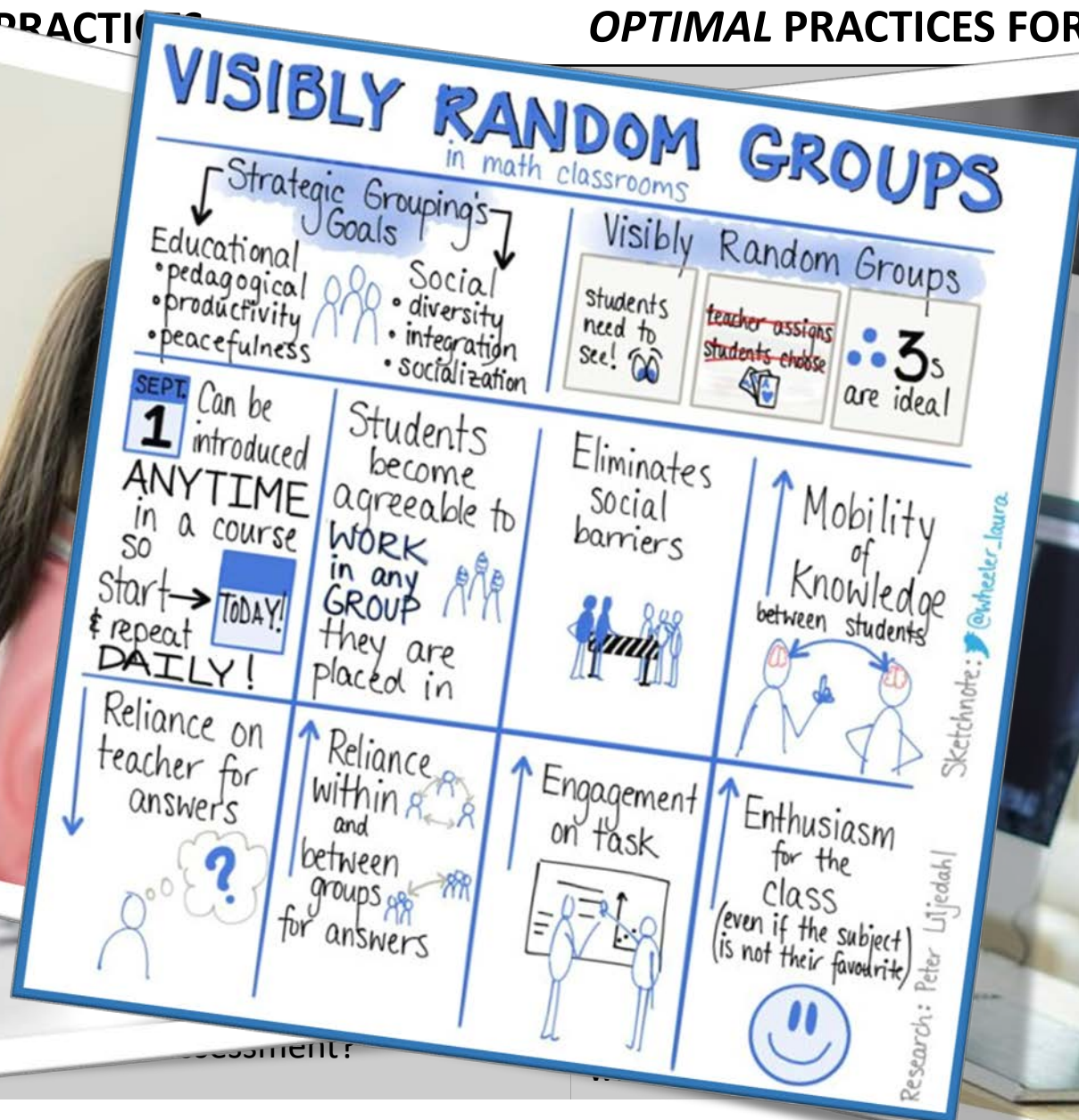
3+3

1+3+3

6+6

Communicate to students where they are and where they are going

Report out based on data (not points)



1 What a

2 How

3

4

5

6

7

8

9

10

11 n

12 W

13 Ho assessment?

14 How we grade?

Report out based on data (s)

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1

Use thinking tasks

2

3

4

5

12 What

13 How

14 How we grade?

where they are ge they are and

Report out based on data (not points)

CLASSROOM

1 What

2 How v

3 Whe

4 How

5

8

9

10

11

12 What we

13 How we use

14 How we grad

Another group has decided to keep their numbers small (and repeat some).
We are using e and o also. Nice!
Our hypothesis broke with 5 we are trying it now with an even number of numbers (i.e. 6 numbers, 8 numbers, etc)



Some letters may make what is happening more transparent.



$(a+b)$ $(b+c)$ $(c+d)$

$a+2b+c$ $b+2c+d$

$a+3b+3c+d$

I wonder what this would look like for 5 numbers?
New hypothesis: When starting with an EVEN number of selected numbers, an odd amount of odds yields an odd final sum. works for 6 - try 1,3,5,2,4,6

