Supporting mathematics teachers to use authentic tasks in their classrooms

Chrissavgi Triantafillou, Despina Potari, and Giorgos Psycharis

National and Kapodistrian University of Athens

Structure of the seminar

- **PART A:** Literature review and theoretical considerations
- **PART B:** Designing and implementing authentic tasks in mathematics classrooms
- **PART C:** Conclusions

- **PART A:** Literature review and theoretical considerations

Realistic Mathematics' Education (RME) approach

- Since 1970s, the RME approach was proved a promising way to fix and improve students' understanding of mathematics concepts.
- In the RME approach, realizable situations and contexts outside of mathematics (the home, school or workplace) provide a resource for mathematizing.
- Thus, context provides the catalyst for developing students' mathematical thinking and reasoning skills (e.g., Gravemeijer & Doorman, 1999)

Authentic tasks in mathematics teaching

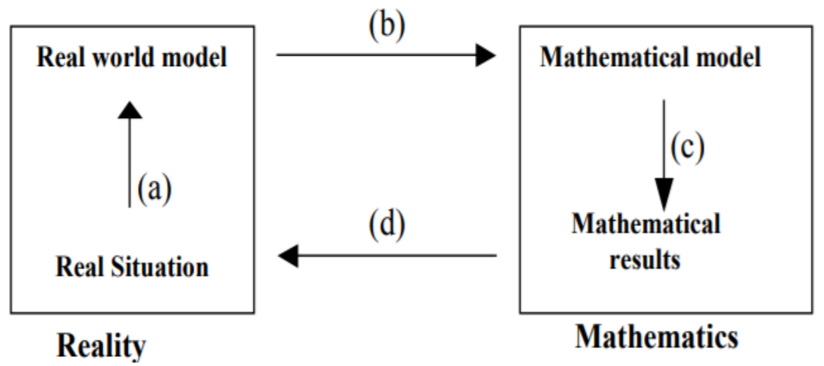
- Furthermore, in recent years there is an interest in mathematics teaching on promoting authentic workplace practices in classrooms .
- Researchers argue on this idea as follows:
  - authentic workplace practices are rich and meaningful and offer students' chances for inquiry activities (Williams & Wake, 2007);
  - engage students with challenging problem solving and modeling practices (Gravemeijer et al., 2017);
  - support students to understand the relevance of mathematics in everyday life, in our environment and for the sciences;
  - support students' development of mathematical reasoning skills (Dierdorp et al., 2011).

What makes a task an authentic task?

- Different terms have been used to label tasks that in some way emulate real life task situations such as authentic tasks, realistic tasks or real-life tasks and in addition many different meanings have been attached to each one of them (Palm, 2007).
- Niss (1992) defines authentic problems as tasks that are recognized by people working in this field *as being a problem they might meet in their daily work.*
- Other researchers realize authenticity as the case that the real situation is simulated with some *reasonable fidelity* in the school situation or that the object (a task/the event/the situation) is *a copy that honestly simulates reality* (Palm, 2007) while Vos (2011) realizes simulation as the opposite of authenticity.
  - Still the issue of distinguishing an authentic from a realistic task is open among the researchers.

The modeling process

- Authentic practices as contexts initiates adequate students' involvement for learning models and modeling.
- Several authors use diagrams to convey mathematical modeling as a process (e.g., Kaiser & Schwarz, 2006).

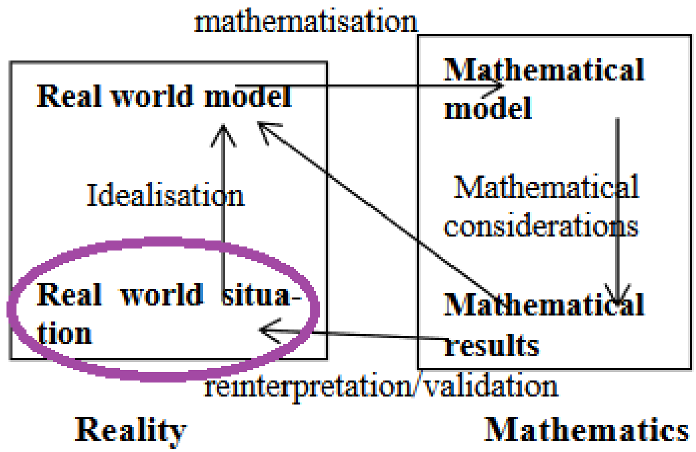


- (a) the real-world situation is **idealized** i.e. *simplified or structured in order to get a real-world model*.
- (b) the real-world model is **mathematized** *i.e. translated into mathematics*
- (c) Mathematical considerations during the mathematical model **produce** mathematical results.
- (d) The adequacy of the results must be checked, i.e. **validated**.

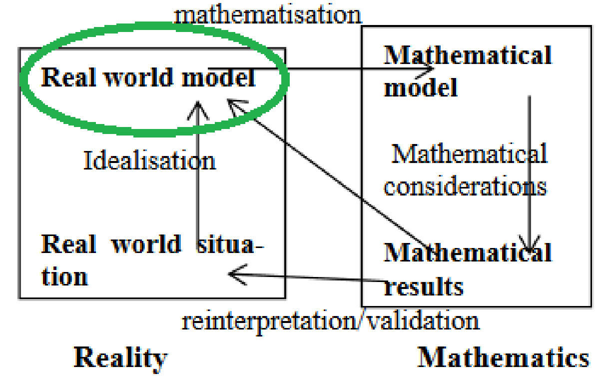
Realistic vs. authentic tasks

- We realize the distinction between authentic and realistic tasks as follows

Authentic task



Realistic task



Authentic tasks are the tasks where the **starting point is the real-world situation**

Realistic tasks are the tasks in which the starting point is the **real -world model** where the authentic situation is simplified/idealized so as to gain a real-world mode

Authentic tasks

- Real world events/phenomena
- **Artifacts use in classroom practice:** Original/authentic objects / phenomena
- **Students:** acting as professionals (e.g., make decisions or engaging in mathematical modeling activities and managing contextual rules/restrictions

High degree of fidelity

Realistic tasks

- **Simulation** of real-world events/phenomena
- **Artifacts use in classroom practice:** simulation of authentic objects or phenomena (e.g., by using digital tools)
- **Students:** acting as professionals (e.g., make decisions or engaging in mathematical modeling activities)

