Physical Learning in Mechanical Network Materials

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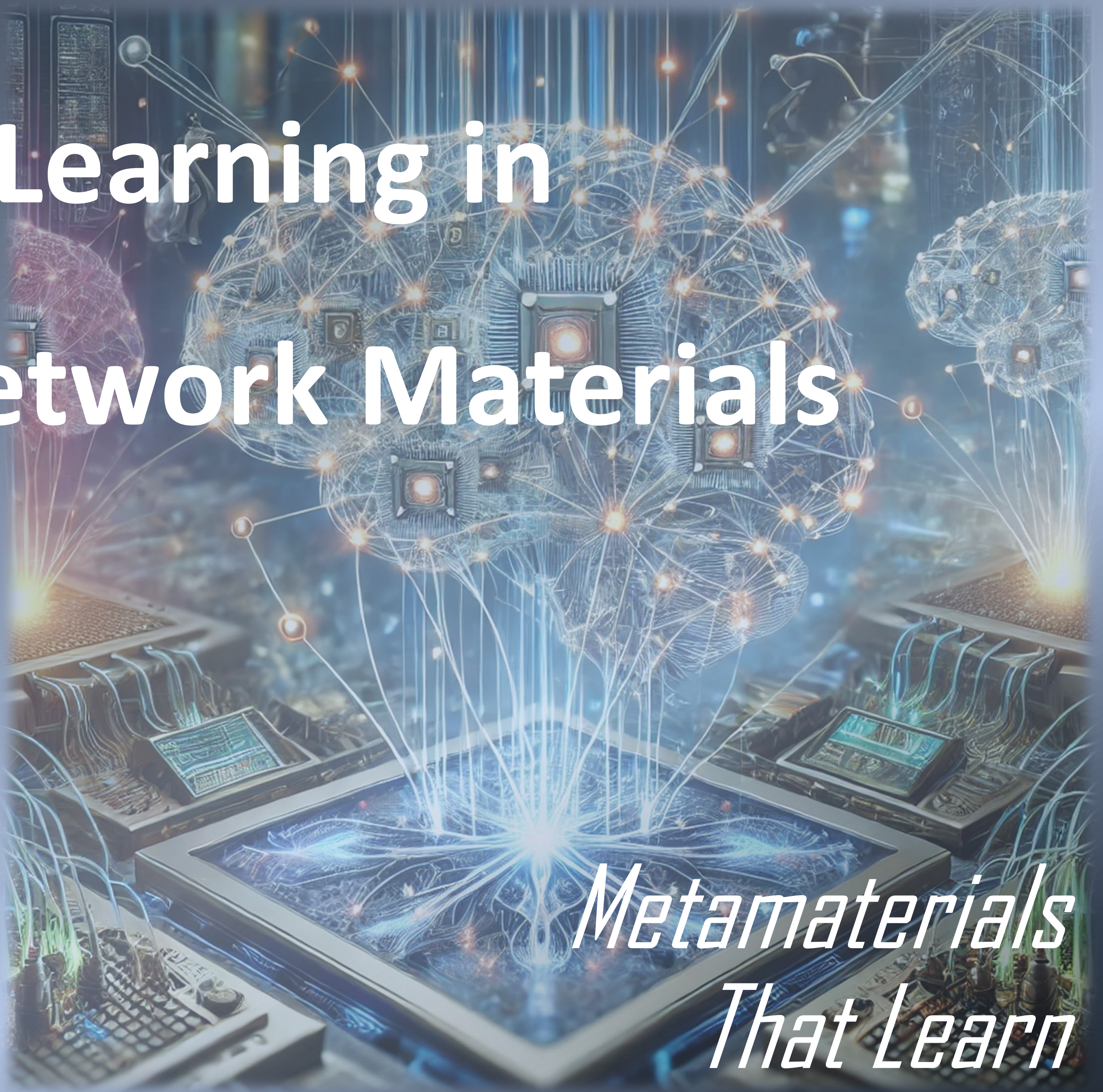
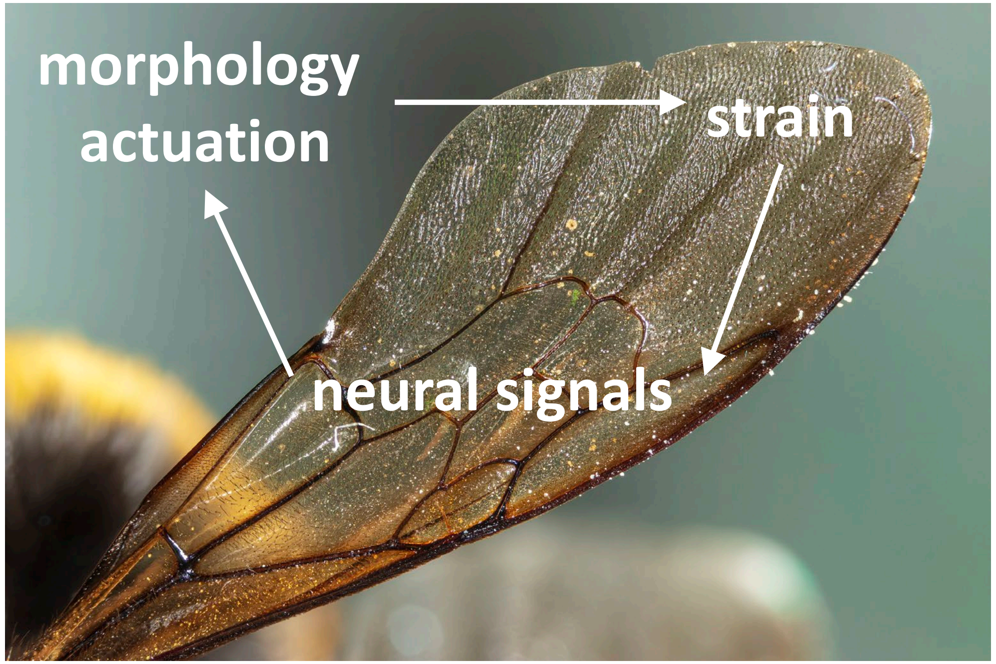
Frontiers of Materials That Learn:
2025 Annual CMMRC Workshop