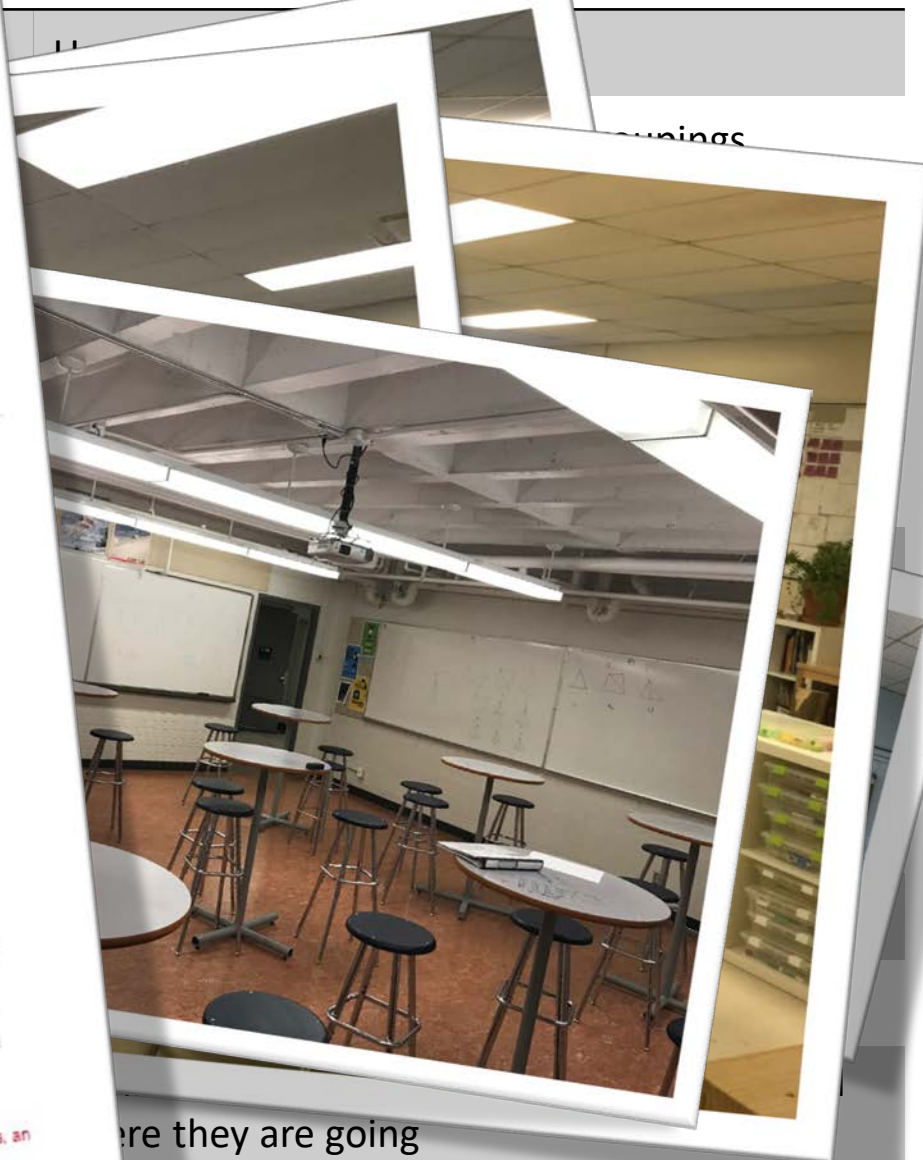
OPTIMAL PRACTICES FOR THINKING

11

12

13 Where they are going

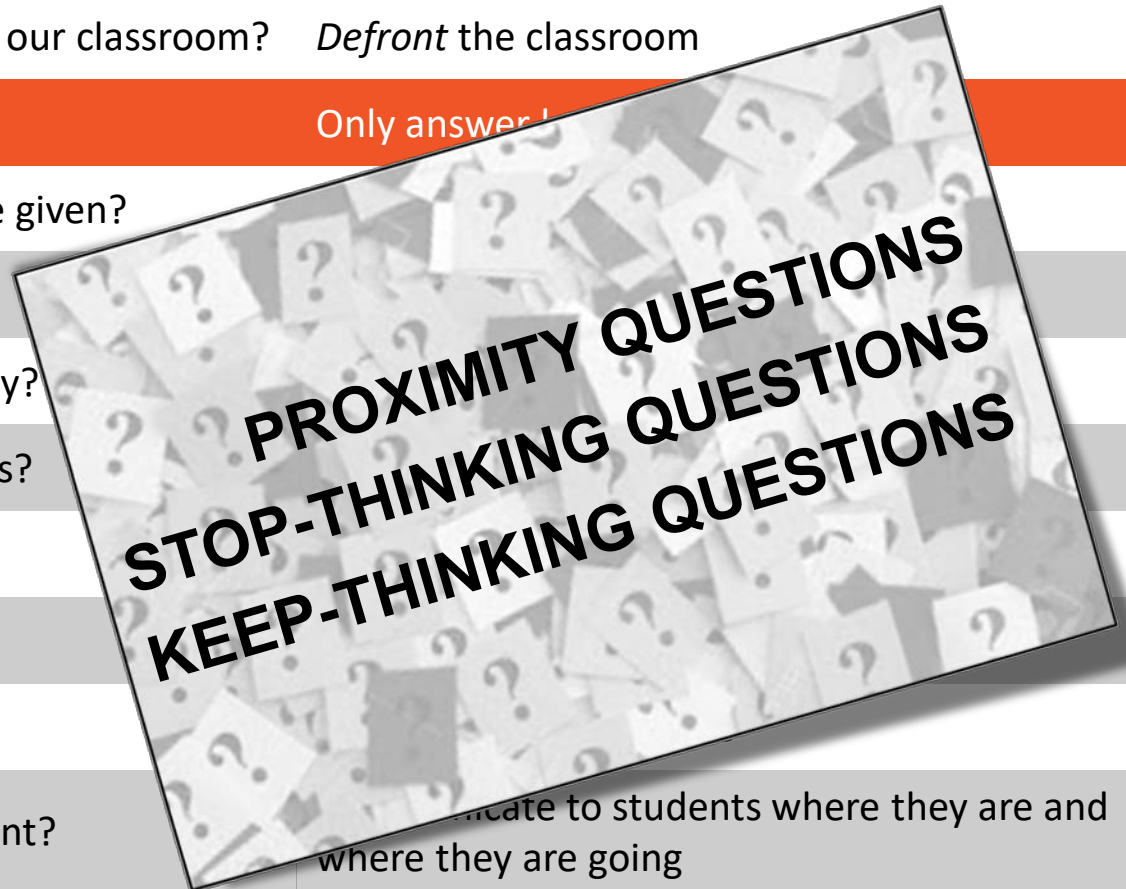
14 report out based on data (not points)



CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

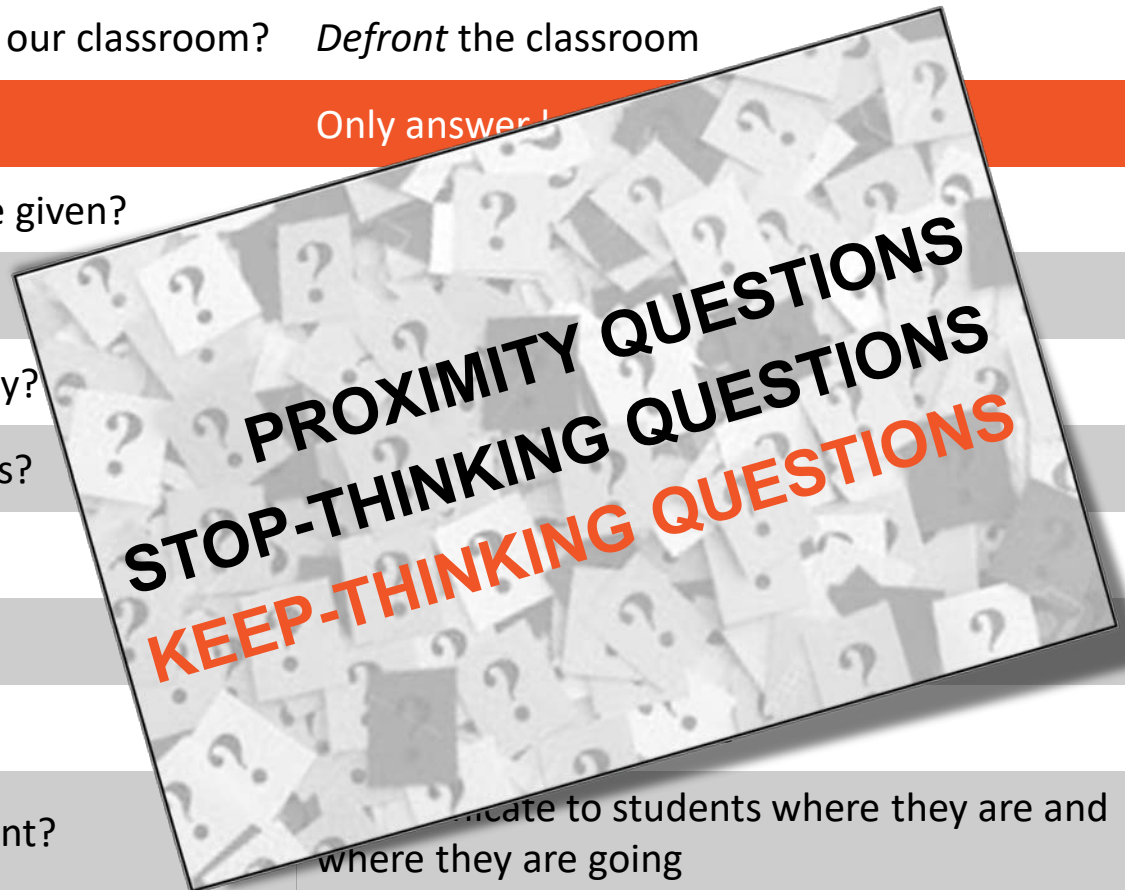
1	What are the types of tasks we use?	Use thinking tasks
2	How we form collaborative groups?	Form frequent visibly random groupings
3	Where students work?	Use vertical non-permanent surfaces
4	How we arrange the furniture in our classroom?	Defront the classroom
5	How we answer questions?	Only answer questions that require thinking
6	When, where, and how tasks are given?	
7	What homework looks like?	
8	How we foster student autonomy?	
9	How we use hints and extensions?	
10	How we consolidate a lesson?	
11	How we give notes?	
12	What we choose to evaluate?	
13	How we use formative assessment?	Communicate to students where they are and where they are going
14	How we grade?	Report out based on data (not points)



CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1	What are the types of tasks we use?	Use thinking tasks
2	How we form collaborative groups?	Form frequent visibly random groupings
3	Where students work?	Use vertical non-permanent surfaces
4	How we arrange the furniture in our classroom?	Defront the classroom
5	How we answer questions?	Only answer questions that require thinking
6	When, where, and how tasks are given?	
7	What homework looks like?	
8	How we foster student autonomy?	
9	How we use hints and extensions?	
10	How we consolidate a lesson?	
11	How we give notes?	
12	What we choose to evaluate?	
13	How we use formative assessment?	Communicate to students where they are and where they are going
14	How we grade?	Report out based on data (not points)



CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1	What are the types of tasks we use?	Use thinking tasks
2	How we form collaborative groups?	Form frequent visibly random groupings
3	Where students work?	Use vertical non-permanent surfaces
4	How we arrange the furniture in our classroom?	
5	How we answer questions?	
6	When, where, and how tasks are given?	
7	What homework looks like?	
8	How we foster student autonomy?	
9	How we use hints and extensions?	
10	How we consolidate a lesson?	Consolidate from the bottom
11	How we give notes?	Use meaningful notes
12	What we choose to evaluate?	Evaluate what you value
13	How we use formative assessment?	Communicate to students where they are and where they are going
14	How we grade?	Report out based on data (not points)

HOW

textbook

board/screen

handout/worksheet

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1 What are the types of tasks we use?