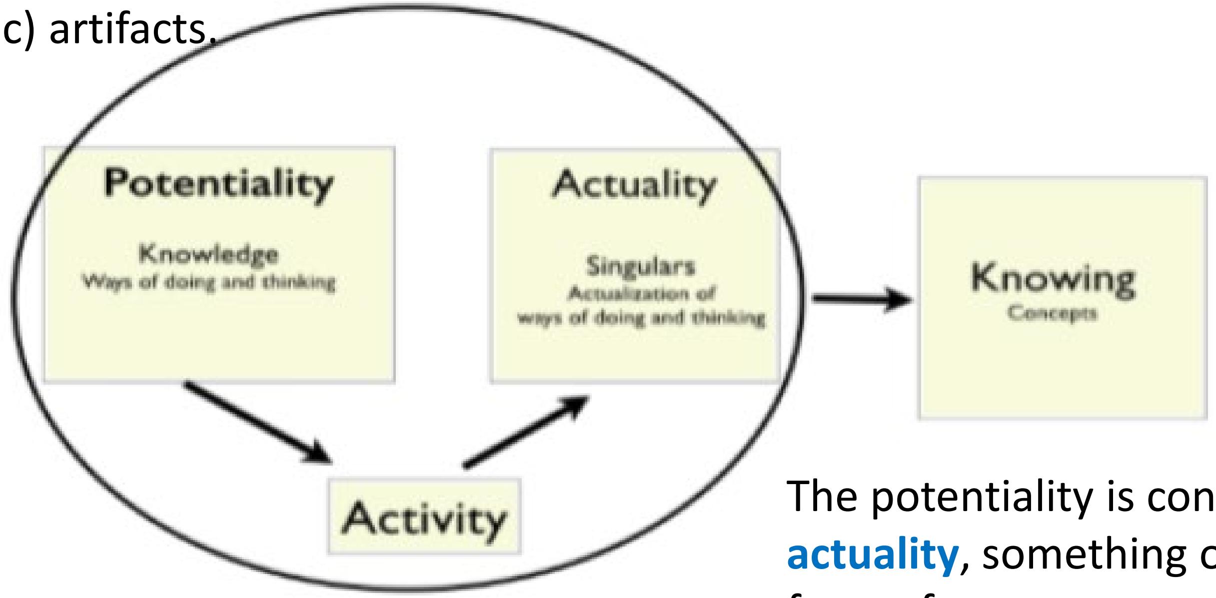
Reasonable degree of fidelity

Theoretical considerations

- Integrating workplace authentic tasks in mathematics teaching was conceptualized through the lens of the following theoretical perspectives
  - Activity Theory (Engeström, 1999) and the Potentiality – Actuality metaphor (Radford, 2015)
  - Boundary Crossing (Bakker & Akkerman, 2014).

The potentiality-actuality metaphor (Radford, 2015)

Potentiality is the definite capacity or the complete totality of possible interpretations of the mathematical objects encoded in the (authentic) artifacts.



The potentiality is contrasted to **actuality**, something occurring in front of us.

Moving from potentiality to actuality is related to subject's participation in a **collective, situated activity** (Engeström, 1999)

Boundary crossing perspective (Bakker & Akkerman, 2014)

- What happens at the boundaries between schools and workplace settings? (Bakker, 2014).
- The integration of workplace to mathematics teaching is rather demanding due to the existed gap between these two practices:
  - The conventional epistemological view of mathematics fails to capture mathematical concepts and skills encountered in the workplace (Triantafillou & Potari, 2010).

    e.g., If viewing the workplace context as non-mathematical might eliminate teachers' opportunities to explore its pedagogical potential.

- **PART B:** Designing and implementing authentic tasks in mathematics classrooms

- We present three school tasks that were designed in two European projects.
  - Karla Lake task (ENSITE): *Deciding on a Lake's restoration and analysis of relevant scientific representations*.
  - The Seismology task(MASCIL): *locating an earthquake epicenter*
  - the Gutter design task (MASCIL): *folding a metal plate in order to maximize the water flow.*

❑ Comparing the three task designs

❑ Presenting teachers' actions while enacted the Seismology task

❑ Presenting students' actions while working on the Gutter design task

https://mascil-project.ph-freiburg.de/
https://icse.eu/ensite/

Designing and implementing authentic tasks while participating in European projects

- **Ensite (*Environmental Socio-Scientific issues in Initial Teacher Education*):** aims to support prospective mathematics and science teachers to acquire teaching skills in order to support their future students at school in becoming responsible citizens.
- **Mascil (*Mathematics and Science for life*):** The project aims to support mathematics and science practicing teachers in using IBL and workplace situations in their teaching.
  - PD courses were designed to support teachers to make sense of the connections between workplace situations and classroom teaching
  - In the PD program mathematics teachers collaborated with teachers in other disciplines in order to co-design and co-teach authentic workplace tasks by using IBL approaches
    - Cycles of designing, implementing and analysing lessons in the spirit of lesson study approaches

- Comparing the three task designs

The Lake Karla task

The story of Karla Lake

Karla lake is located in the central part of Greece.

In the beginning of the 20th century the lake had a rich biodiversity.

The Lake was drained in the early 1960s.

The Lake has been re-flooded in recent years.

Resource 1 – Yearly water balance of Karla Lake

Resource 2 –The Lake Karla's water level

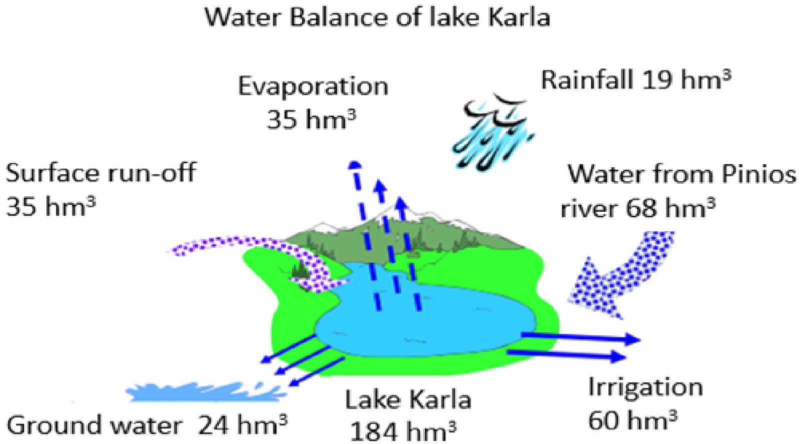


Fig. 1 *The figure shows the yearly average values of some key functional characteristics of Karla Lake.*