UNIVERSITY OF MICHIGAN



Why do we need materials to learn?

Integrating learning with adaptive functionality: materials that resemble living systems



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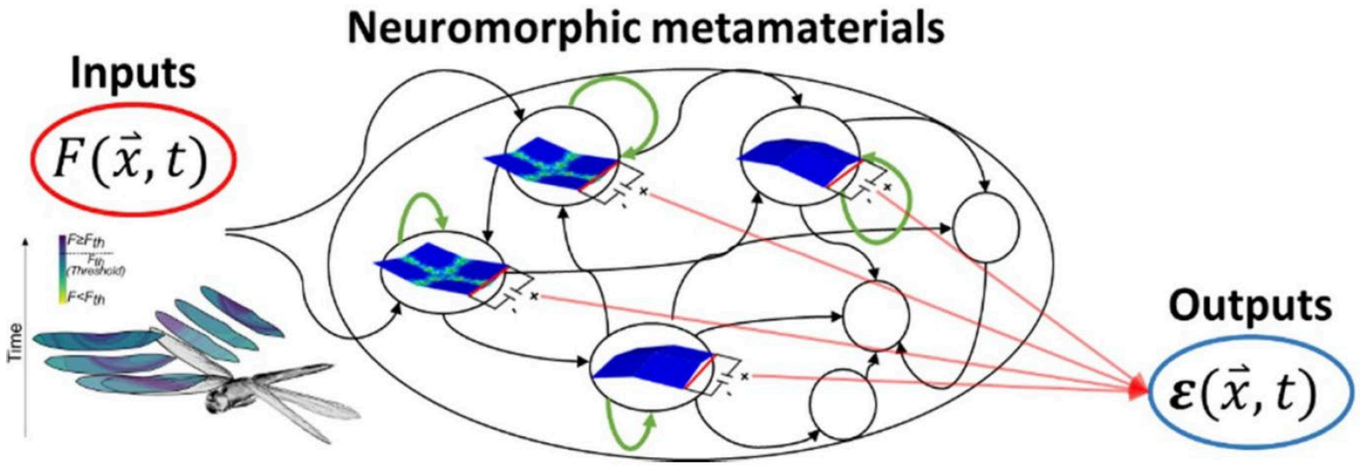
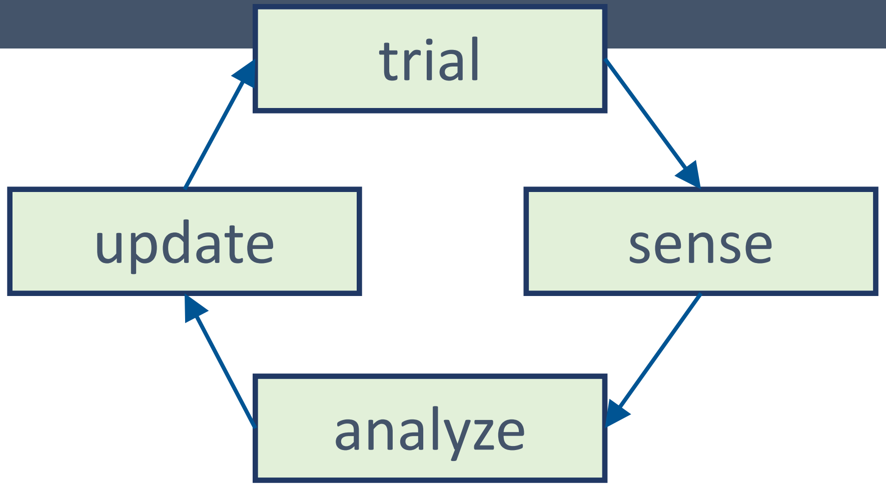
Aiello et al Curr. Opin. Insect Sci. 48 8 (2021)



Arrieta and Sarles, in *Roadmap on embodying mechano-intelligence and computing in functional materials and structures,* Alu et al, Smart Mater. Struc. 34 063501 (2025)

- **"Learning"**: generalize & adapt to unpredictable new environments
- Intrinsic material processes for efficiency

How to make materials learn?