2 How we

3 Where st

4 How we a

5 How we a

6 When, whe

7 What home

8 How we fos

9 How we use

10 How we cons

11 How we give

12 What we choo

13 How we use fo

14 How we grade?

Use the

any paycheck
are

groupings

Tax Man begins with 12 paychecks for yourself. Once you choose factors of the number you make every choice that you make the tax man a paycheck, the remaining paychecks.

Example:

Turn 1: Take 3

Turn 2:

Turn 3:

You have
the remaining

Total:

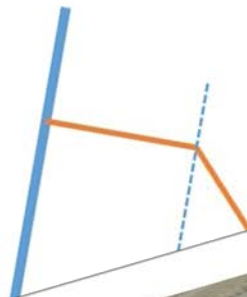
\$8 + \$

Questions:

Can you get more

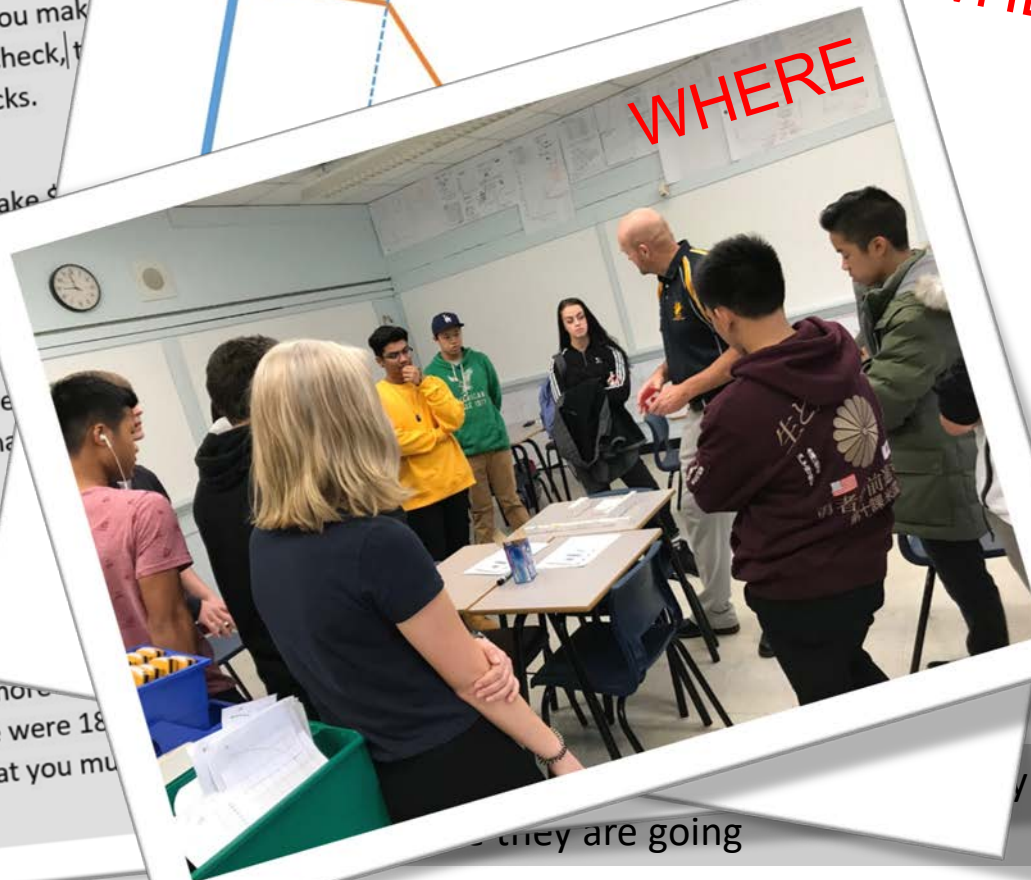
What if there were 18

strategies that you mu



WHEN

WHERE



Report out based on data (not points)

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1 What are the types of +

2

3

4

5

6

7

8

9

10 How

11 How

12 What

13 How v

14 How we grade?

Home

1. Complex Trinomials

- $6x^2 - 7x - 3$
- $12x^2 + 5x - 3$
- $8x^2 - 10x - 3$
- $6x^2 - 23x + 7$
- $12x^2 - 5x - 2$

2. Difference of Squares

- $16x^2 - 1$
- $9x^2 - 4$
- $4x^2 - 9$
- $25x^2 - 81$
- $100x^2 - 9$

3. Simple Trinomials

- $x^2 - 12x + 35$
- $x^2 - 11x + 30$
- $x^2 + 2x - 35$
- $x^2 - 2x - 35$
- $x^2 - 12x + 20$

4. Common Factor

- $4x^2 + 4x - 30$
- $3x^2 + 3x - 18$
- $2x^2 - 2x - 12$
- $2x^2 - 24x + 40$
- $3x^2 - 6x - 105$

$$\begin{array}{r} 16x^2 - 1 \\ \hline 16x^2 - 4x + 4x - 1 \\ \hline (-4x + 1)(-4x - 1) \end{array}$$

$$\begin{array}{r} 4x^2 + 4x - 30 \\ \hline 2x^2 + 2x - 15 \\ \hline 2x^2 + 2x - 15 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3x^2 + 3x - 18 \\ \hline 3x^2 + 3x - 18 \\ \hline 0 \end{array}$$



CLASSROOM PRACTICES

1 What are the types of tasks we use?

2 How we form



12 W

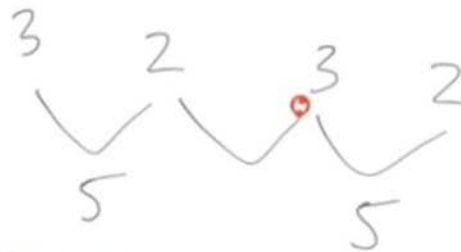
13 How

assessment?

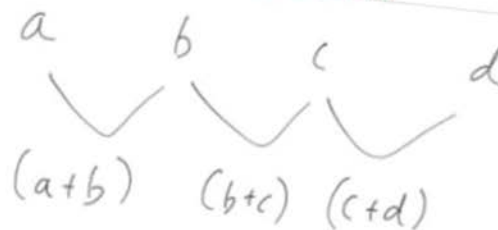
14 How we grade?

OPTIMAL PRACTICES FOR THINKING

Another group has decided to keep their numbers small (and repeat some).
We are using e and o also. Nice!
Our hypothesis broke with 5 we are trying it now with an even number of numbers (i.e. 6 numbers, 8 numbers, etc)



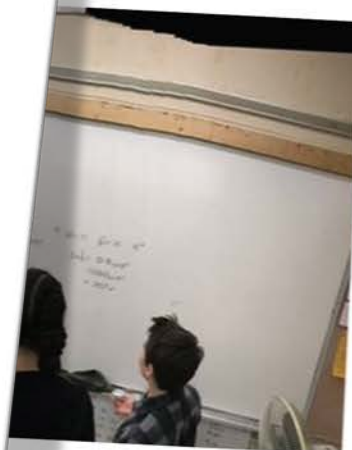
Some letters may make what is happening more transparent.



$$a+2b+c \quad b+2c+d$$

$$a+3b+3c+d$$

I wonder what this would look like for 5 numbers?
New hypothesis: When starting with an EVEN number of selected numbers, an odd amount of odds yields an odd final sum. works for 6 - try 1,3,5,2,4,6



ere they are and

Report on

t points)