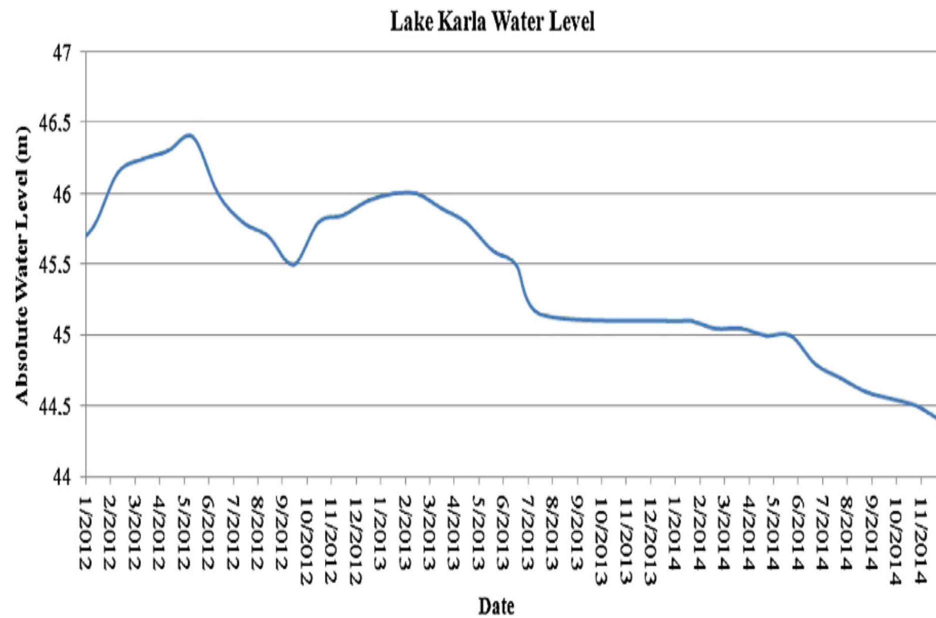


Fig. 2*: The graph shows Lake Karla's water level in the years 2012-2014*

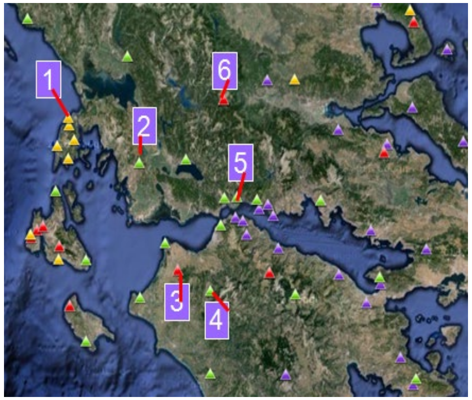
Task

– **Reflect on the advantages and disadvantages involved in two core decisions related to the drainage and restoration of Karla Lake**.

– **Identify ways to increase Lake Karlas' water level.**

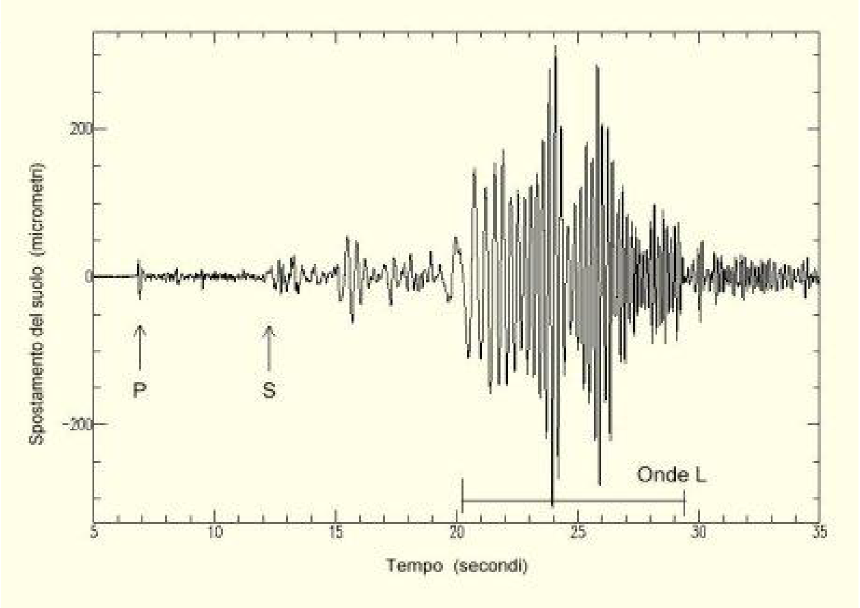
The Seismology task

- “You are seismologists in the Geodynamics Institute. You are going to use authentic data about an earthquake that really happened so as **to locate its epicenter.**
  - Date: Nov., 5, 2014; Time 14: 22: 24(GTM),



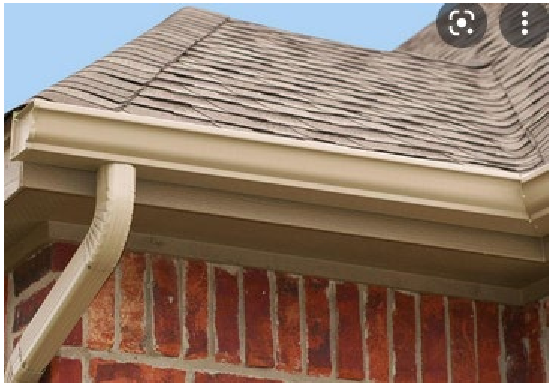
The geographical map indicates 6 seismic stations

Data source: The National Geodynamics Institute.



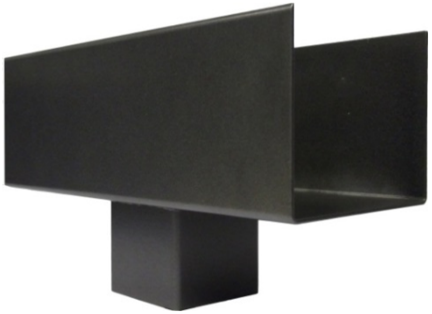
The seismogram indicates and records the s (secondary) and p (primary) seismic waves

The Gutter task



Gutters are metal plates that collect rainwater and let it go through pipes to the ground.

Gutter designers are aiming to fold the metal sheet in order to maximize water flow.



The task

You work as a designer in a company, and you provide assistance related to the construction of gutters.

Your company has undertaken a project to propose the way of bending some metal plates, so as to create a gutter that holds the maximum quantity of water.

The task assigned to you is to **indicate the best way of folding a rectangular metal plate in order to construct a gutter in which the quantity of water is maximized**.