Elements of "in situ learning":

- **"Local rule":** only local measurements are needed to find what should change to learn something
- **"Physical update" (neuroplasticity):** learning/adaptive degrees of freedom change physically in the material

Recent advances:

- **Contrastive learning:** comparing states under different conditions (Movellan, "Contrastive Hebbian learning in the continuous Hopfield model," in *Proceedings of the 1990 Connectionist Models Summer School*)
- **Equilibrium propagation:** nudging towards desired state (Scellier & Bengio, Front. Comput. Neurosci. 11, 24 (2017))
- **Coupled learning:** coupled twin systems under different conditions (Dillavou et al Phys. Rev. App. 18, 014040 (2022))

Challenging to combine with "backpropagation": main algorithmic framework for current ML

This talk: mechanical networks that learn

Shuaifeng Li (UM)

Article | Open access | Published: 09 December 2024

Training all-mechanical neural networks for task learning through in situ backpropagation

Shuaifeng Li & Xiaoming Mao

Nature Communications **15**, Article number: 10528 (2024) | Cite this article

arXiv > cond-mat > arXiv:2503.07796

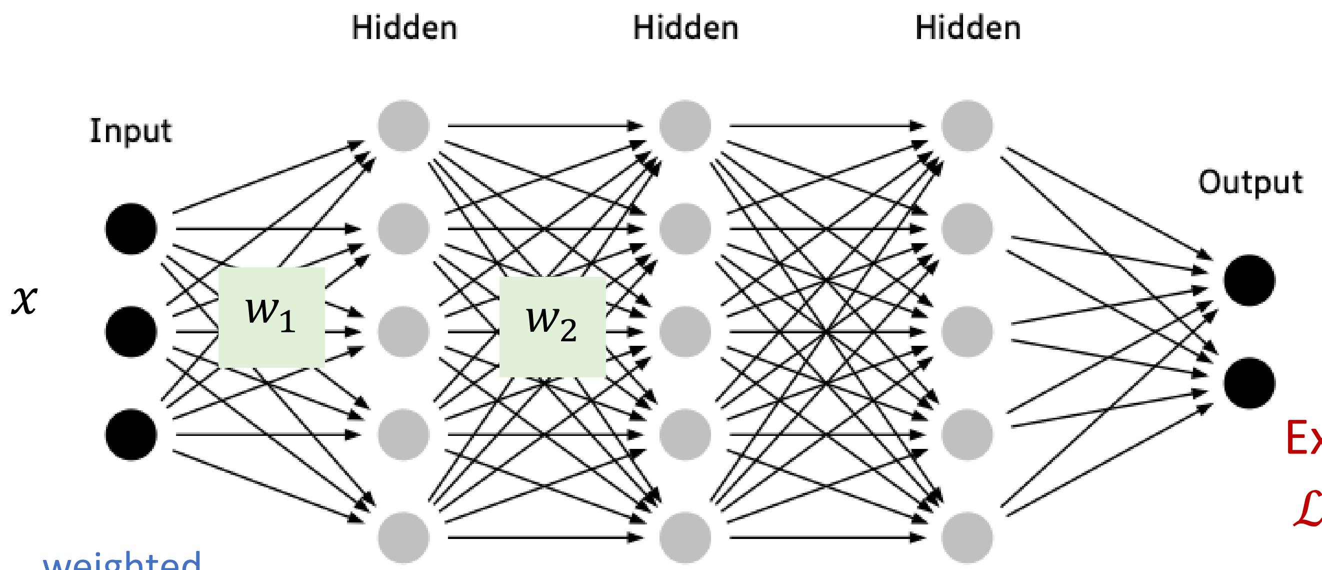
Condensed Matter > Disordered Systems and Neural Networks

[Submitted on 10 Mar 2025]

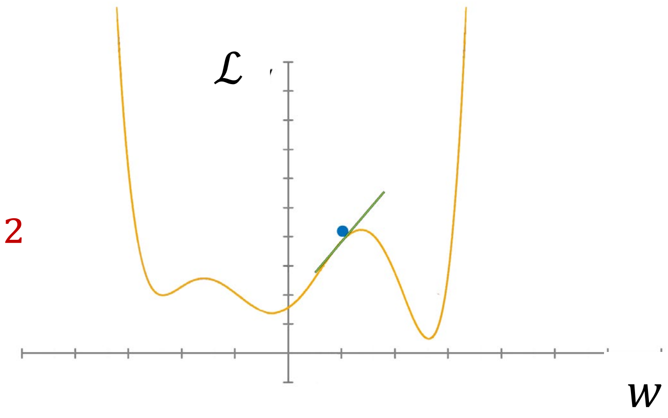
Topological mechanical neural networks as classifiers through in situ backpropagation learning