CLASSROOM PRACTICES

PRACTICES FOR THINKING

1 What are

2

3

4

5

6

7

8

9

10

11

12

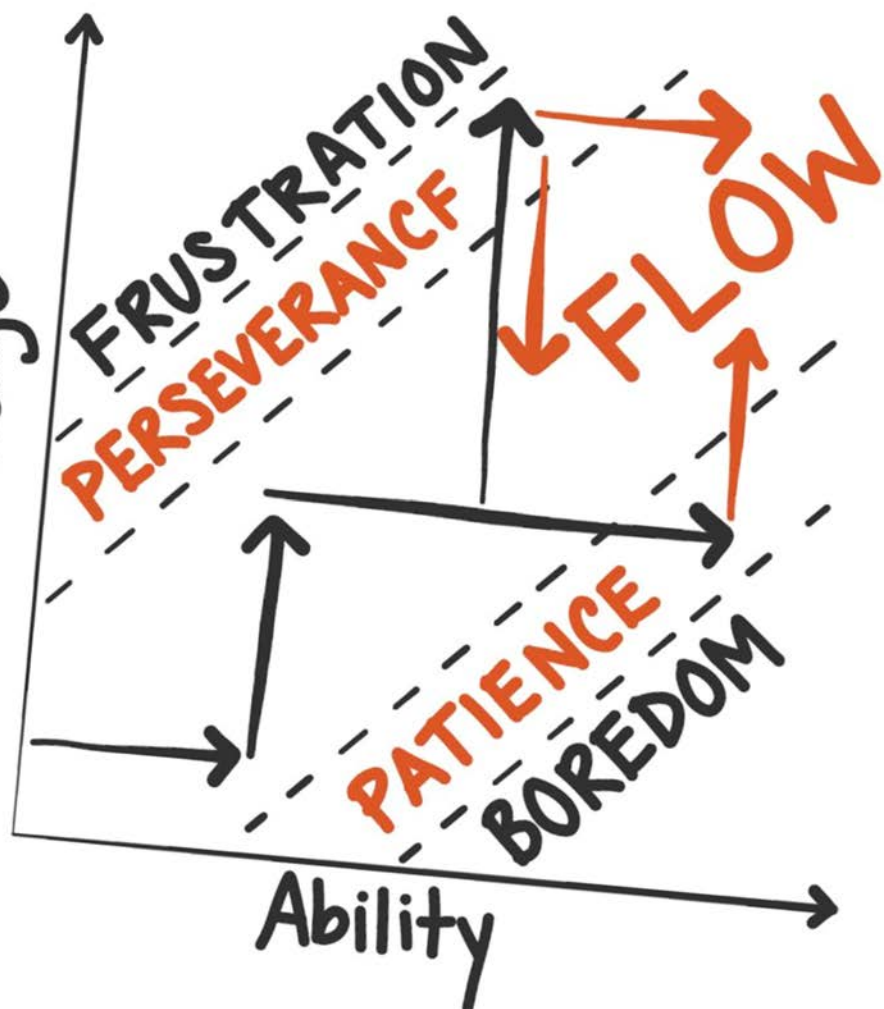
13

14

Challenge

1. $(x + 2)$
2. $($
3. $($
4. $($
5. $($
6. $($
7. $($
8. $($
9. $($
10. $($
- 11.
- 12.
- 13.
- 14.
- 15.

Challenge



Ability

1. $\sqrt{18} =$

18. ...

(not points)

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

1 What are the types of tasks we use?

Use thinking tasks

2 How we form collabor

frequent

3 Where students

4

5

6

7

8

9

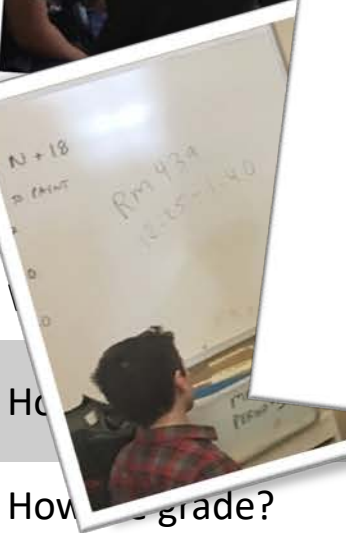
10

11

12

13

14



Challenge

Time spent



ts where they are and

Report out based on data (not points)

CLASSROOM

PRACTICES FOR THINKING

1 What are

2 How we

3 Vocabulary/Definition

Big Ideas/Concepts

4

5

6

7

8 H

9 H

Procedures

Examples

10 Ho

11 Ho

12 Wh

13 How

14 How

TRIGONON

Similar tria

$$= 2/(x-2)^2 - 1$$

MT - Measurement & Trigonometry

SON CAN YOU

Hydrogenase

$$a^2 + b^2 = c^2$$

Pythagoras

Use to find missing
sides/angles:

Report out based on (not points)