Explore the task in Casyopée (digital environment)

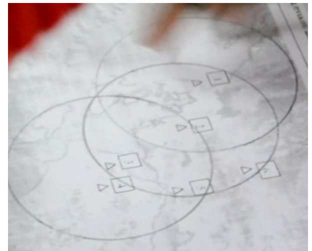
Task design dimensions

The Lake Karla task

- Real life phenomenon: Drainage and restoration of a Lake.
- **Artifacts use in classroom practice:** Scientific representations published in relevant papers
- **Students:** interpreting scientific resources; and commenting on professionals' decision making;
- Mathematical models: **Provided**

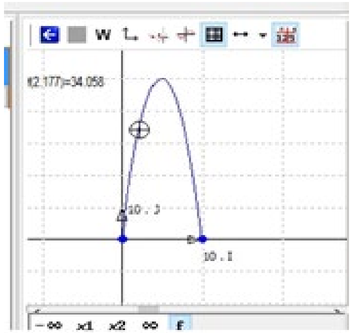
The seismology task

- Real life phenomenon: Earthquakes
- **Artifacts use in classroom practice:** geographical map; seismograms; authentic data from the National Geodynamics Institute.
- **Students:** acting as a seismologists; interpret scientific models in order to locate the epicenter;
- Mathematical models: **required**



The Gutter task

- Real-life phenomenon: the gutter design
- **Artifacts use in classroom practice:** digital tools (Casyopée) to explore the manipulation of covarying quantities.
- **Students:** acting as Gutter designers; finding the optimal design for gutter cross-sectional area
- Mathematical models: **required** (L+ 2l= C (C the side of the sheet of metal); the optimal shape is for L=2l and l= C/4, L=C/2).



- Presenting teachers’ actions while enacted the Seismology task

Operationalizing the theoretical constructs in enacting lessons on authentic tasks

Potentiality	Knowledge of Mathematics, Science and workplace contexts (earthquakes and seismography - Concepts and methods), pedagogical content knowledge related to inquiry based learning and teaching using workplace/authentic tasks.
Activity	The design and the enactment of inquiry based lessons based on workplace/authentic tasks.
Actuality	The way that the design and enactment were operationalized by each teacher in the Mascil context.
Knowing	Professional learning in designing and enacting inquiry based lessons integrating workplace/authentic tasks

Research questions

- (1) How does the potentiality to link the workplace situation to inquiry-based mathematics teaching develop in PD meetings?
- (2) In what ways the same authentic task is actualized by different mathematics teachers as they design and enact inquiry-based mathematics teaching?

Focusing on two teachers' actualizations

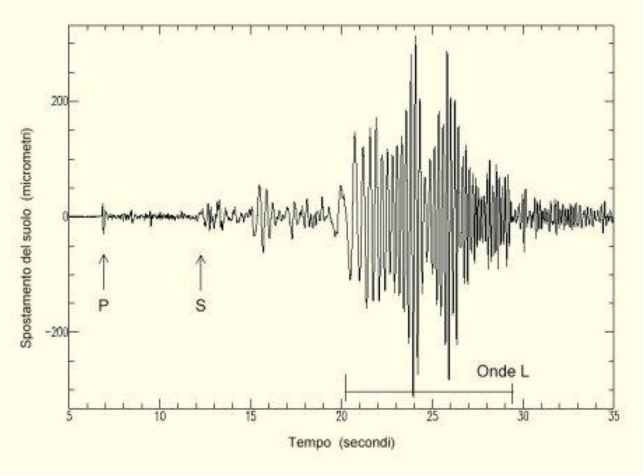
- Teaching actions goals and tools, boundaries between workplace and mathematics practices, boundary actions were identified in the main phases of the lesson
- Comparing across the teachers –looking for ways of enactment

Teacher A: Introducing the workplace and scientific context

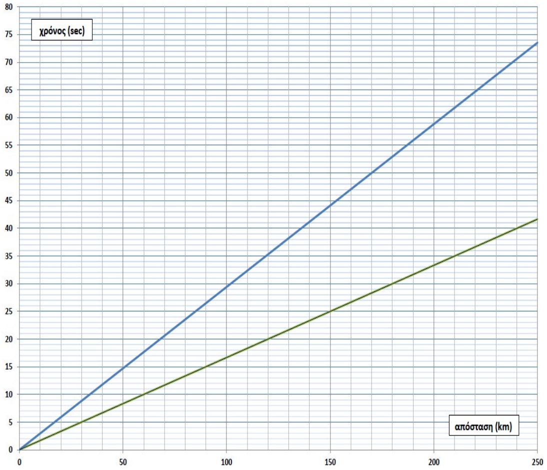
- ***ACTIONS –GOALS***
- ***Exposition/Providing Information*** – use of different tools and resources to orient students toward the phenomenon (a PPT file, videos, authentic representations (seismograph and seismogram)
- ***Teacher questioning*** (prompting students to observe and explain representations and to make links with prior experiences)
- ***Building on students' ideas*** (revoicing and confirming students' responses; synthesizing students' ideas addressing critical aspects of the phenomenon and the underlying scientific concepts)

Teacher A: Developing a mathematical model -Working with the seismogram to identify the distance of the epicenter (workplace tool)

- The teacher asks the students to interpret the seismogram and find relevant data.
- ***Teacher questioning*** (Prompting students to express and justify their claims and clarify their ideas; focused questioning)
  - The students find difficult to recognize the time difference between the S and P waves (the key idea for calculating the distance of the epicenter from the seismic station)
  - T: “Can you conclude from the diagram what time the earthquake happened? How far?”
- ***Building on students’ ideas*** (revoicing students’ explanations, synthesizing)
- ***Exposition/ Providing information***
- ***Establishing the workplace context***
  - T: “We are in a research institute, we are doing our internship. The scientists work there and I do not think that they just guess. They follow some methods.”
- The students start relating the time, the speed and the distance but not correctly.



Vp = 6 km/s and Vs = 3,4 km/s



Teacher A: Developing a mathematical model - Working with the distance-time graph (mathematical/scientific tool)

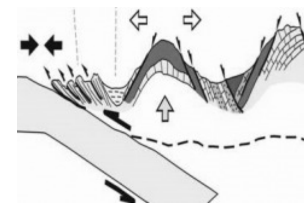
- ***Teacher questioning*** (asking students to interpret the graph by making links to their prior knowledge; Focused questioning to point out to key mathematical ideas (proportionality, slope of the graphs).
  - The students respond correctly to the teacher's questions and identify features of the graph
- ***Exposition/Demonstrating the method*** of finding a distance for a given time difference by placing the ruler on the graph. The students seem to be confused about the method
- ***Establishing the workplace context*** – Role playing, providing information about the phenomenon. Pointing students' attention to specific characteristics (the difference in time) of the graph relating it with the workplace situation and tools (the seismogram). (***boundary action***)
  - Students cannot use the method and prefer to use their own inaccurate methods.