Shuaifeng Li, Xiaoming Mao

Backpropagation in neural networks: exact gradients



Loss function: $\mathcal{L}(\mathrm{y})$
difference between output and the desired output



Example:
$\mathcal{L}(\mathrm{y}) = (y - y_{\mathrm{target}})^2$

weighted input: $z_1 = w_1 x$ $\quad z_2 = w_2 a_1$ $\quad \ldots \quad z_L = w_L a_{L-1}$

activation: $a_1 = f(z_1)$ $\quad a_2 = f(z_2)$ $\quad y = f(z_L)$

Forward pass

$\frac{\partial \mathcal{L}}{\partial w_l} = \delta_l (a^{l-1})^T$, with $\delta_l = (W_{l+1}^T \delta^{l+1}) \odot f'(z_l)$

Backward pass

Materials don't have an external processor to do this?

Adjoint method: exact gradient from local info

Optimization problem: $\min_{k} \mathcal{L}(u(k))$

Subject to: $Du = F$

Forward problem

$\mathcal{L}$: Loss function

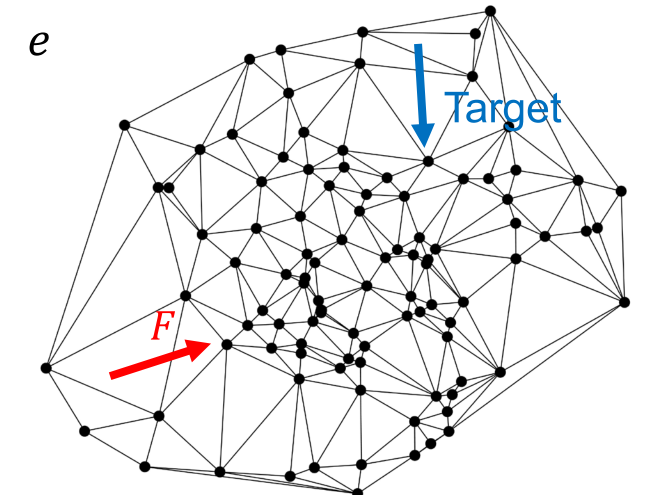
k: Spring constant ("weight")

u: Node displacement (output)

F: Force (input)

D: Dynamical matrix of the network

C: Compatibility matrix



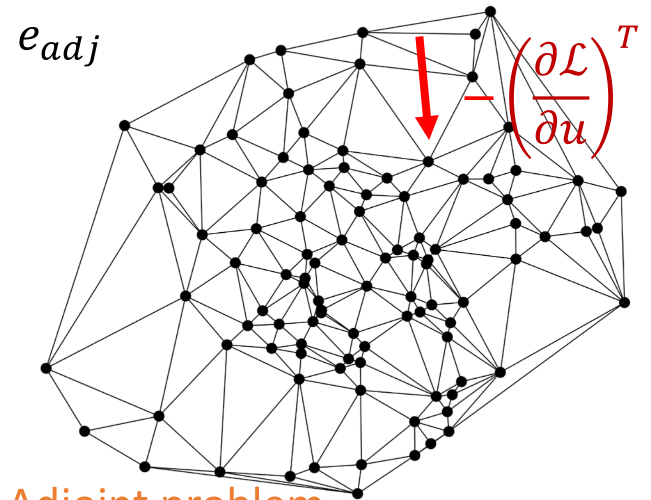
Forward problem

$$\nabla\mathcal{L} = \frac{d\mathcal{L}}{dk} = \frac{\partial\mathcal{L}}{\partial u}\frac{du}{dk}$$

$$= \frac{\partial\mathcal{L}}{\partial u}\left(-D^{-1}\frac{dD}{dk}u\right) = u_{adj}^T\frac{dD}{dk}u$$

$$= u_{adj}^T\frac{d(C^TKC)}{dk}u \; = e_{adj} \circ e$$

$$\frac{d}{dk}(Du) = \frac{d}{dk}(F) \;\rightarrow\; \frac{du}{dk} = -D^{-1}\frac{dD}{dk}u$$



Adjoint problem

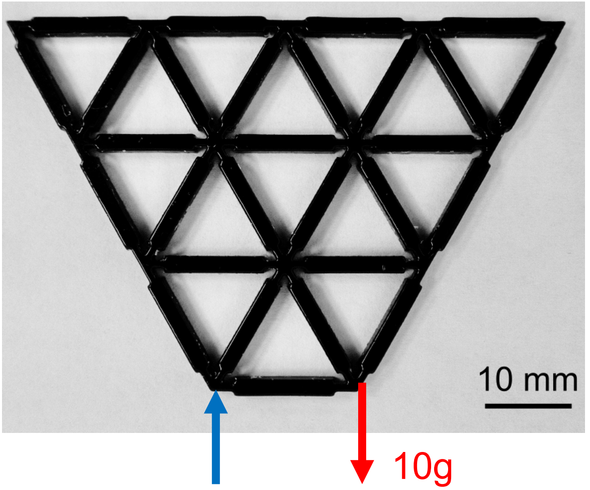
In-situ backpropagation using only **two** simulations or experiments regardless of the network size