CLASSROOM PRACTICES

1 What are the

PRACTICES FOR THINKING

FRACTIONS

BASIC

INTERMEDIATE

ADVANCED

FRACTIONS

Definitions

BASIC

INTERMEDIATE

ADVANCED

OUT
OF

Add and subtract proper fractions

✓X✓✓✓

Add and subtract mixed fractions

✓✓

XXH✓✓

XXNX_cX_c

4

Multiply and divide proper fractions

NXH✓X

✓✓✓

XGHX

4

Multiply and divide mixed fractions

✓✓

XS✓✓

XNGX_o

4

Solve order of operation tasks with proper and mixed fractions

XH✓✓✓

XHHX

NNX

4

Solve contextual problems involving fractions

✓S✓

XHX_c

XNN

4

Estimate solutions for problems involving fractions

✓✓

XH✓✓

GXXH_c

4

XHV

N✓✓✓

4

CLASSROOM PRACTICES

OPTIMAL PRACTICES FOR THINKING

BUILDING THINKING CLASSROOMS in MATHEMATICS

GRADES K-12

14 TEACHING
PRACTICES
FOR ENHANCING
LEARNING



PETER LILJEDAHL

FOREWORD BY TRACY JOHNSTON ZAGER
ILLUSTRATIONS BY LAURA WHEELER

CORWIN Mathematics

Use thinking tasks

Form frequent visibly random groupings

Use vertical non-permanent surfaces

Defront the classroom

Only answer keep thinking questions

Give tasks early, standing, and verbally

Give check your understanding questions

Be intentionally less helpful

Create and manage *flow*

Consolidate from the bottom

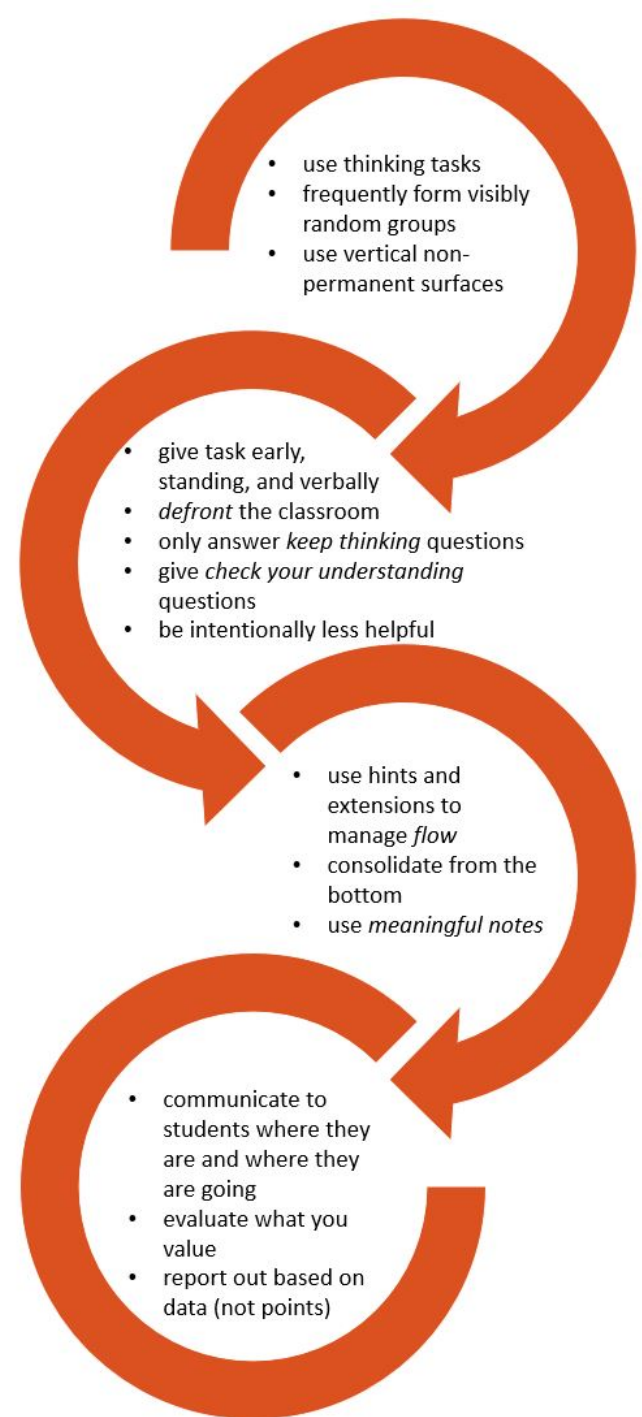
Use meaningful notes

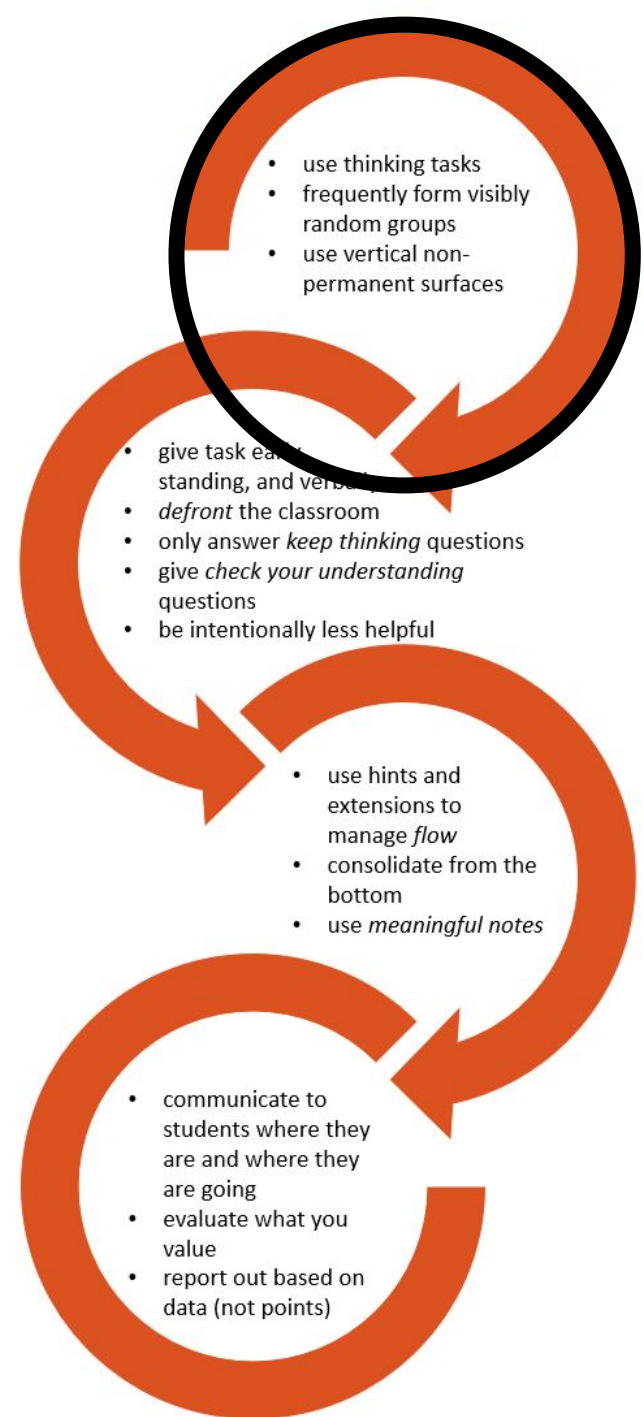
Evaluate what you value

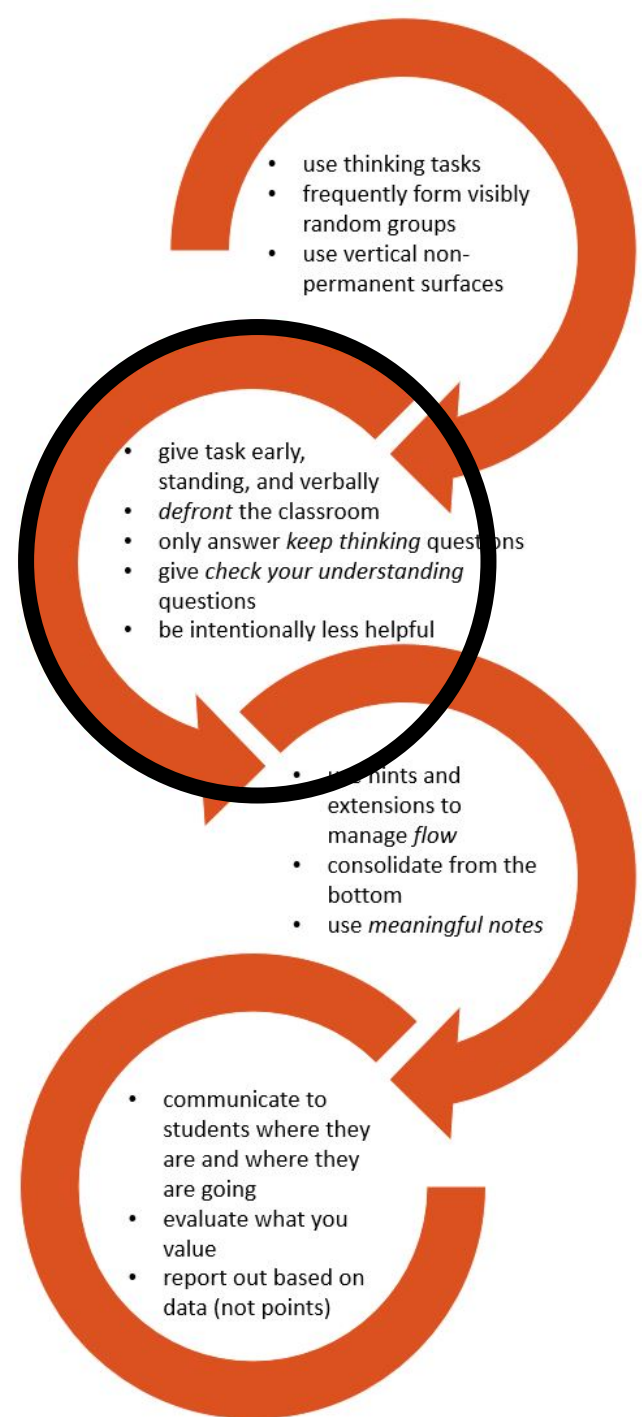
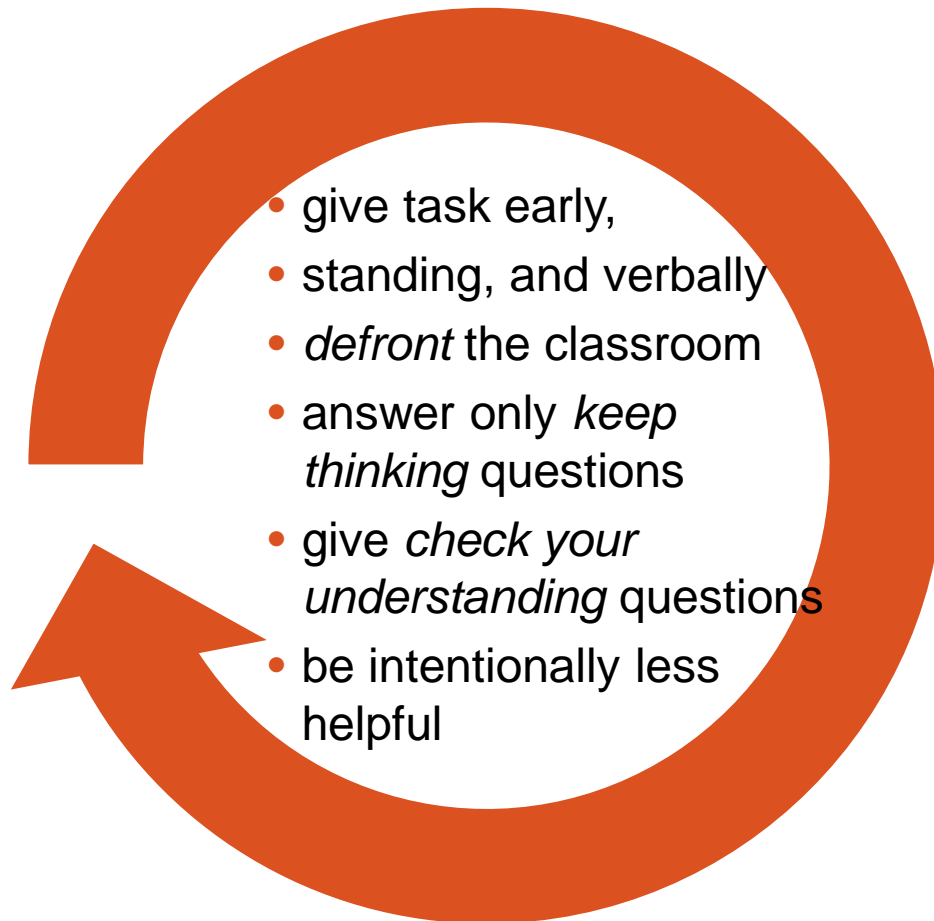
Communicate to students where they are and where they are going