Teacher A: Developing a mathematical model - Working with the intersection of circles to identify the epicenter

- A student identifies two possible positions of the epicenter
  - *T: From which station you have the data? Where you are?*
  - *S2: It can be here and there (showing different places of the epicenter)*
  - *T: Ah! So, it is not one place. How many are they?*
  - *S1: Two.*
  - *T: Are they two?*
  - *S2: It can be also perpendicular. We do not know.*
  - *T: How many places can work as epicenters?*
  - *S2: Infinite. They can be in a circle*
  - *T: Ok. Take the compass and draw it.*
- The discussion continues and the teacher challenges students to understand that one circle is not enough. He points students' attention to the data they have from different stations (***establishing workplace context***) and asks them to calculate the place of the epicenter from this information and draw the three circles. Some students cannot make sense. The teacher rephrases students' ideas and asks for further explanations (***building on students' ideas, teacher questioning***).

Teacher B: Introducing the workplace and scientific context

- ***Exposition: Providing information*** about aspects of the phenomenon of earthquakes
  - Internal forces that shape the earth's surface;
  - Causes of the natural phenomenon
    - *T: "what are earthquakes and what causes them to happen"*
  - The nature of the different seismic waves: **p** (primary) & **s** (secondary) and their role in locating the epicenter.



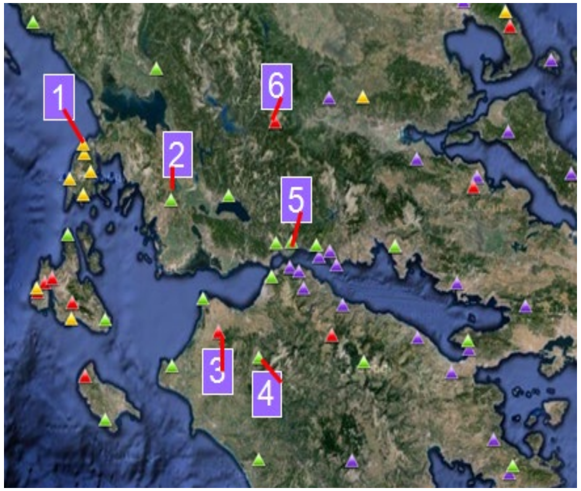
- ***Teacher questioning***: Relating the seismic waves with students' personal experiences
  - *T: "What do you feel when an earthquake is happening*?"
- ***Exposition: Providing relevant scientific representations*** Monitoring the seismic activity in Greece in real time

Teacher B: Developing a mathematical model: providing formulas and data

- ***Exposition: Providing information and authentic data***
- the velocities of p (V_P) and s (V_S) seismic waves; (V_p = 6 km/s; V_s = 3,4 km/s)
- the mathematical formula

  $D = \Delta t \cdot (V_P \cdot V_S) / (V_P - V_S)$

  which indicates the distance D in Km between a specific seismic station and the epicenter in relation to the velocities of the seismic waves and the different time they arrive in the station;
- a geographical map indicating 6 seismic stations in the central and west part of Greece;
- the specific measures recorded in the seismographs of the seismic stations during the specific earthquake



| Frequency band used in inversion (Hz)

| 0.04 - 0.05 -- 0.08 - 0.09

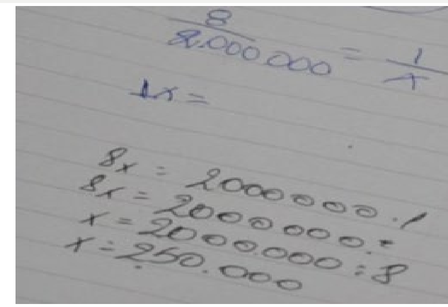
| Stations-Components Used-Distance

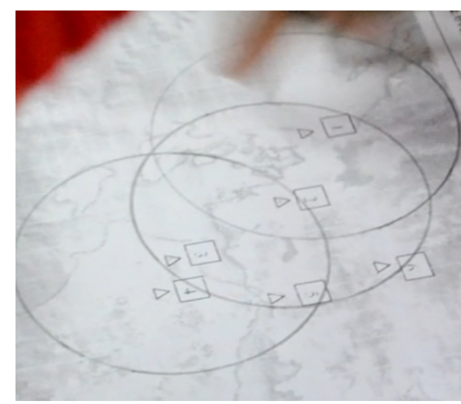
Station	NS	EW	Z	D(km)
LKD2	-	+	+	78
PDO	+	+	+	82
RLS	+	-	+	86
DRO	+	-	+	109
EFP	+	+	+	129
EVR	+	+	+	147

1.LKD2 Λευκάδα
2.PDO Πρόδρομος
3.RLS Ρίολος, Κάτω Αχαΐα
4..DRO Δροσιά
5.EFP Ευπάλιο, Ν. Φωκίδας
6.EVR Ευρυτανία

Teacher B: Developing a mathematical model: simulating the workplace action

- ***Establishing context***. Attributing students the role of the seismologist
- ***Guiding students to use specific mathematical methods***
  - Use the data from a particular seismic centre and the formula $D = \Delta t \, . \, (VP \, . \, VS) / (VP - VS)$ to calculate the distance D in Km of the epicenter from the seismic station
  - Use the map scale to calculate the distance on the map
  - Draw three circles on the map using the data from 3 seismic stations to identify the epicenter





Comparing the two cases

- Different ways of handling the modeling process in the classroom
  - Level of inquiry (guided vs. exploratory)
  - The tools and representations used in developing the mathematical model (formulas vs. seismogram)

What do we learn about enacting authentic tasks in the classroom?

- Boundary actions are important to be established to support the modeling process.
- In case of students' difficulty to make sense of the task the teacher establishes the context
- Establishing context, focused questioning, synthesizing students' ideas seem to facilitate the identification of key mathematical ideas
- Links between workplace representations and mathematical ones (seismogram and time-distance graph) are not transparent to the students.