Adjoint problem: $Du_{adj} = -\left(\frac{\partial\mathcal{L}}{\partial u}\right)^T$

Li & Mao, Nat. Comm. 15, 10528 (2024)

Experimental measurement of the gradient

3D printed network

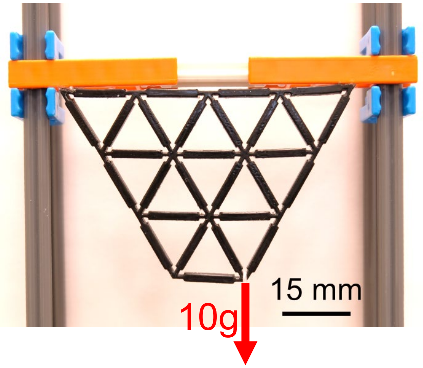


u_L

$$\mathcal{L} = (u_L - u_{L,\text{target}})^2$$

$\nabla\mathcal{L}$?

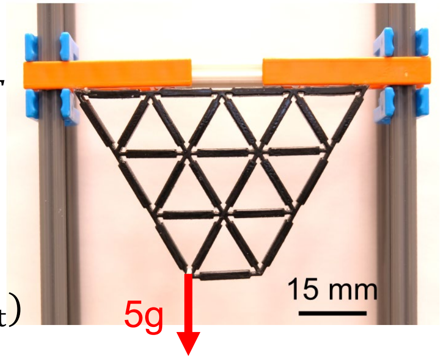
Forward field



$$-\left(\frac{\partial \mathcal{L}}{\partial u}\right)^T$$

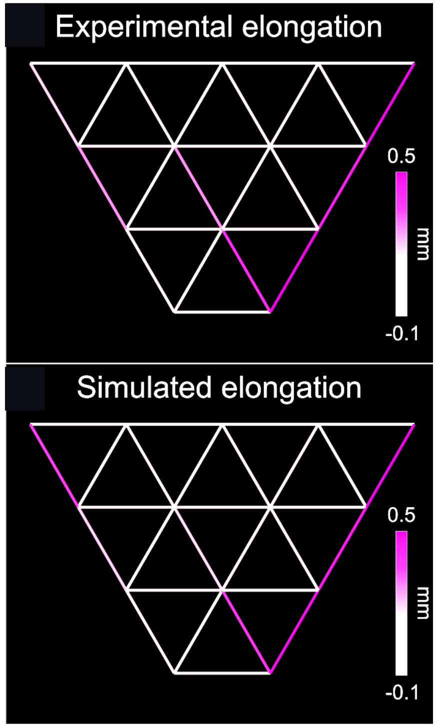
$$-2(u_L - u_{L,\text{target}})$$

Adjoint field



Gradient of the loss

$\nabla\mathcal{L}$



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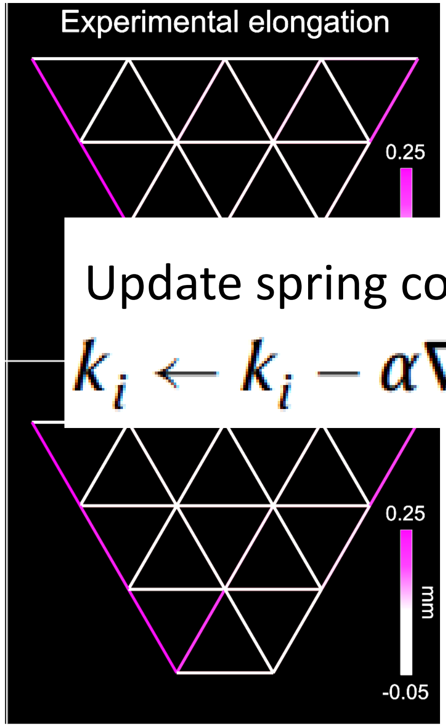

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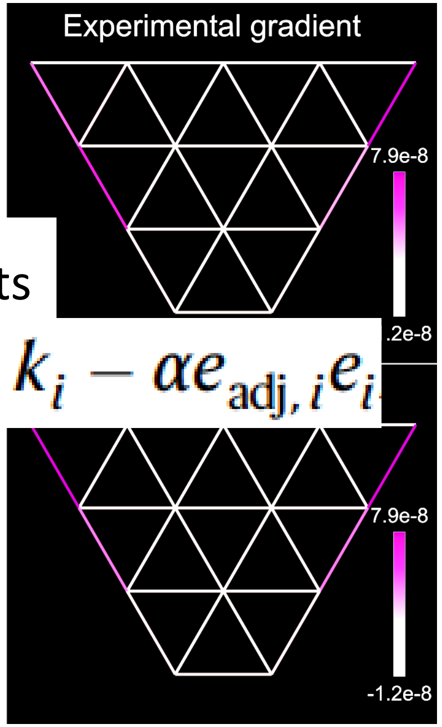