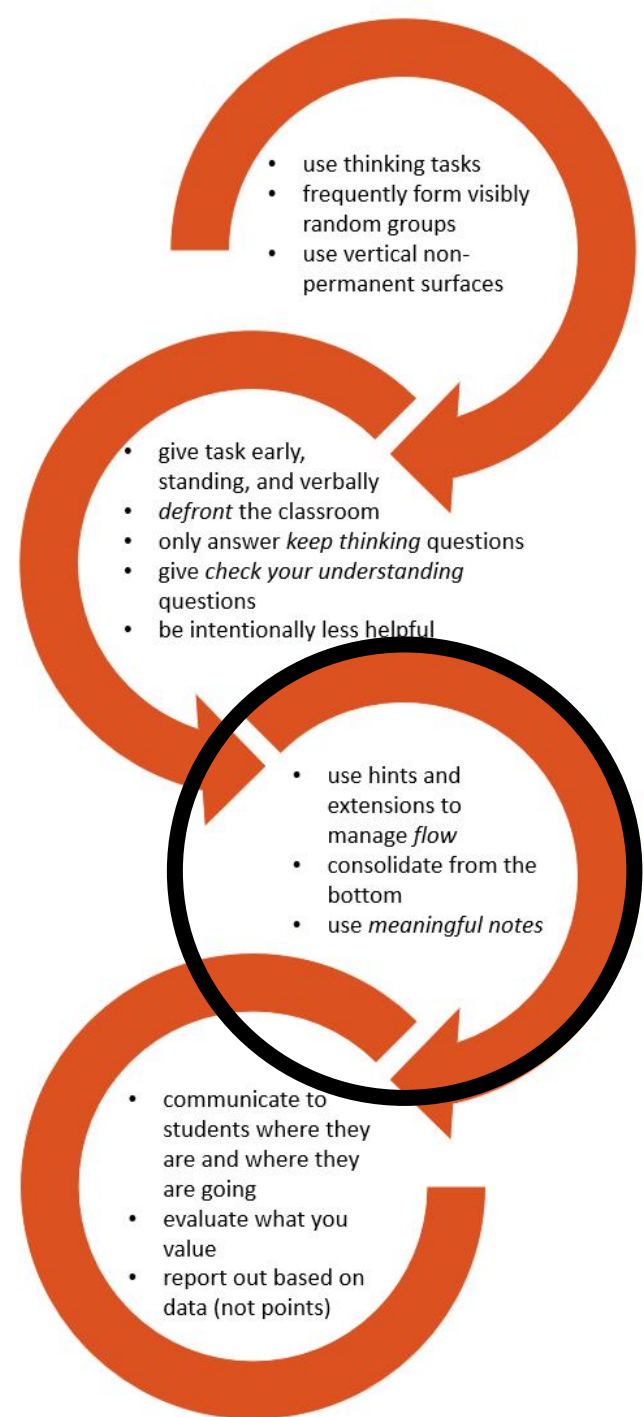
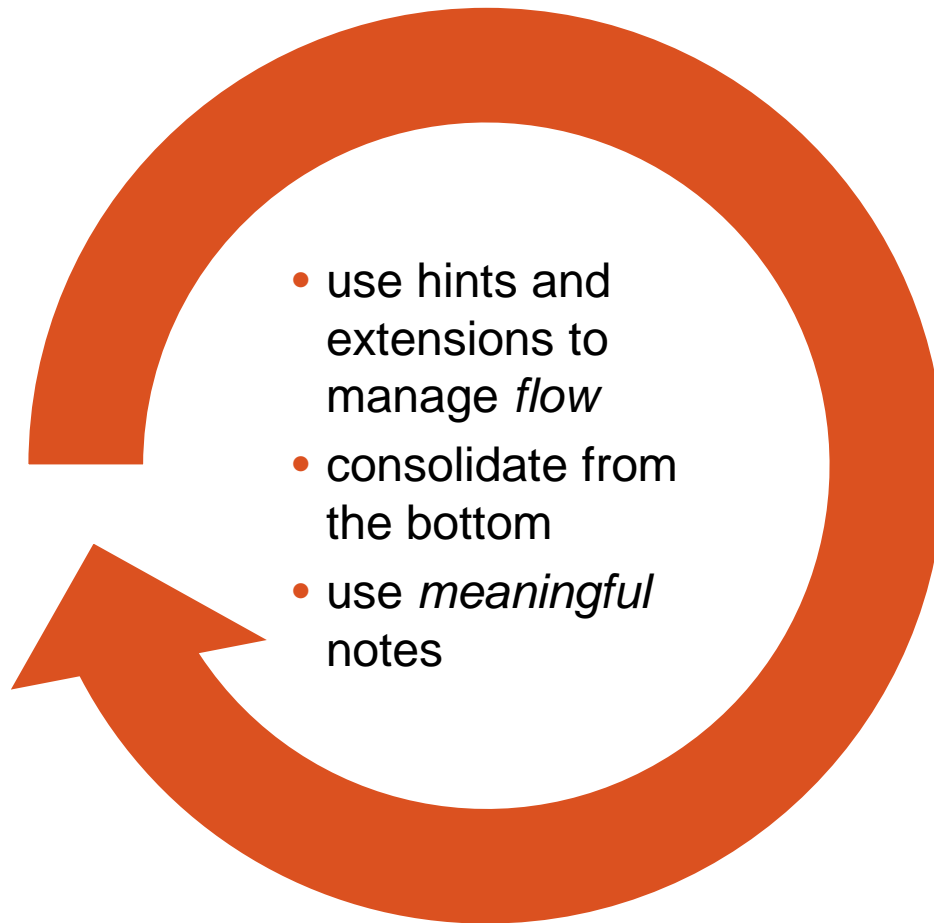
Report out based on data (not points)