- Presenting students' actions while working on the Gutter design task

The Gutter task

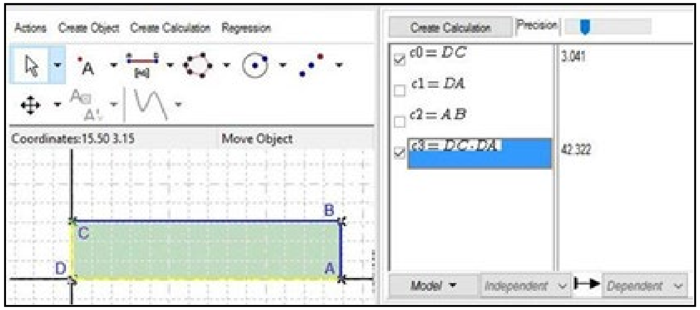
- Study aim: investigate students' evolving conceptualization of function as covariation at the upper secondary level
- Research focus
  - ways that the students treat the covarying quantities in the different models involved in the complex path from physical context to algebra
    - a function exists first as a dependency between physical objects, then between geometrical objects, then between quantities and finally, as a mathematical function
  - attention to the connections between these models

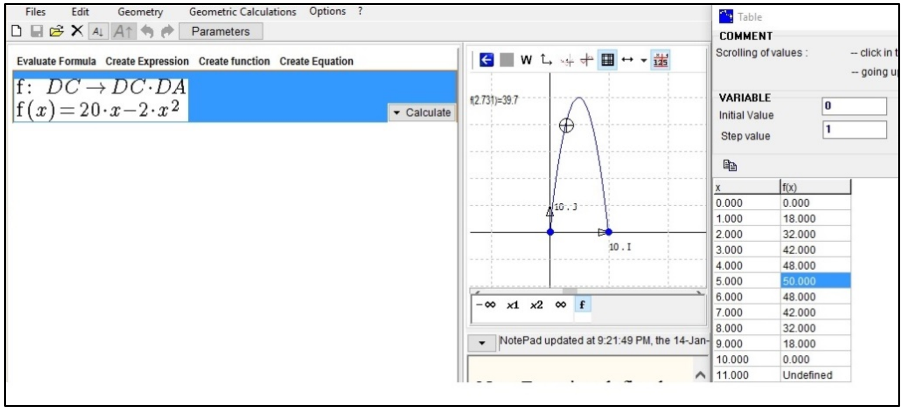
(Psycharis et al., 2021)

The digital environment

Casyopée (https://casyopee.math.univ-paris-diderot.fr/?lng=en)

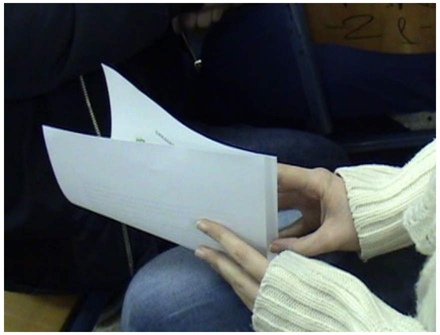
- Dynamic Geometry (DG) window and a symbolic window with registers: tabular, graphic and symbolic
- Students can work together on a geometric model as well as on the final algebraic model
  - observe covarying quantities,
  - define independent and dependent quantities,
  - check if two covarying quantities are in functional dependency,
  - work with interconnected representations of function.





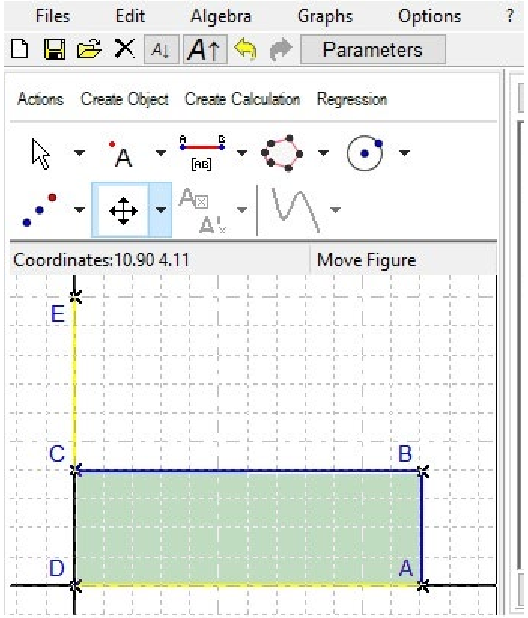
Models involved in the modeling process (1)

- ***Paper model***
  - stays close to reality
  - a rectangular sheet of paper can be folded in order to make a rectangular cross-section
  - allows students to appreciate sensually how the choice of the folds influences water circulation
  - covariation is between concrete entities (i.e. folds, flux), but the variations are difficult to appreciate because of the poor dynamicity of the model



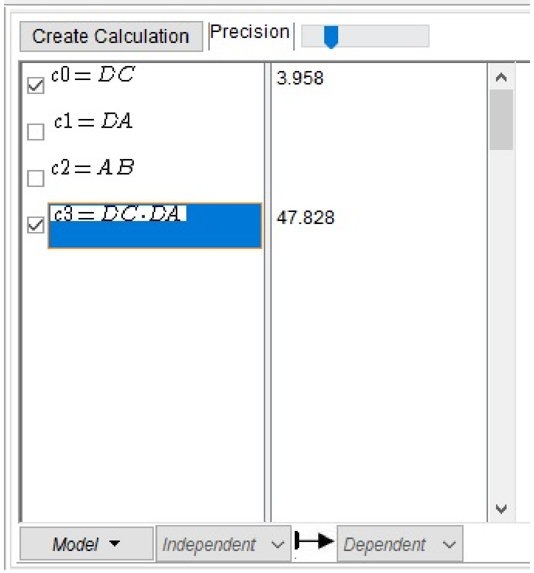
Models involved in the modeling process (2)

- ***Dynamic Geometry model***
  - adds dynamicity and interactivity
  - helps students become aware of variations
  - a single free point has to be created that allows constructing the other "dependent" points of the rectangle
    - constraints have to be set on this point and on the other points in order to reflect the real cross-section's constraints
    - while moving the free point, one can observe the variations of the rectangle



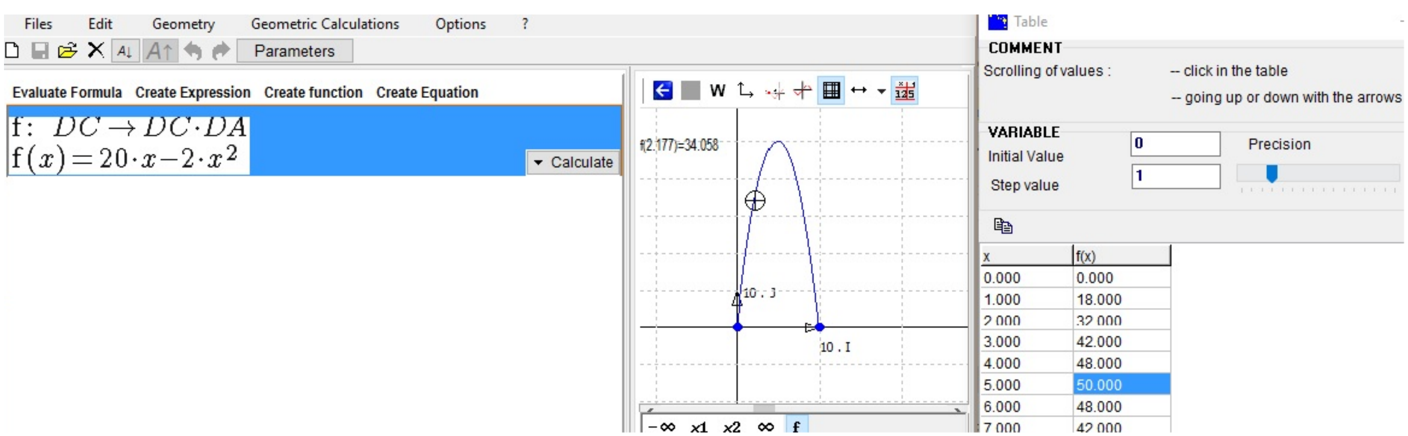
Models involved in the modeling process (3)