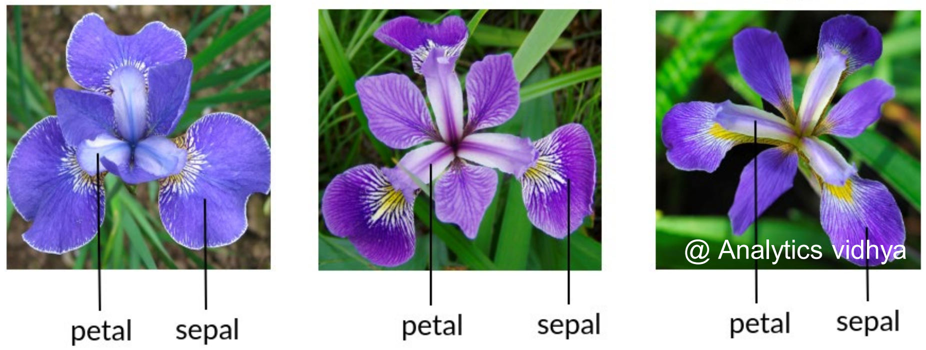
Update spring constants

$$k_i \leftarrow k_i - \alpha\nabla\mathcal{L}_i = k_i - \alpha e_{\text{adj},i} e_i$$

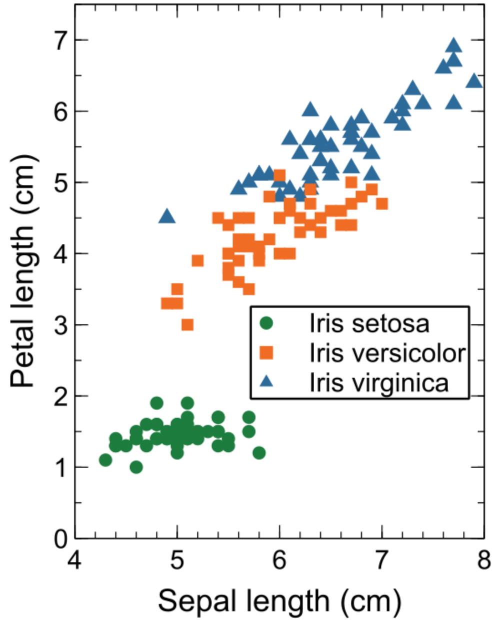
Li & Mao, Nat. Comm. 15, 10528 (2024)

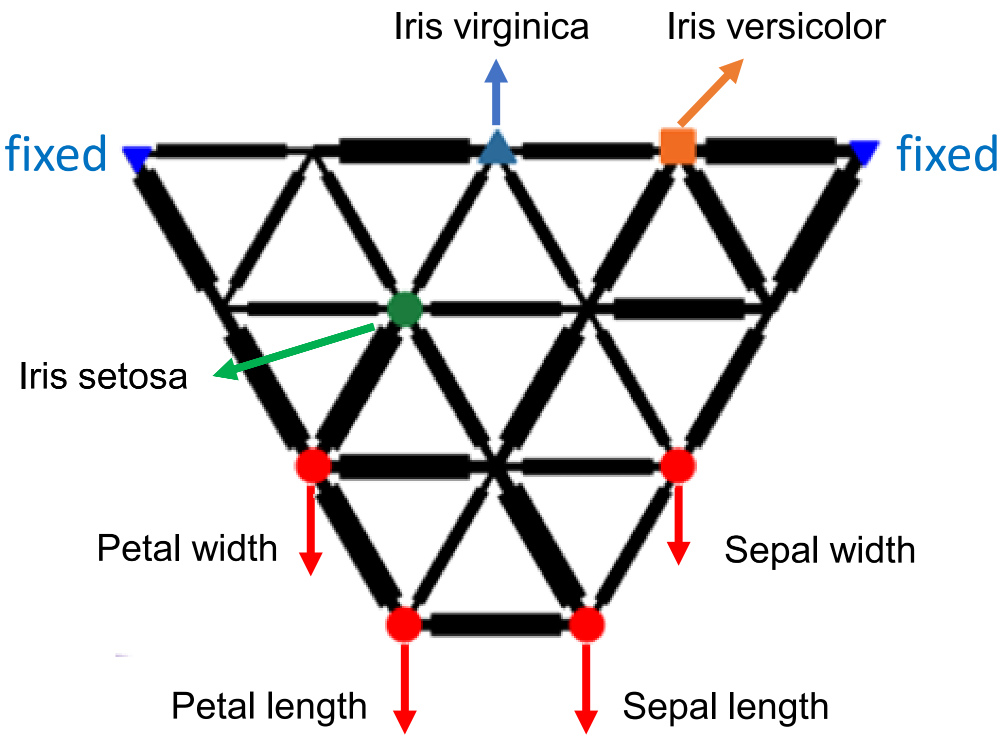
Machine learning task: classification

iris setosa | iris versicolor | iris virginica



Petal length, Petal width, Sepal length, Sepal width





Indicator: the node with largest displacement

Li & Mao, Nat. Comm. 15, 10528 (2024)