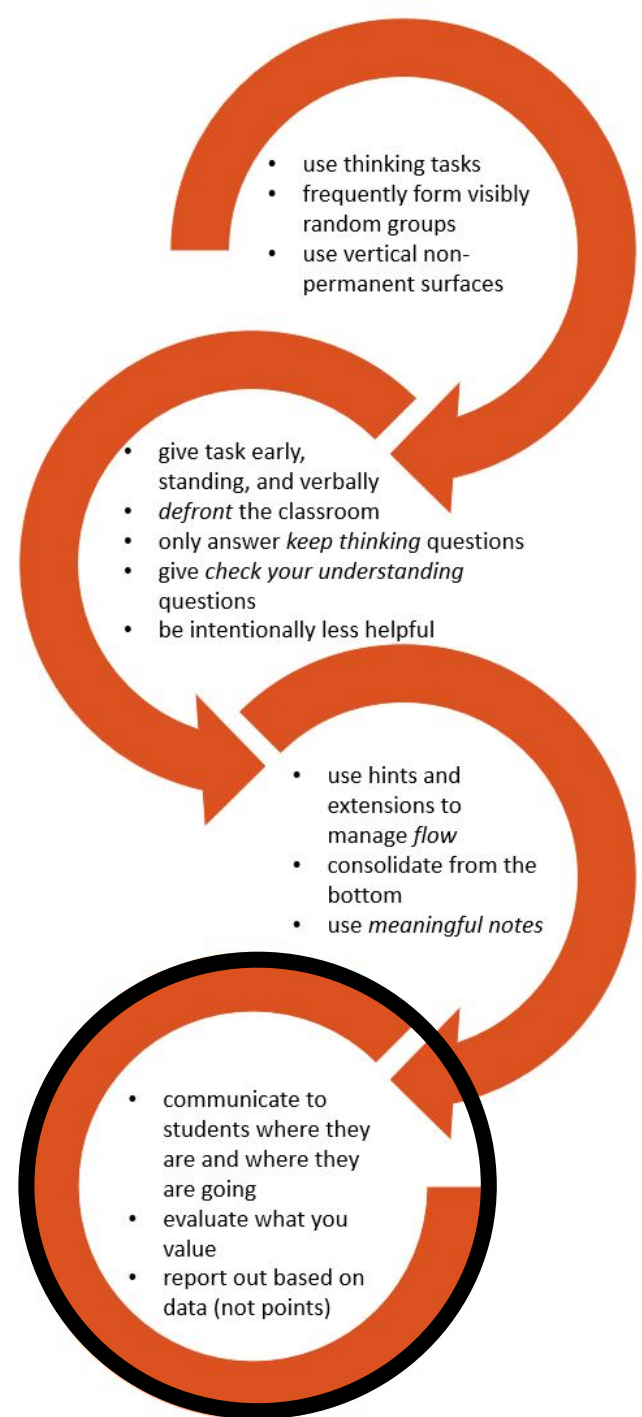
WHERE TO START?



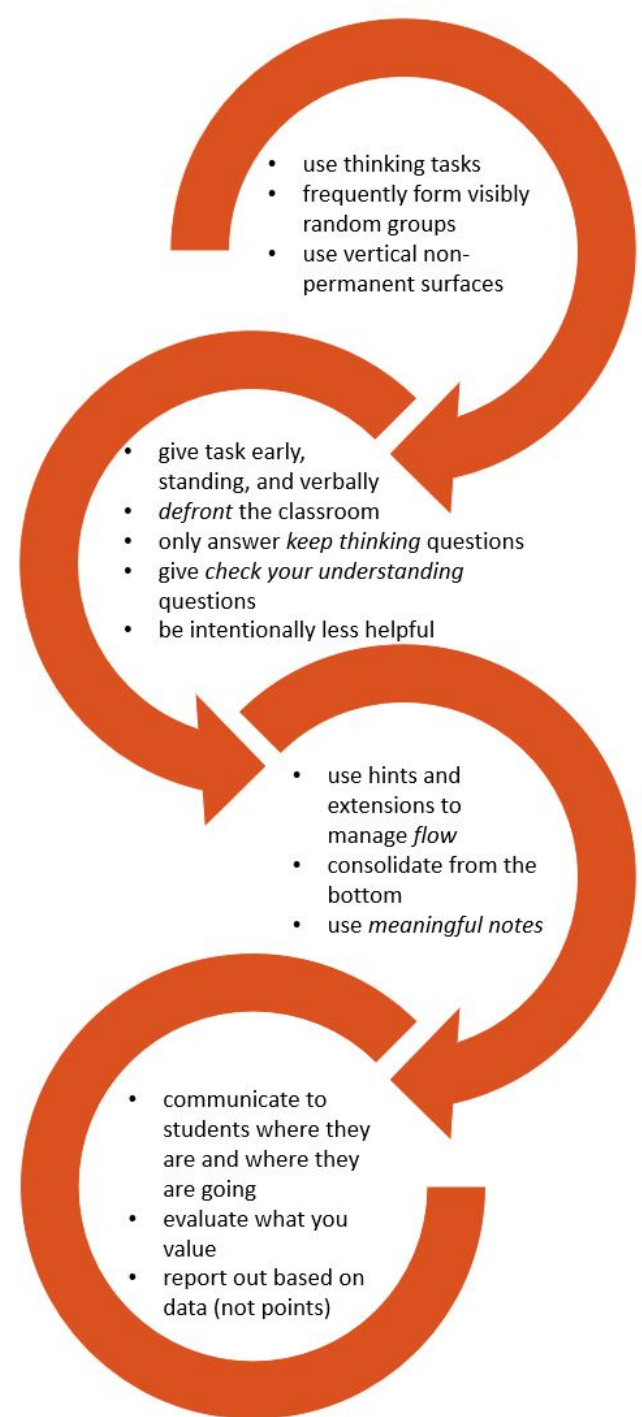









BUILDING THINKING CLASSROOMS



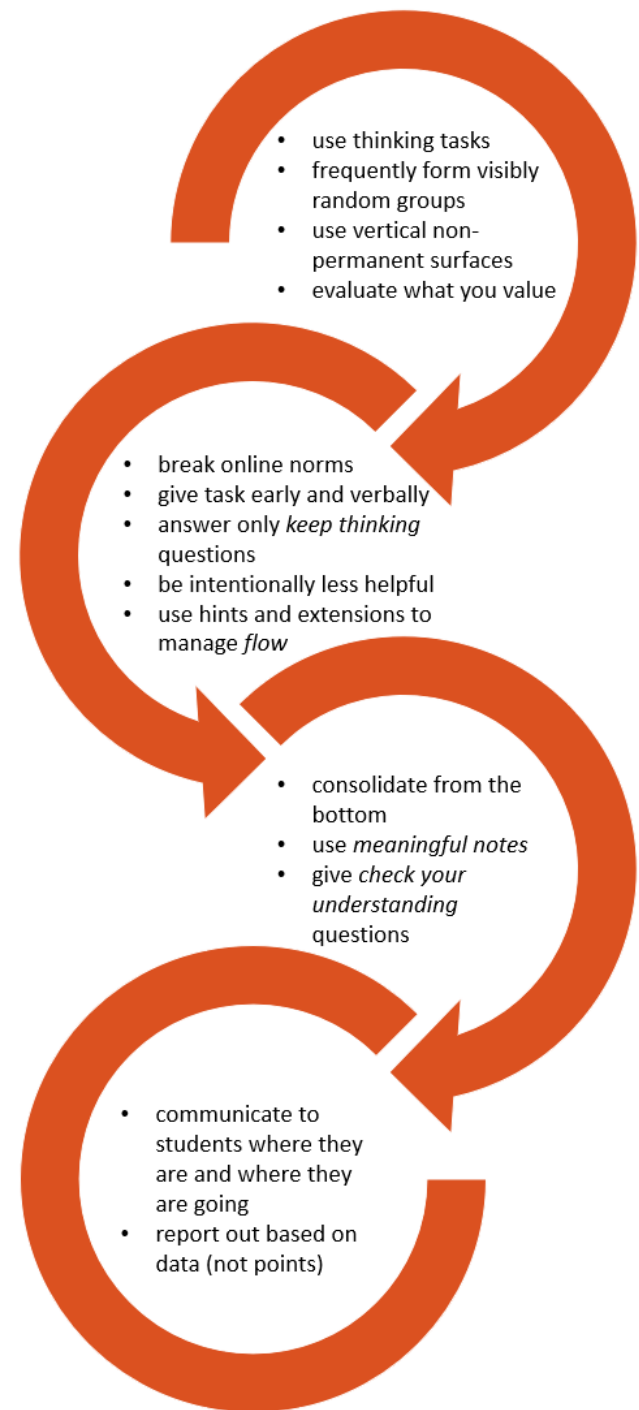
BUILDING THINKING CLASSROOMS

year II

- 
- use thinking tasks
 - use vertical non-permanent surfaces
 - frequently form visibly random groups
 - give task early, standing, and verbally
 - *defront* the classroom
 - answer only *keep thinking* questions
 - be intentionally less helpful
 - consolidate from the bottom
 - use hints and extensions to manage *flow*