- ***Measures model***
  - a result of the quantification process as the covariation appears between quantities' measures rather than geometrical entities
  - use of automatic modelling functionality of the software



Models involved in the modeling process (4)

- ***Functions model***
  - function is exported as a result of a selected pair of two covarying quantities (e.g., a length of a rectangle and an area)
    - not an obvious step for students as it requires a robust understanding of the situation and a strong mathematical background (i.e., selection of independent/dependent variables)
  - the students can work with different function representations



Context

Implementation

- One class of twenty 11th grade students (groups of two or three)
- 14 teaching sessions (45 minutes each) over 2 months
- At the time of the study, the students had been taught about function as correspondence, monotonicity and extreme points

The task given to students

(1) experiment with folding a piece of paper (10cmX20cm) to explore and notice dependencies

(2) construct a dynamic DG figure that models the situation in Casyopée and explore

(3) use the software to propose quantities in functional dependency

(4) obtain a mathematical function modelling the dependency and use algebraic techniques

Identifying dependencies (paper model)

Episode 1

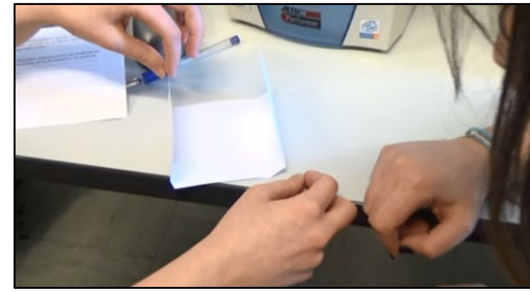
[*S3 folds a sheet of paper with a very small vertical edge*]

79 S3: What about if I fold the metal sheet like this? I want to have the maximum water volume.

80 T: Can you explain your idea?

81 S3: Actually, we need to concentrate on the changes. As the length grows the height diminishes and the area changes. So we have to find out a proportion for which the height and length will be exactly what we need.

82 S1: Basically, the maximum product.



- S3's experimentation through folding

- explains her idea of the relationship between the bottom and the vertical edge and their influence on the cross sectional area

-constructs her understanding of covariation

- S1 makes sense of the need for a 'maximum product' as a criterion for maximizing the water flow (line 82).

Modeling in dynamic geometry (DG model)

Episode 2

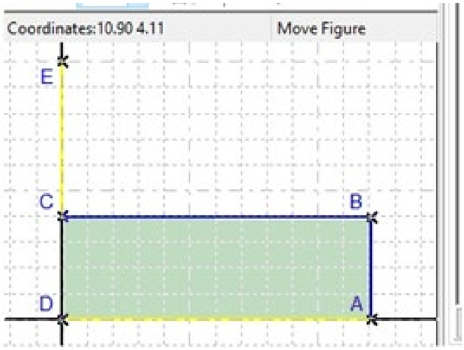
67 T: You need one point for the lower part of the gutter [*i.e. D*] and one point which describes the maximum folding [*i.e. E*].

68 S4: Then we need another point [*i.e. free point C*] between these two points to describe every time the different folding.

69 S3: I propose to put point D in (0;0).

70 S4: We have to create a point E as (0;10) in case we fold the metal plate in the middle so that we get a segment for positioning the free point C.

After creating the point C, the students observed the folding in order to find an expression for the x-coordinate of A. Most of the groups attempted to find it through solving the equation x+x+y=20 for y. Students from different groups commented: "I tried to create A, as (20-2*x ; 0) but it did not work!", "We created A as (20-2*yC;0) and it worked!", "As for us, we created A as (20-2*DC;0) and it worked also!".



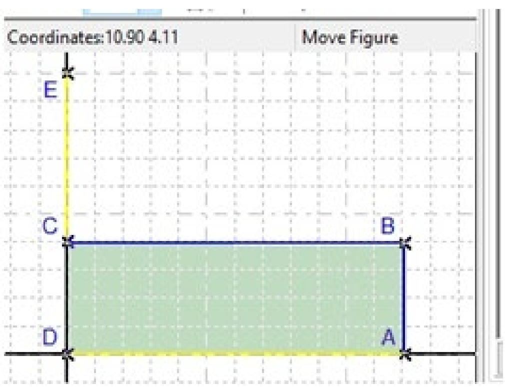
- Need to distinguish the point that 'causes' the dynamic change of the construction
- Faulty and successful attempts to relate the coordinates of point A to measures dependent on point C
- Progressive coordination of preceding sensual manipulation of the paper sheet with the notation structures of DG

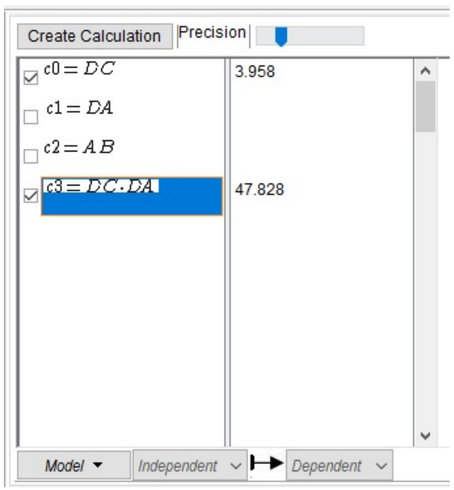
Quantification of variations (Measures model)

Episode 3 (part A)

11 S5: Look at the area here [*i.e. to the geometrical calculation DC x DA in the tab*]. We see that the maximum area is 50 and as we change this value... [of DC] … Okay. We cannot say that it is the maximum. If we change the point C in this straight line [*segment DE*] the area continuously decreases and maximizes when it [*DC*] gets its maximum value.

12 S6: Look here, it says 50 and we have the maximum value of segment DC. While we move down point C, we see that the area is decreasing too.