Machine learning task: classification

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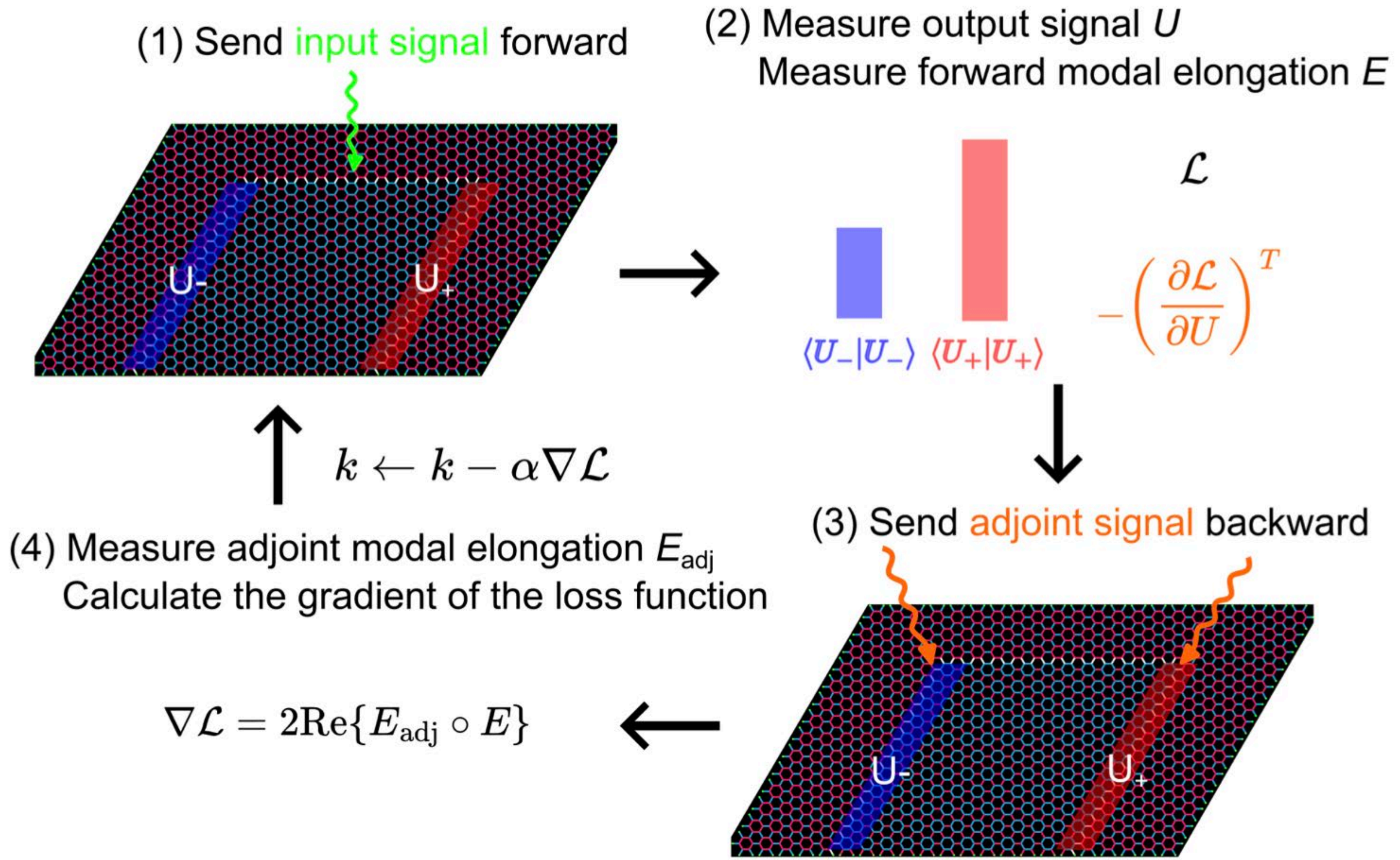
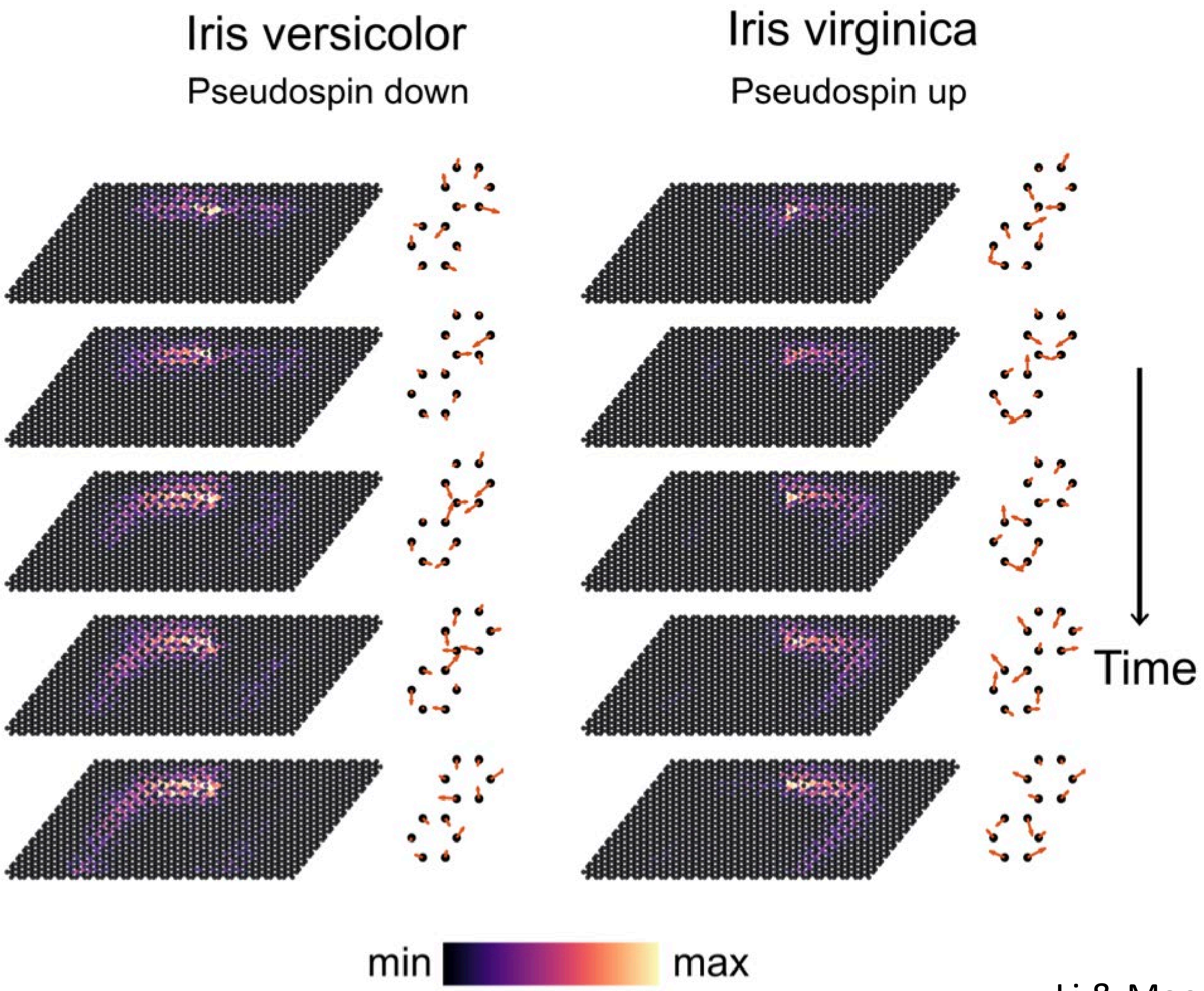
In situ backpropagation using topological states