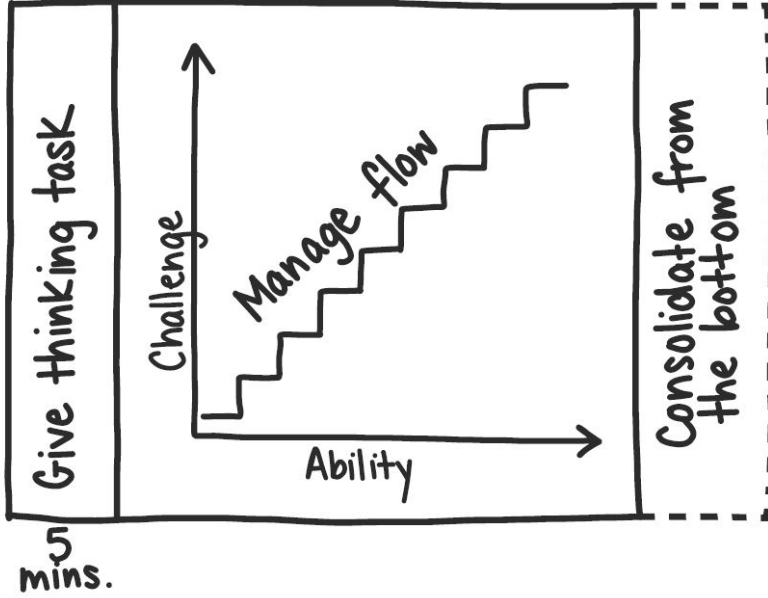
- use *meaningful notes*
- give *check your understanding* questions
- communicate to students where they are and where they are going
- evaluate what you value
- report out based on data (not points)

BUILDING THINKING CLASSROOMS online

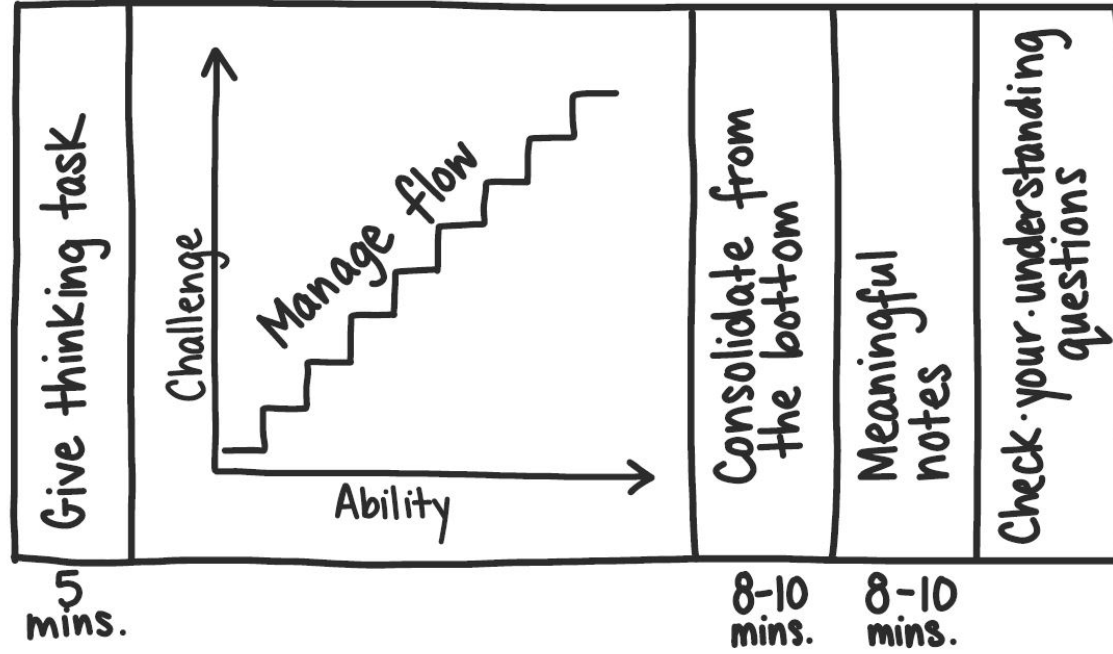
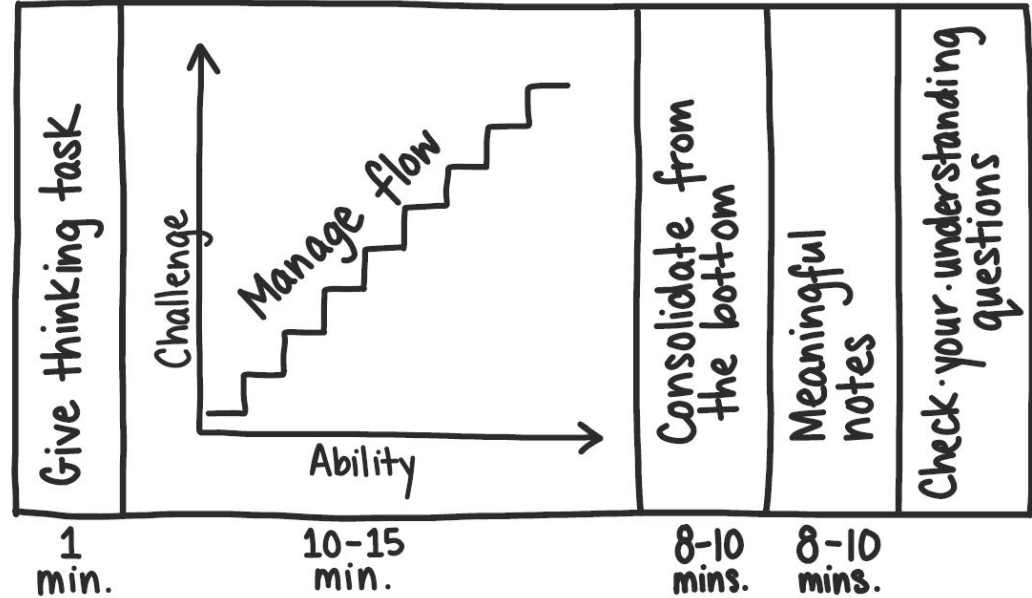




DAY 1



DAY 2





THANK YOU!



liljedahl@sfu.ca



[@pgliljedahl](https://twitter.com/pgliljedahl) | [#thinkingclassroom](https://twitter.com/thinkingclassroom)



[www. buildingthinkingclassrooms.com/podcasts](http://www.buildingthinkingclassrooms.com/podcasts)



THANK YOU



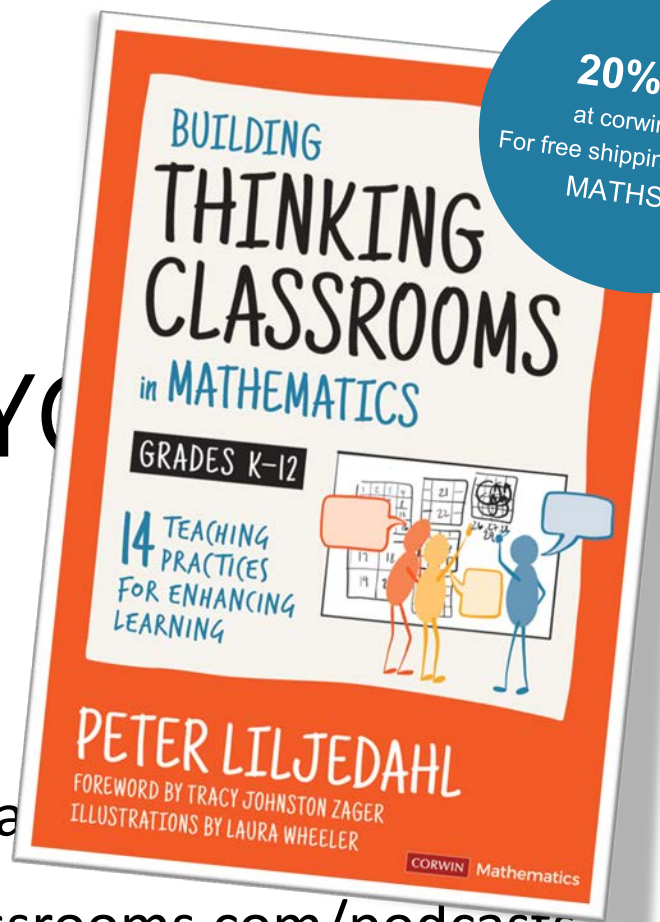
liljedahl@sfu.ca



[@pgliljedahl](https://twitter.com/pgliljedahl) | [#thinkingclassrooms](https://twitter.com/#thinkingclassrooms)



www.buildingthinkingclassrooms.com/podcasts



20% off

at corwin.com

For free shipping use code:

MATHSHIP