- By moving the point C in Dynamic Geometry, S5
  - observes continuously changes in the numerical values of the relevant measures (Geometric Calculation tab) and
  - links the two covarying magnitudes (i.e. DC, DC x DA) to find a solution to the problem (line 11)
- S6 makes sense of the direction of the change of these magnitudes stating that the area decreases as the length of DC decreases too (line 12)

Distinction between independent-dependent variables (Measures model)

Episode 3 (part B)

[The students select two measures (DC*DA, DC) as a pair of independent – dependent variables]

180 S6: It [*automatic modelling in Casyopée*] cannot calculate a function.

81 T: Why? What do you think about it?

82 S5: It [*DC*DA*] is dependent.

83 T: So, what?

84 S5: I will put here the area [*as dependent variable*] and here [*as independent one*] something that is independent of the rest of the others. That is, DC.

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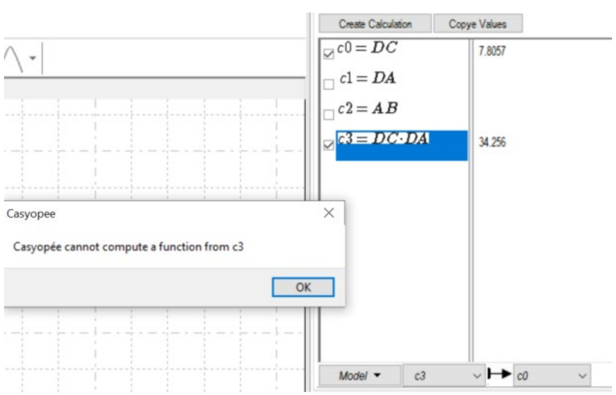


- The provided feedback mediates students' correct selection of the pair independent-dependent variables (line 82).
- DC as a coherent element 'generating' the change of the area of the rectangle ABCD in a rather primitive sense ("independent of the rest of the others") (line 84).

Expressing covariation through the use of variables (Functions Model)

Episode 4

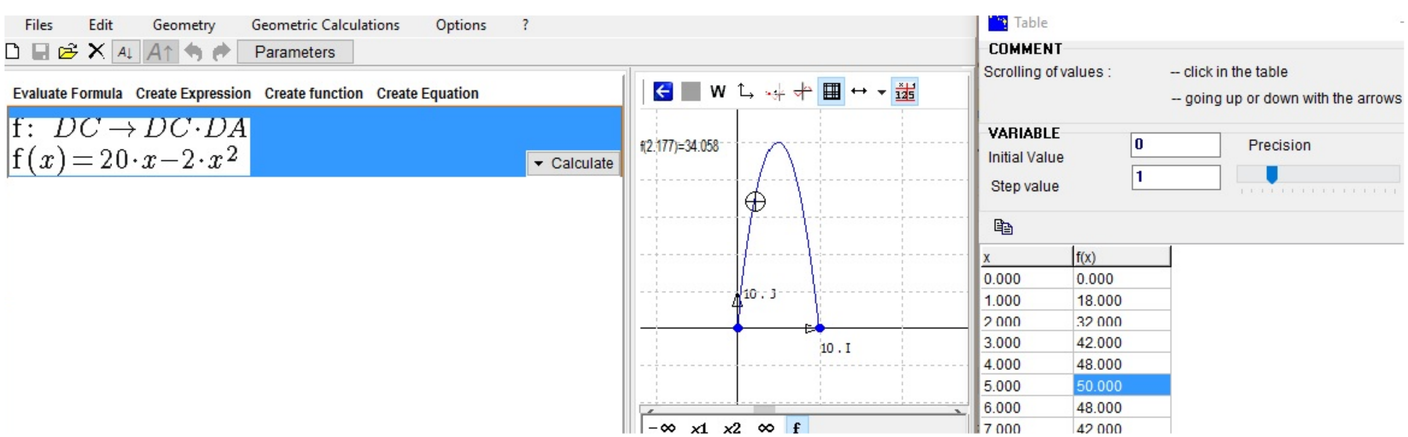
41 S1: From the table we see the maximum value at 5…

42 S2: It shows the area for each value that x takes with the restrictions we set.

43 S1: If we change the step it shows us the area in relation to the side DC that changes by 0.5. We see that 5 remains the value of the side DC so as to have the maximum area. We notice that for the different values of x the area changes and reaches its maximum in DC [equal to 5]

44 S3: Wait. For the various values of x, the area changes and finds a maximum for x = 5 with the area equal to f(5) = 50.

- Work with variables
- S2 helps S1 to conceptualize the variation of DC as the variation of the independent variable x (line 43).
- S3 conceptualizes function as covariation by relating the changes in the two columns of the table in terms of independent and dependent variables (line 44).



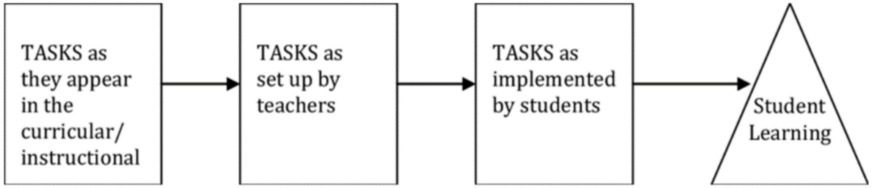
Connections between models

- Progressive character of the connections students develop while working in the different models
  - DG model: building on their experience of folding the paper model the students recognize key functionalities of the DG system in order to model a variable rectangle
  - Measures model: associating a moving point in the DG figure with variations of measures ("If we change the point C…")
  - Functions model: linking the measure model and the function model, recognizing mathematical variables as representing quantities, making sense of the variations of the function as representing covariation of measures.
- Wider connections: considering a table of the function (function model) together with notions that exist in other models (area, side…):

  > "We see in the table that the area is maximized when the coordinate of the free point C [i.e. yC] is 5. That is, we have the maximum area when one side [of the gutter] is half of the other."

- **PART C:** Conclusions

Transforming authentic tasks in the classrooms



Mathematics and Science Tasks Framework (Stein & Smith, 1998)

- Addressing the demands of task design
  - being aware of the balance between authenticity and realistic elements
  - considering the complexity of introducing the authentic context in the class
  - identifying multiple models involved in the modeling process (e.g., manipulatives, digital environment)
- Managing task enactment
  - addressing students' difficulty to make sense of the task
  - establishing context through boundary actions
  - linking authentic and mathematical representations (e.g., transparency)
  - facilitating the identification of key mathematical ideas (e.g., questioning, synthesizing)
- Addressing students' learning outcomes
  - issues in developing/making sense of mathematical models based on authentic or realistic situations
  - the nature of students' mathematics in authentic tasks and realistic ones
  - the mediating role of specially designed digital tools

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Thank you!