Topologically protected edge states: robust to damage



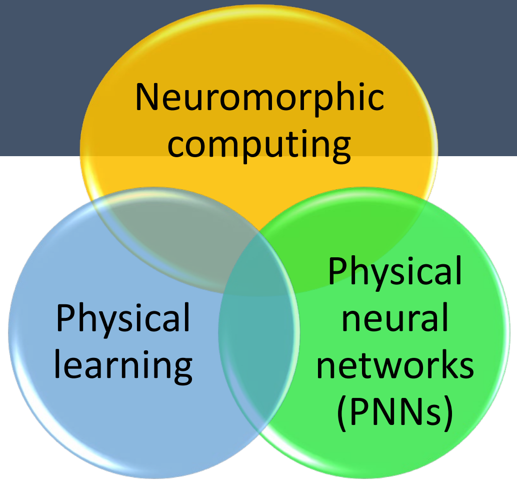
Li & Mao, arXiv:2503.07796 (2025)

In situ backpropagation using topological states



Li & Mao, arXiv:2503.07796 (2025)

What's next?



Physical neural networks & neuromorphic computing

- Computing device for ML
- Physical platforms: optical, E&M, acoustic, chemical, …
- Promise for orders of magnitude improvement of energy efficiency

New paradigm for materials and manufacturing

- Materials that can "update" and "adapt" for new environments
- Algorithms for broad contexts
- Parallel with biological functions

Thank You!

- Algorithm for backpropagation in mechanical neural networks
- Experimental demonstration of obtaining $\partial\mathcal{L}/\partial k$
- Embed learning in real materials